Faithful mapping of model classes to mathematical structures

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Abstraction in OO specification languages

- Abstraction is indispensable
  - to specify types with no implementation
  - to support subtyping and information hiding
- Two-tiered specification languages (e.g. Larch) directly provide mathematical structures for abstraction
- One-tiered specification languages (e.g. JML) provide *model classes* for abstraction

Model classes

- Provide OO interface for mathematical concepts
- Used as immutable types
- Equipped with contracts (not shown)

```
package org.jmlspecs.models;

public final /*@ pure @*/ class JMLObjectSet {
   public JMLObjectSet();
   public JMLObjectSet(Object e);

   public boolean has(Object elem);
   public boolean isEmpty();
   public boolean isSubset(JMLObjectSet s2);

   public JMLObjectSet insert(Object elem);
   public JMLObjectSet remove(Object elem);
   …
}
```

Use of model classes

```
package java.util;
//@ import org.jmlspecs.models.JMLObjectSet;

public interface Set extends Collection {
  //@ public model instance JMLObjectSet  _set;

  /*@   public normal_behavior
    @      ensures  contains(o);
    @*/
  public boolean add(Object o);

  /*@   public normal_behavior
    @      ensures  \result ==  _set.has(o);
    @*/
  /*@ pure @*/ public boolean contains(Object o);
  …
}
```

Handling of model classes – pure methods

For verification, model classes need to be encoded in underlying theorem prover

By encoding pure methods [DarvasMüller06,JacobsPiessen06]

- pure methods encoded as uninterpreted functions
- functions axiomatized based on pure-method contracts

Problems

- theorem provers optimized for their own theories, rather than encodings of pure methods
- difficult to ensure consistency of resulting axiom system

Handling of model classes – direct mappings

For verification, model classes need to be encoded in underlying theorem prover

By direct mappings [LeavensEA05,Charles06,LeavensEA07]

- map model classes directly to theories of provers
- map pure methods to functions of selected theories
- mapping based on signature

Problems

- mapping ignores contracts
- possible mismatch between contract and semantic meaning of selected function
- leads to unexpected results during verification and runtime assertion checking

Faithful mapping of model classes to structures

- Our contribution is an approach that
  - follows idea of direct mappings
  - takes contracts into account
  - formally proves that mappings are semantically correct
  - allows identification and checking of redundant specs

- Approach
  - leads to better quality of model class specifications
  - eliminates semantic mismatches

Specifying and proving faithfulness of mappings

Approach consists of 3 stages:

1. Specifying mapping

2. Proving consistency: what can be proven using contracts can also be proven using theory of theorem prover

3. Proving completeness: what can be proven using the theory of theorem prover can also be proven using contracts

Correctness of mapping

Specifying mappings

- Introducing new JML clause: **mapped_to**
- Clause attached to a class
  - specifies theorem prover, theory, and type to which class is mapped

```
//@ mapped_to(“Isabelle”, “HOL/Set”, “ ’a set”);
public final /*@ pure @*/ class JMLObjectSet
```

- Clause attached to a method
  - specifies prover and term to which a call of the method is mapped

```
//@ mapped_to(“Isabelle”, “this Un s2”);
public JMLObjectSet union(JMLObjectSet s2);
```

Proving consistency

1. Turn each invariant and method specification into a lemma in language of selected theory
2. Prove lemmas using selected theory

```
/*@ public normal_behavior
  @   ensures
  @     (\forall Object e; ;
  @       \result.has(e) <==>
  @          this.has(e) || (e == elem));
  @*/
//@ mapped_to("Isabelle","insert elem this");
public JMLObjectSet insert(Object elem);
```

```
theory consistent imports Set:
lemma
∀ this, elem. ∀ e. e : (insert elem this) = (e : this  ∨  e = elem)
apply(auto)
```

Proving completeness

Create theory file as follows

1. Turn each pure method into a function symbol

```
public boolean
 isProperSubset(JMLObjectSet s2);
```

method specification into axiom

```
s.isProperSubset(s2) ==
  (s.isSubset(s2) && !s.equals(s2))
```

lemma

$A < B == A <= B \ \& \ \neg A=B$

axioms created in step 2.

theory complete:

consts

isProperSubset: ‘a set x ‘a set => bool

…

axiom

ax_isPropSub:

$\forall$s,s2,e1,e2. isProperSubset(s,s2) =
 (isSubset(s,s2) $\wedge$ $\neg$equals(s,s2))

…

lemma

$\forall$A,B. isProperSubset(A,B) =
 (isSubset(A,B) $\wedge$ $\neg$equals(A,B))

apply(simp **add**: ax_isPropSub)

…

Guarantees

Consistency

- selected theory is *model* for model class
- model-class specification is free of contradictions provided that theory is free of contradictions
- can show consistency of recursive specifications

- Completeness
  - extracted axiom system is complete relative to theory

Case study

- Mapped JMLObjectSet to Isabelle's HOL/Set theory
- Considered 17 members:
  - 2 constructors, 9 query methods, and
    6 methods that return new JMLObjectSet objects
  - made several simplifications
- Total of 380 lines of Isabelle code
  - 100 for consistency, 110 for completeness, and
    170 for equivalence proof (see later)
  - all code written manually

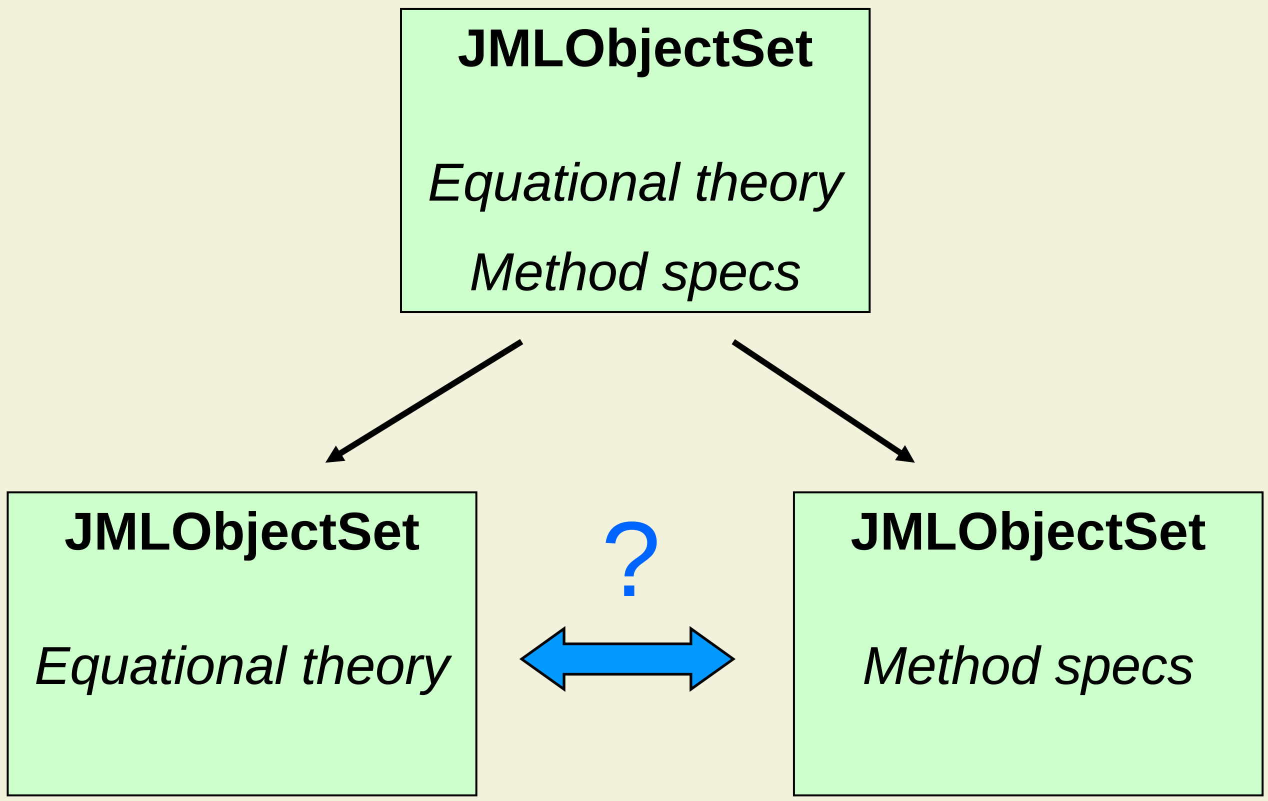
Case study – Division of specifications

Specification of JMLObjectSet expressed by equational theory and method specifications

```
/*@ public invariant
  @   (\forall JMLObjectSet s2; s2 != null;
  @     (\forall Object e1, e2; ;
  @       equational_theory(this, s2, e1, e2) ));
  @
  @ public normal_behavior
  @     ensures \result <==>
  @       s.insert(e1).has(e2) ==
  @         (e1 == e2 || s.has(e2));
  @   also  ...
  @ static public pure model boolean
  @   equational_theory(JMLObjectSet s,
  @     JMLObjectSet s2, Object e1, Object e2);
  @*/
```

Case study – Division of specifications

Specification of JMLObjectSet expressed by equational theory and method specifications



Case study – Specifying the mapping

- Mapping of model-class methods to function symbols of HOL/Set mostly straightforward

- Some interesting cases

```
//@ mapped_to("Isabelle","this - {elem}");
public JMLObjectSet remove(Object elem);
```

```
//@ mapped_to("Isabelle","SOME x. x : this");
public Object choose();
```

```
public int int_size();
```

Case study – Consistency

- Performed both for equational theory and method specifications
- Revealed one unsound equation in equational theory

```
s.insert(e1).remove(e2).
    equals(e1 == e2 ? s : s.remove(e2).insert(e1))
```

Not true if e1 == e2 and s contains e1!

- Possibility for high degree of automation
  - generation of lemmas based on few simple syntactic substitutions
  - lemmas proved automatically by Isabelle's tactics

Case study – Equivalence of specifications

- Inspected relation of equational theory and method specifications: equivalent? one stronger than the other?
- Answer: not equivalent and none stronger!
  - needed to add new specifications or strengthen some

From equations over isEmpty

new JMLObjectSet().isEmpty()   and   !s.insert(e1).isEmpty()

could not derive

```
/*@ public normal_behavior
  @    ensures \result == (\forall Object e; ; !this.has(e));
  @*/
public boolean isEmpty();
```

Case study – Completeness

- Performed both for equational theory and method specifications
- Most Isabelle definitions expressed by set comprehension
  - JML supports construct on syntax level
  - axiomatized construct based on Isabelle’s definition (correct and provides connection to model class)
- Most definitions easily mapped back and proved
  - could not map back some function symbols
- Lower degree of automation
  - lemma and proof generation only partially possible

Mismatching classes and structures

- Pure method cannot be mapped to semantically equivalent term of selected theory
  - no guarantee that specification is consistent and method corresponds to some mathematical operation
  - for instance, method int_size
  - need to pick other theory (e.g. HOL/Finite_Set)

- Function symbol of selected theory cannot be mapped to expression of model class
  - no isomorphism **but** *observational faithfulness*: mapping of all client-accessible pure methods faithful
  - for instance, function image
  - sufficient result for sound use of **mapped_to** clauses

Conclusions

- Improvements over previous work
  - formally proving semantic correspondence between mapped entities
  - better specifications for model classes: consistent and complete, redundancy identifiable and checkable
  - ensuring consistency of specifications even in the presence of recursion

- Case study
  - revealed incorrect specification
  - identified missing specifications
  - identified relation between equational theory and method specifications

Future work

- Tool support
  - typechecking of **`mapped_to`** clauses
  - (partial) generation of proof scripts
  - use of mappings in program verification system

- More case studies
  - with more complex structures (e.g. sequence)
  - with structures that have no directly corresponding theory (e.g. stack)