Reasoning about Iterators with Separation Logic

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Overview

- Multiple iterators traversing a collection in parallel
- Safe changes to the collection (e.g. caching) OK; only *logical* state needs to be immutable
- Can separately check client and implementation for conformance to abstract interface

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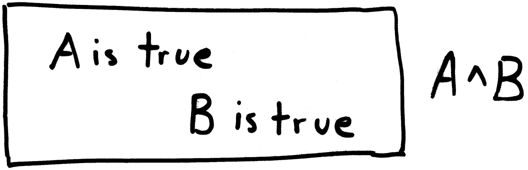
- Can separately check client and implementation for conformance to abstract interface
- Specification language developed in collaboration with John Reynolds, Jonathan Aldrich, and Lars Birkedal

Conjunction, Regular and Separating



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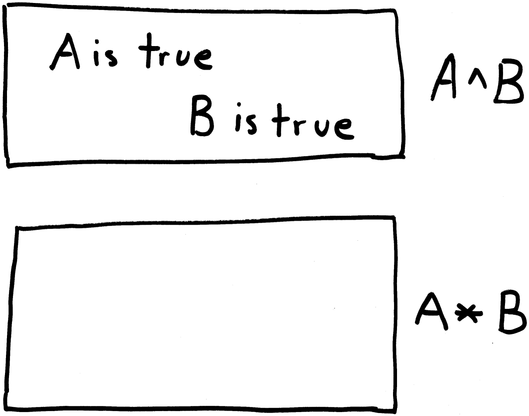

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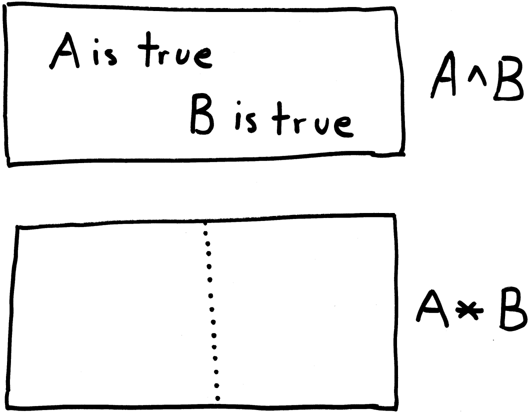

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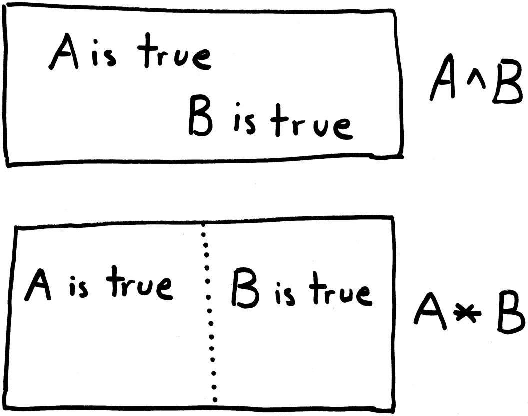

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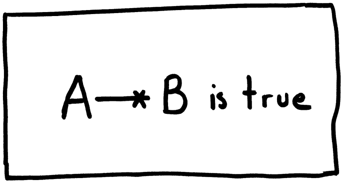

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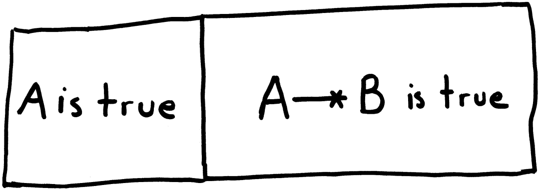

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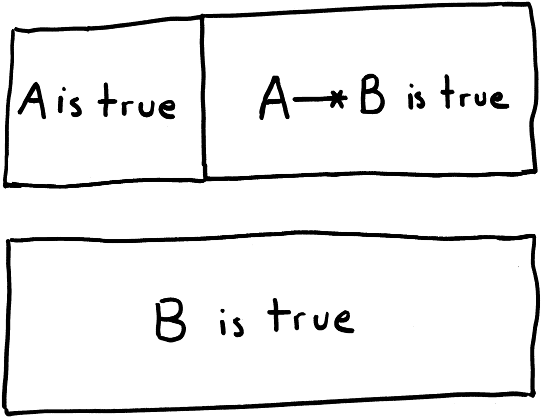

The Iterator Protocol, In Separation Logic

$$\exists coll : (\tau_c \times \text{seq} \;\; \times \text{prop}) \Rightarrow \text{prop}.$$

$\{\top\}$ `new_coll()` $\{a : \tau_c.\ \exists P.\ coll(a, [], P)\}$ and

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$\forall P, c, xs.\ \{coll(c, xs, P)\}$
$\quad \texttt{empty}(c)$
$\quad \{a : \text{bool}.\ coll(c, xs, P)\}$ and

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$\forall P, c, xs.\ \{coll(c, xs, P)\}$
$\texttt{empty}(c)$
$\{a : \text{bool}.\ coll(c, xs, P)\}$ and

$\forall P, c, x, xs.\ \{coll(c, xs, P)\}$
$\texttt{add}(c, x)$
$\{a : 1.\ \exists P'.\ coll(c, x :: xs, P')\}$ and

The Iterator Protocol, In Separation Logic

$$\exists iter : (\tau_i \times \tau_c \times \text{seq} \quad \times \text{prop}) \Rightarrow \text{prop}.$$

$$\begin{array}{l} \forall c, xs, P.\ \{coll(c, xs, P)\} \\ \qquad\qquad \texttt{new\_iter}(c) \\ \qquad\qquad \{a : \tau_i.\ iter(a, c, xs, P)\} \ \text{ and} \end{array}$$

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$$\texttt{new\_iter}(c)$$
$$\{a : \tau_i.\ iter(a, c, xs, P)\} \text{ and}$$

$$\forall i, c, xs, P.\ \{iter(i, c, xs, P)\}$$
$$\texttt{next}(i)$$
$$\{a : 1 + \text{nat}.\ iter(i, c, xs, P)\} \text{ and}$$

The Iterator Protocol, In Separation Logic

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$$\begin{aligned} \forall i, c, xs, P.\ & \{iter(i, c, xs, P)\} \\ & \texttt{next}(i) \\ & \{a : 1 + \text{nat}.\ iter(i, c, xs, P)\} \text{ and} \end{aligned}$$

$$\begin{aligned} \forall i, c, xs, P.\ \{iter(i, c, xs, P) \supset\ & coll(c, xs, P) * \\ & coll(c, xs, P) \mathrel{-\!\!*} iter(i, c, xs, P)\} \end{aligned}$$

A Client Program

$$\{coll(c, xs, P)\}$$

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let $b = \texttt{empty}(c)$;

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$\{coll(c, xs, P)\}$
$\text{let } b = \texttt{empty}(c);$
$\{coll(c, xs)\}$
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$\{iter(i_1, c, xs, P)\}$

A Client Program

$\{coll(c, xs, P)\}$
let b = `empty`(c);
$\{coll(c, xs)\}$
let i_1 = `new_iter`(c);
$\{iter(i_1, c, xs, P)\}$
$\{(coll(c, xs, P) * (coll(c, xs, P) -\!\!* \; iter(i_1, c, xs, P))\}$

A Client Program

$\{coll(c, xs, P)\}$

$\text{let } b = \texttt{empty}(c);$

$\{coll(c, xs)\}$

$\text{let } i_1 = \texttt{new\_iter}(c);$

$\{iter(i_1, c, xs, P)\}$

$\{(coll(c, xs, P) * (coll(c, xs, P) -\!\!* \; iter(i_1, c, xs, P))\}$

$\text{let } i_2 = \texttt{new\_iter}(c);$

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$$\{(coll(c, xs, P) * (coll(c, xs, P) \mathrel{-\!\!*} iter(i_1, c, xs, P))\}$$
$$\text{let } i_2 = \texttt{new\_iter}(c);$$
$$\{iter(i_2, c, xs, P) * (coll(c, xs, P) \mathrel{-\!\!*} iter(i_1, c, xs, P))\}$$
$$\{coll(c, xs, P) * \\ (coll(c, xs, P) \mathrel{-\!\!*} iter(i_1, c, xs, P)) * \\ (coll(c, xs, P) \mathrel{-\!\!*} iter(i_2, c, xs, P))\}$$

A Client Program

1 $\{coll(c, xs, P)\}$

2 $\texttt{let } b = \texttt{empty}(c);$

3 $\{coll(c, xs)\}$

4 $\texttt{let } i_1 = \texttt{new\_iter}(c);$

5 $\{iter(i_1, c, xs, P)\}$

6 $\{(coll(c, xs, P) * (coll(c, xs, P) \mathrel{-\!\!*} iter(i_1, c, xs, P))\}$

7 $\texttt{let } i_2 = \texttt{new\_iter}(c);$

8 $\{iter(i_2, c, xs, P) * (coll(c, xs, P) \mathrel{-\!\!*} iter(i_1, c, xs, P))\}$

9 $\{coll(c, xs, P) *$
$(coll(c, xs, P) \mathrel{-\!\!*} iter(i_1, c, xs, P)) *$
$(coll(c, xs, P) \mathrel{-\!\!*} iter(i_2, c, xs, P))\}$

10 $\texttt{let } b' = \texttt{empty}(c);$

A Client Program

1 $\{coll(c, xs, P)\}$

2 $\texttt{let } b = \texttt{empty}(c);$

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5 $\{iter(i_1, c, xs, P)\}$

6 $\{(coll(c, xs, P) * (coll(c, xs, P) -\!\!* \; iter(i_1, c, xs, P))\}$

7 $\texttt{let } i_2 = \texttt{new\_iter}(c);$

8 $\{iter(i_2, c, xs, P) * (coll(c, xs, P) -\!\!* \; iter(i_1, c, xs, P))\}$

9 $\{coll(c, xs, P)*$
$(coll(c, xs, P) -\!\!* \; iter(i_1, c, xs, P))*$
$(coll(c, xs, P) -\!\!* \; iter(i_2, c, xs, P))\}$

10 $\texttt{let } b' = \texttt{empty}(c);$

11 $\{coll(c, xs, P)*$
$(coll(c, xs, P) -\!\!* \; iter(i_1, c, xs, P))*$
$(coll(c, xs, P) -\!\!* \; iter(i_2, c, xs, P))\}$

A Client Program, Continued

$$
\begin{aligned}
11\ \{&coll(c, xs, P) *\\
&(coll(c, xs, P) \mathrel{-\!\!*} iter(i_1, c, xs, P)) *\\
&(coll(c, xs, P) \mathrel{-\!\!*} iter(i_2, c, xs, P))\}
\end{aligned}
$$

A Client Program, Continued

11 $\{coll(c, xs, P) *$
$(coll(c, xs, P) -\!\!* \; iter(i_1, c, xs, P)) *$
$(coll(c, xs, P) -\!\!* \; iter(i_2, c, xs, P))\}$

12 $\{iter(i_1, c, xs, P)) *$
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13 let $v = \texttt{next}(i_1)$;

A Client Program, Continued

11 $\{coll(c, xs, P) *$
$(coll(c, xs, P) -\!\!* \; iter(i_1, c, xs, P)) *$
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11 $\{coll(c, xs, P) *$
$(coll(c, xs, P) \multimap iter(i_1, c, xs, P)) *$
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A Client Program, Continued

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$(coll(c, xs, P) \mathrel{-\!\!*} iter(i_2, c, xs, P))\}$

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A Client Program, Continued

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16 $\{iter(i_2, c, xs, P)) *$
$(coll(c, xs, P) \mathrel{-\!\!*} iter(i_1, c, xs, P))\}$

17 let $v = \mathtt{next}(i_2)$;

A Client Program, Continued

11 $\{coll(c, xs, P) *$
$(coll(c, xs, P) -\!\!* \; iter(i_1, c, xs, P)) *$
$(coll(c, xs, P) -\!\!* \; iter(i_2, c, xs, P))\}$

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A Client Program, Continued Again

$$18\ \{iter(i_2, c, xs, P) *$$
$$(coll(c, xs, P) -\!* \ iter(i_1, c, xs, P))\}$$

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A Client Program, Continued Again

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20 $\mathtt{add}(c, x)$

A Client Program, Continued Again

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$(coll(c, xs, P) \mathrel{-\!\!*} iter(i_2, c, xs, P))\}$

20 $\texttt{add}(c, x)$

21 $\{\exists Q.\ coll(c, xs, Q) *$
$(coll(c, xs, P) \mathrel{-\!\!*} iter(i_1, c, xs, P)) *$
$(coll(c, xs, P) \mathrel{-\!\!*} iter(i_2, c, xs, P))\}$

Questions?

Any questions?

Implementing the Module – Invariants

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$$\begin{array}{ll} linked\_list(c, x :: xs) & \equiv \exists c'.\ c \hookrightarrow \text{cons}(x, c') * linked\_list(c', xs) \\ linked\_list(c, []) & \equiv c \hookrightarrow \text{nil} \end{array}$$

Iterator Invariants

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$$\begin{aligned} iter(i, c, xs, P) \equiv \exists l, n, xs_1, xs_2.& \\ & (P \wedge (seg(\mathsf{fst}\ c, l, xs_1) \ * coll(l, xs_2))) * \\ & i \hookrightarrow l \ * \mathsf{snd}\ c \hookrightarrow n \wedge \\ & xs = xs_1 \cdot xs_2 \end{aligned}$$

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& i \hookrightarrow l \; * \text{snd } c \hookrightarrow n \; \wedge \\
& xs = xs_1 \cdot xs_2 \\
seg(l, l', x :: xs) \equiv \exists l''. \; l \hookrightarrow \text{cons}(x, l'') * seg(l'', xs) & \\
seg(l, l', []) \qquad \equiv l = l' &
\end{aligned}$$