Total Correctness of Recursive Functions Using JML4 FSPV

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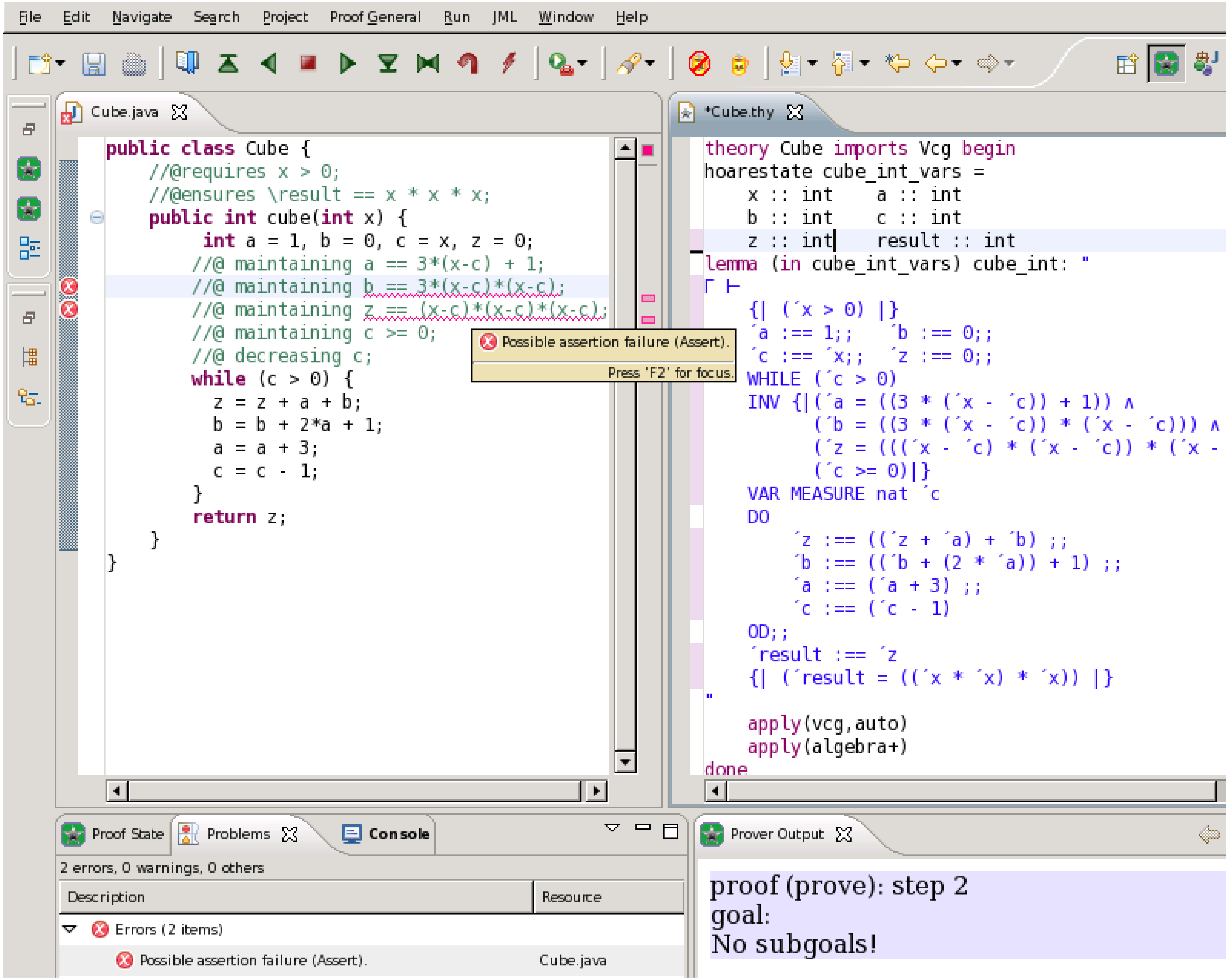
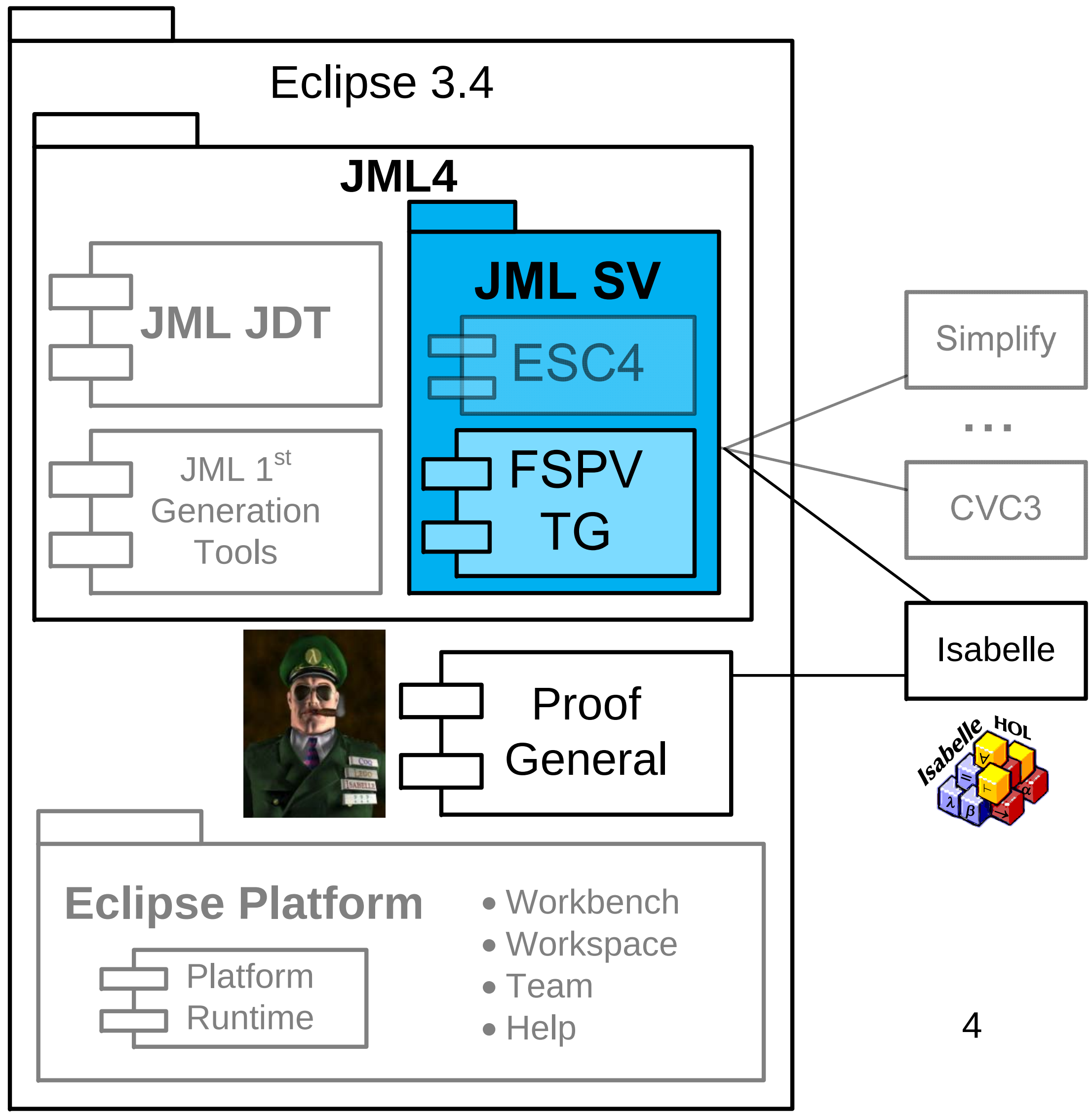
Research Context

- Java Modeling Language (JML)
- Program Verification
  - Aging 1st generation tools
- Integrated Verification Environment
  - JML4 …


File Edit Navigate Search Project Proof General Run JML Window Help
Cube.java
public class Cube {
//@requires x > 0;
//@ensures \result == x * x * x;
public int cube(int x) {
int a = 1, b = 0, c = x, z = 0;
//@ maintaining a == 3*(x-c) + 1;
//@ maintaining b == 3*(x-c)*(x-c);
//@ maintaining z == (x-c)*(x-c)*(x-c);
//@ maintaining c >= 0;
//@ decreasing c;
while (c > 0) {
z = z + a + b;
b = b + 2*a + 1;
a = a + 3;
c = c - 1;
}
return z;
}
}
Possible assertion failure (Assert).
Press 'F2' for focus.
*Cube.thy
theory Cube imports Vcg begin
hoarestate cube_int_vars =
x :: int a :: int
b :: int c :: int
z :: int result :: int
lemma (in cube_int_vars) cube_int: "
Γ ⊢
{| (´x > 0) |}
´a :== 1;; ´b :== 0;;
´c :== ´x;; ´z :== 0;;
WHILE (´c > 0)
INV {|(´a = ((3 * (´x - ´c)) + 1)) ∧
(´b = ((3 * (´x - ´c)) * (´x - ´c))) ∧
(´z = (((´x - ´c) * (´x - ´c)) * (´x -
(´c >= 0)|}
VAR MEASURE nat ´c
DO
´z :== ((´z + ´a) + ´b) ;;
´b :== ((´b + (2 * ´a)) + 1) ;;
´a :== (´a + 3) ;;
´c :== (´c - 1)
OD;;
´result :== ´z
{| (´result = ((´x * ´x) * ´x)) |}
"
apply(vcg,auto)
apply(algebra+)
done
Proof State
Problems
Console
2 errors, 0 warnings, 0 others
Description
Resource
Errors (2 items)
Possible assertion failure (Assert).
Cube.java
Prover Output
proof (prove): step 2
goal:
No subgoals!

Eclipse-based IVE

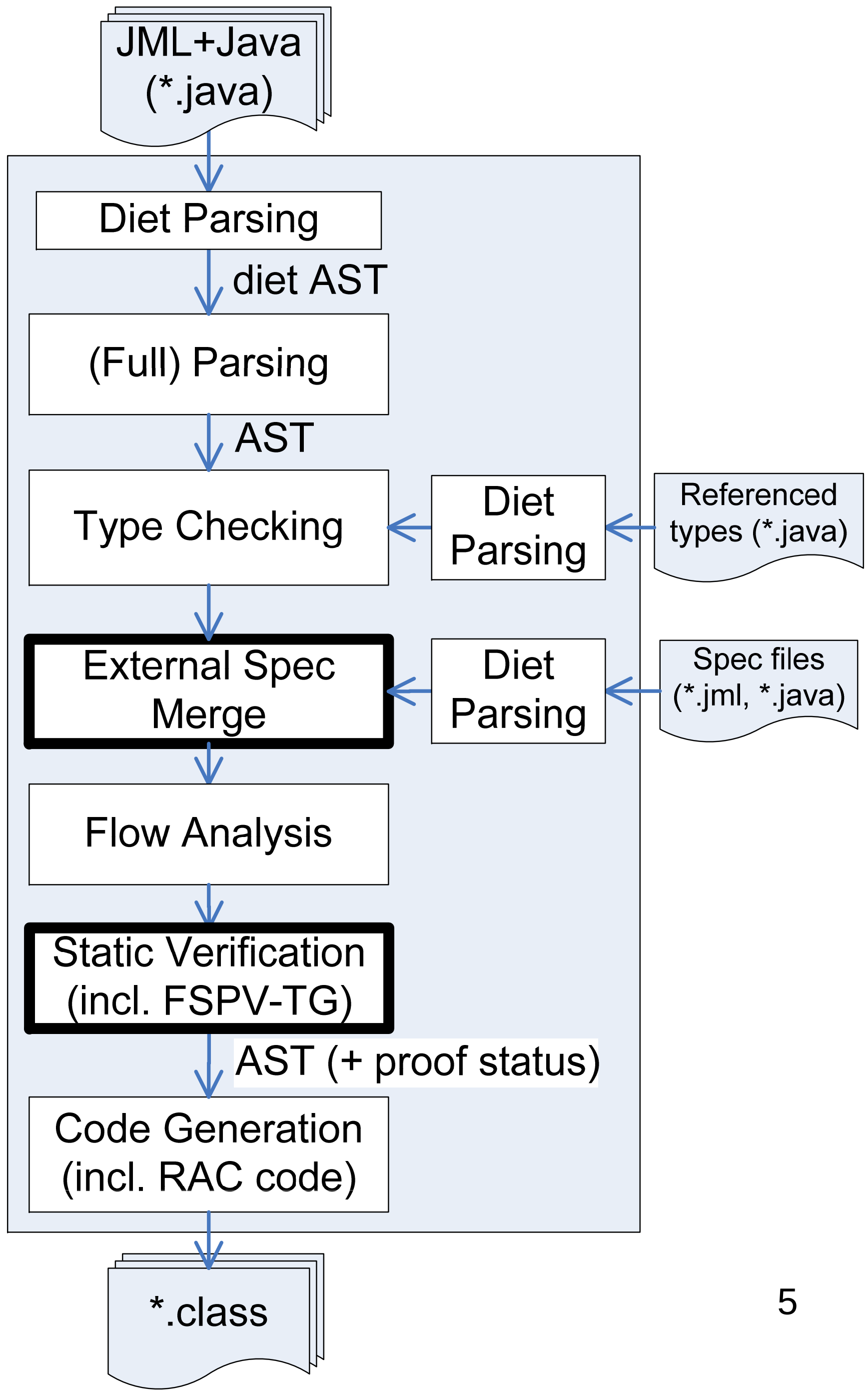
- Next-Generation Research Platform
- Integrates existing tools
  - RAC (jml & jmlc)
  - ESC (ESC/Java2)
- ESC4 & FSPV-TG



JML4 Extends Eclipse JDT

Java Development Tooling (JDT)

- JDT/JML4 Core Phases



Isabelle

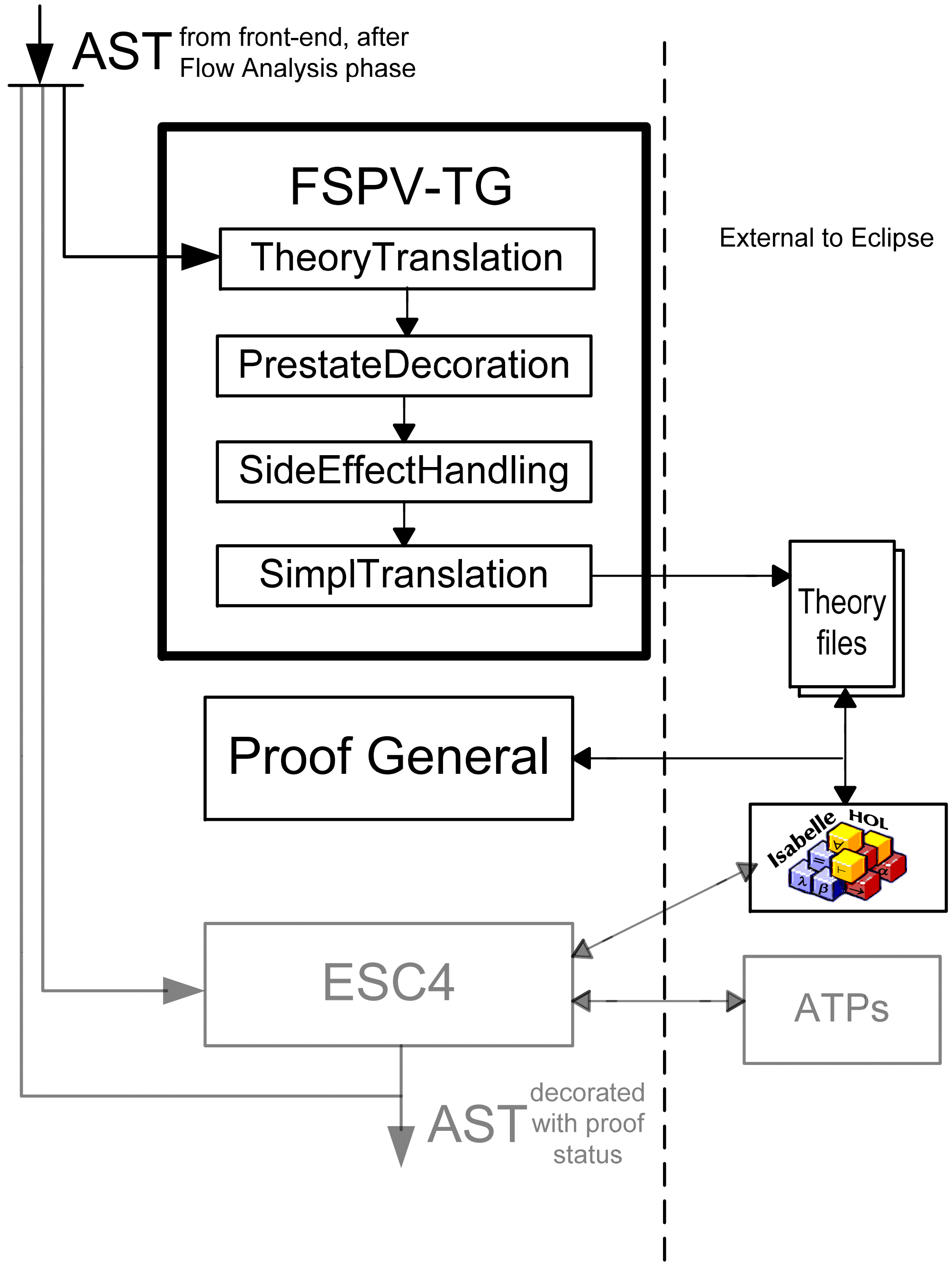
- Theorem prover framework
  - Isabelle/Pure (meta-logic)
  - classical reasoner
  - simplifier
- Provides machinery for defining new logics
  - existing logics can be extended
  - syntax can be added as sugar

Isabelle/Simpl

- Extends Isabelle/HOL
- Hoare Rules for
  - Partial & Total Correctness
- VCs generated within Simpl
  - through vcg and vcg_step tactics
- Proven Sound and Complete
- Expressive
  - Global and local variables, exceptions, abrupt termination, procedures, breaks out of loops, procedures, references and heap

Dataflow in FSPV-TG

- Translate JML to Hoare triples
- Store info for \old
- Remove side effects from expressions
- Generate Simpl



McCarthy's 91 Function

```
public class McCarthy {
 //@ requires n >= 0;
 //@ ensures \result == (100 < n ? n-10 : 91);
 //@ measured_by 101 - n;
 public static int f91(int n) {
  if(100 < n)
   return n - 10;
  else
   return f91(f91(n + 11));
 }
}
```

McCarthy's 91 Simpl Theory

```
theory McCarthy imports Vcg begin
  procedures
    McCarthy_f91_int(n::int | result'::int)
  "IF 100 < ´n
   THEN
     ´result' :== ´n - 10
   ELSE
     CALL McCarthy_f91_int(´n + 11) ≫ n1.
       ´result' :== CALL McCarthy_f91_int(n1)
   FI"
  lemma (in McCarthy_f91_int_impl) McCarthy_f91_int_spec:
    "∀n σ. Γ ⊢\<^sub>t
       {|σ. n=´n ∧ ´n≥0|}
       ´result' :== PROC McCarthy_f91_int(´n)
       {| ´result'=(if 100 < n  then  n - 10 else 91 ) |}"
   apply(hoare_rule HoareTotal.ProcRec1
    [where r="measure (λ (s,p). nat (101 - \<^bsup>s\<^esup>n) )"])
   apply(vcg)
   apply(auto)
  done
end
```

McCarthy’s 91 Simpl Theory

```
theory McCarthy imports Vcg begin
  procedures
    McCarthy_f91_int(n::int | result'::int)
  "IF 100 < ´n
   THEN
     ´result' :== ´n - 10
   ELSE
     CALL McCarthy_f91_int(´n + 11) ≫ n1.
       ´result' :== CALL McCarthy_f91_int(n1)
   FI"
  lemma (in McCarthy_f91_int_impl) McCarthy_f91_int_spec:
    "∀n σ. Γ ⊢\<^sub>t
       {|σ. n=´n ∧ ´n≥0|}
       ´result' :== PROC McCarthy_f91_int(´n)
       {| ´result'=(if 100 < n  then  n - 10 else 91 ) |}"
   apply(hoare_rule HoareTotal.ProcRec1
    [where r="measure (λ (s,p). nat (101 - \<^bsup>s\<^esup>n) )"])
   apply(vcg)
   apply(auto)
  done
end
```

Method Translation

Total Correctness Proof

Components of an Isabelle/Simpl Theory

- States

```
hoarestate vars =
    x::τ1
    …
```

- Procedures

```
procedures
   N (x::τ1, y::τ2, …| z::τ3)
   where v::τ4 … in B
```

- Hoare Tuples

$\Gamma, \Theta \vdash$ {|*P*|} *B* {|*Q*|}, {|*R*|}

$\Gamma, \Theta \vdash_t$ {|*P*|} *B* {|*Q*|}, {|*R*|}

Hoarestate

- Define global and local variables
- Statespaces
  - Modeled as a function from abstract names to abstract values
  - Organizes
    - distinctness of names and
    - projection/injection of concrete values into the abstract one.
- Locales
  - Support modular reasoning
  - Allows multiple inheritance of other locales
  - Allows for renaming components

McCarthy 91 Function

```
public static int f91(int n) {
    if(100 < n)
        return n - 10;
    else
        return f91(f91(n + 11));
}
```

```
procedures
  McCarthy_f91_int(n::int | result'::int)
"IF 100 < ´n
 THEN
   ´result' :== ´n - 10
 ELSE
   CALL McCarthy_f91_int(´n + 11) ≫ n1.
     ´result' :== CALL McCarthy_f91_int(n1)
 FI"
```

McCarthy 91 Function

```
public static int f91(int n) {
    if(100 < n)
        return n - 10;
    else
        return f91(f91(n + 11));
}
```

Input

Name

Output

```
procedures
 McCarthy_f91_int(n::int | result'::int)
"IF 100 < ´n
 THEN
   ´result' :== ´n - 10
 ELSE
   CALL McCarthy_f91_int(´n + 11) ≫ n1.
     ´result' :== CALL McCarthy_f91_int(n1)
 FI"
```

McCarthy 91 Function

```
public static int f91(int n) {
    if(100 < n)
            return n - 10;
    else
            return f91(f91(n + 11));
}
```

```
procedures
 McCarthy_f91_int(n::int | result'::int)
"IF 100 < ´n
 THEN
   ´result' :== ´n - 10
 ELSE
   CALL McCarthy_f91_int(´n + 11) ≫ n1.
     ´result' :== CALL McCarthy_f91_int(n1)
 FI"
```

Input

Output

Name

Simpl Variable

Binder Variable

Inner Rec.

Outer Rec.

Generate for Simpl

- a hoarestate(`McCarthy_f91_int_impl`)
  - statespace, locale
- Functions
  - Copying the actual to formal parameters
  - Updating global variables
  - Copying the formal result parameter

McCarthy 91- Proving Correctness

```
//@ requires n >= 0;
//@ ensures \result == (100<n ? n-10 : 91);
//@ measured_by 101 - n;
public static int f91(int n)
```

```
lemma (in McCarthy_f91_int_impl) McCarthy_f91_int_spec:
  "∀n σ. Γ ⊢\<^sub>t
     {|σ. n=´n ∧ ´n≥0|}
     ´result' :== PROC McCarthy_f91_int(´n)
     {| ´result'=(if 100 < n  then  n - 10 else 91 ) |}"
 apply(hoare_rule HoareTotal.ProcRec1
  [where r="measure (λ (s,p). nat (101 - \<^bsup>s\<^esup>n) )"])
 apply(vcg)
 apply(auto)
done
```

McCarthy 91- Proving Correctness

```
//@ requires n >= 0;
//@ ensures \result == (100<n ? n-10 : 91);
//@ measured_by 101 - n;
public static int f91(int n)
```

locale

Name

```
lemma (in McCarthy_f91_int_impl) McCarthy_f91_int_spec:
  "∀n σ. Γ ⊢\<^sub>t
     {|σ. n=´n ∧ ´n≥0|}
     ´result' :== PROC McCarthy_f91_int(´n)
     {| ´result'=(if 100 < n  then  n - 10 else 91 ) |}"
 apply(hoare_rule HoareTotal.ProcRec1
  [where r="measure (λ (s,p). nat (101 - \<^bsup>s\<^esup>n) )"])
 apply(vcg)
 apply(auto)
done
```

McCarthy 91- Proving Correctness

```
//@ requires n >= 0;
//@ ensures \result == (100<n ? n-10 : 91);
//@ measured_by 101 - n;
public static int f91(int n)
```

locale

Name

```
lemma (in McCarthy_f91_int_impl) McCarthy_f91_int_spec:
  "∀n σ. Γ ⊢\<^sub>t
     {|σ. n=´n ∧ ´n≥0|}
     ´result' :== PROC McCarthy_f91_int(´n)
     {| ´result'=(if 100 < n  then  n - 10 else 91 ) |}"
 apply(hoare_rule HoareTotal.ProcRec1
  [where r="measure (λ (s,p). nat (101 - \<^bsup>s\<^esup>n) )"])
 apply(vcg)
 apply(auto)
done
```

Pre

Statement

Post

McCarthy 91- Proving Correctness

```
//@ requires n >= 0;
//@ ensures \result == (100<n ? n-10 : 91);
//@ measured_by 101 - n;
public static int f91(int n)
```

```
lemma (in McCarthy_f91_int_impl) McCarthy_f91_int_spec:
  "∀n σ. Γ ⊢\<^sub>t
     {|σ. n=´n ∧ ´n≥0|}
     ´result' :== PROC McCarthy_f91_int(´n)
     {| ´result'=(if 100 < n  then  n - 10 else 91 ) |}"
 apply(hoare_rule HoareTotal.ProcRec1
  [where r="measure (λ (s,p). nat (101 - \<^bsup>s\<^esup>n) )"])
 apply(vcg)
 apply(auto)
done
```

prestate

locale

Name

Pre

Statement

Post

McCarthy 91- Proving Correctness

```
//@ requires n >= 0;
//@ ensures \result == (100<n ? n-10 : 91);
//@ measured_by 101 - n;
public static int f91(int n)
```

prestate | locale | Name

```
lemma (in McCarthy_f91_int_impl) McCarthy_f91_int_spec:
  "∀n σ. Γ ⊢\<^sub>t
     {|σ. n=´n ∧ ´n≥0|}
     ´result' :== PROC McCarthy_f91_int(´n)
     {| ´result'=(if 100 < n  then  n - 10 else 91 ) |}"
 apply(hoare_rule HoareTotal.ProcRec1
  [where r="measure (λ (s,p). nat (101 - \<^bsup>s\<^esup>n) )"])
 apply(vcg)
 apply(auto)
done
```

Pre | Statement | Post | VC generator | measure | auto

Fibonacci

```
//@ public static native int fib_spec(int
  n);
//@ requires n>=0;
//@ ensures \result ==  fib_spec(n);
//@ measured_by n;
public static /*@ pure */ int fib(int n) {
  if(n == 0)
    return 0;
  else if (n == 1)
    return 1;
  else
    return fib(n-1) + fib(n-2);
}
```

Fibonacci

```
function fib_spec :: "int ⇒ int" where
"fib_spec n =
  (if n = 0 then 0 else
    (if n=1 then 1 else
      (if n < 0 then arbitrary
        else (fib_spec (n - 1)) + (fib_spec (n - 2)))))"
by(pat_completeness, auto)
termination by (relation "measure (λn. nat n)",auto)

lemma (in Fibonacci_fib_int_impl) Fibonacci_fib_int_spec:
"∀n σ. Γ ⊢\<^sub>t
     {|σ. ´n=n ∧ ´n≥0|}
     ´result' :== PROC Fibonacci_fib_int(´n)
     {|´result'=fib_spec(n)|}"
apply(hoare_rule HoareTotal.ProcRec1
  [where r="measure (λ (s,p). nat \<^bsup>s\<^esup>n )"])
by(vcg,auto)
```

Fibonacci

```
class Fibonacci {
  //@ requires n>=0;
  //@ ensures \result == (n==0)? 0 : (n==1) ? 1
  //@  : fib_spec(n-1)+fib_spec(n-2);
  //@ measured_by n;
  //@ public static pure model
  //@                      int fib_spec(int n);

  //@ requires n>=0;
  //@ ensures \result == fib_spec(n);
  //@ measured_by n;
  public static /*@ pure */ int fib(int n) {
    ...
```

Ackermann

```
//@ public static native int ack_spec(int n);
//@ requires n >= 0 && m >= 0 ;
//@ ensures \result == ack_spec(n,m);
public static int ack(int n, int m) {
  if(n == 0)
    return m + 1;
  else if(m == 0)
    return ack(n-1, m);
  else
    return ack(n-1, ack(n, m-1));
}
```

Ackermann

```
lemma (in Ackermann_ack_int_int_impl) Ackermann_ack_int_int_spec:
  "∀n m σ. Γ ⊢\<^sub>t
     {|σ. n=´n ∧ ´n≥0 ∧ m=´m ∧ ´m≥0 |}
     ´result' :== PROC Ackermann_ack_int_int(´n, ´m)
     {|´result' =  (ack_spec n m) |}"
 apply(hoare_rule HoareTotal.ProcRec1
    [where r="measures [λ(s,p). nat \<^bsup>s\<^esup>n,
                              \<lambda>(s,p). nat \<^bsup>s\<^esup>m]"] )
 apply((auto|vcg)+,case_tac "nat n",auto,case_tac "nat n",auto)
 by (case_tac "nat m",auto)
```

Milestones

- FSPV TG
  - Supporting functional subset of JML+Java
- Study the adequacy of Isabelle/Simpl
  - Non-recursive Programs
    - Cube, Factorial, Summation
  - Total Correctness of Recursive Programs
    - Factorial, Fibonacci, McCarthy's 91, Ackermann
  - Classes
    - Point

Related Work

- LOOP
  - VerifiCard project (Industry)
- JACK
  - Banking Application (Academia)
- Krakatoa/Why

Comparison Table

	LOOP	JACK	Krakatoa/Why	FSPV-TG Simpl
Maintained	✗	✗	✓	✓
Open Source	✗	✓	✓	✓
Proven Sound	✓	✗	✓	✓
Proven Complete	✗	✗	✗	✓
Above two proofs done	in PVS	N/A	by hand	in Isabelle
VC generation done in prover	✗	✗	✗	✓
Termination of Recursive Functions	✗	✗	✗	✓

Future Work

- Update the translator to reflect current state.
- Case Study
- Object Oriented Support
  - Inheritance
  - Behavioral Subtyping
- Additional Language Elements
  - Exceptions
  - Loops with break and continue statements
- JML
  - Revise measured_by clause (see Ackermann)

Thank you!

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Fields + Memory

```
hoarestate globals_memory =
 alloc::"ref list"
 free::nat
hoarestate globals_Point = globals_memory +
 XCoord :: "ref ⇒ int"
 YCoord :: "ref ⇒ int"
definition sz where "sz ≡ 2::nat"
```

Total Correctness Proof for the f91 Function

```
//@ requires n >= 0;
//@ ensures \result == (100<n ? n-10 : 91);
//@ measured_by 101 - n;
public static int f91(int n)
```

```
lemma (in McCarthy_f91_int_impl) McCarthy_f91_int_spec:
  "∀n σ. Γ ⊢\<^sub>t
     {|σ. n=´n ∧ ´n≥0|}
     ´result' :== PROC McCarthy_f91_int(´n)
     {| ´result'=(if 100 < n  then  n - 10 else 91 ) |}"
```

Constructor

```
procedures (imports globals_Point)
 Point_Point_int_int(x::int, y::int | result'::ref)
 "´result':==NEW sz [´XCoord:==0,´YCoord:==0];;
  ´result'→´XCoord :== ´x;;
  ´result'→´YCoord :== ´y"
```

- Constructors defined as a regular procedure
- Allocate memory using the `NEW` operator
  - Providing a size (sz) and
  - A list of with the fields and their initialization
- Assignments of input values to fields

Method Calls

```
procedures (imports globals_Point)
 Point_tester_Point(P::ref|result'::ref)
 "´P :== CALL Point_Point_int_int(10,11);;
   CALL Point_move_int_int(´P,1,0);;
  ´result' :== ´P"
```

Point Class

```
Public class Point
 public int XCoord;
 public int YCoord;

 public Point(int x, int y) {
  XCoord = x;
  YCoord = y;
 }
 //@ensures XCoord==\old(XCoord)+dx;
 //@ensures YCoord==\old(YCoord)+dy;
 public void move(int dx, int dy) {
  XCoord += dx;
  YCoord += dy;
 }
 //@ requires P == null;
 //@ ensures \result != null;
 //@ ensures \result.XCoord == 11;
 //@ ensures \result.YCoord == 11;
 public static Point tester(Point P){
  P = new Point(10,11);
  P.move(1,0);
  return P;
 }
}
```

Point Class Simpl Theory

```
theory Point imports HeapList Vcg
begin
 hoarestate globals_memory =
  alloc::"ref list"
  free::nat
 hoarestate globals_Point = globals_memory +
  XCoord :: "ref ⇒ int"
  YCoord :: "ref ⇒ int"
 definition sz where "sz ≡ 2::nat"

 procedures (imports globals_Point)
  Point_Point_int_int(x::int, y::int | result'::ref)
  "´result':==NEW sz [´XCoord:==0,´YCoord:==0];;
   ´result'→´XCoord :== ´x;;
   ´result'→´YCoord :== ´y"
 lemma (in Point_Point_int_int_impl) Point_Point_int_int_spec:
  "∀x y. Γ⊢\<^sub>t
        {|´x = x ∧ ´y = y ∧ sz ≤ ´free|}
        ´result' :== PROC Point_Point_int_int(´x,´y)
        {|´result' ≠ Null ∧
          ´result'→´XCoord = x ∧ ´result'→´YCoord = y|}"
 by(vcg,auto)

 procedures (imports globals_Point)
  Point_move_int_int(this::ref, dx::int, dy::int)
  "´this→´XCoord :== ´this→´XCoord + ´dx;;
   ´this→´YCoord :== ´this→´YCoord + ´dy"
 lemma (in Point_move_int_int_impl) Point_move_int_int_spec:
  "∀dx dy x y σ. Γ⊢\<^sub>t
    {|σ.´this→´XCoord = x ∧ ´this→´YCoord = y ∧
        ´dx = dx ∧ ´dy = dy ∧ ´this ≠ Null |}
    PROC Point_move_int_int(´this, ´dx, ´dy)
    {|´this =  \<^bsup>σ\<^esup>this ∧
      ´this→´XCoord =  x + dx ∧ ´this→´YCoord =  y + dy|}"
 by(vcg, auto)

 procedures (imports globals_Point)
  Point_tester_Point(P::ref|result'::ref)
  "´P :== CALL Point_Point_int_int(10,11);;
    CALL Point_move_int_int(´P,1,0);;
   ´result' :== ´P"
 lemma (in Point_tester_Point_impl) Point_tester_Point_spec:
  "∀x y. Γ⊢\<^sub>t
    {| sz ≤ ´free ∧ ´P = Null |}
    ´result' :== PROC Point_tester_Point(´P)
    {| ´result' ≠ Null ∧
       ´result'→´XCoord = 11 ∧ ´result'→´YCoord = 11 |}"
 by(vcg, auto)
end
```

Point Class Simpl Theory

fields

Constructor

move

tester

```
theory Point imports HeapList Vcg
begin
 hoarestate globals_memory =
  alloc::"ref list"
  free::nat
 hoarestate globals_Point = globals_memory +
  XCoord :: "ref ⇒ int"
  YCoord :: "ref ⇒ int"
 definition sz where "sz ≡ 2::nat"

 procedures (imports globals_Point)
  Point_Point_int_int(x::int, y::int | result'::ref)
  "´result':==NEW sz [´XCoord:==0,´YCoord:==0];;
   ´result'→´XCoord :== ´x;;
   ´result'→´YCoord :== ´y"
 lemma (in Point_Point_int_int_impl) Point_Point_int_int_spec:
  "∀x y. Γ⊢\<^sub>t
        {|´x = x ∧ ´y = y ∧ sz ≤ ´free|}
        ´result' :== PROC Point_Point_int_int(´x,´y)
        {|´result' ≠ Null ∧
          ´result'→´XCoord = x ∧ ´result'→´YCoord = y|}"
 by(vcg,auto)

 procedures (imports globals_Point)
  Point_move_int_int(this::ref, dx::int, dy::int)
  "´this→´XCoord :== ´this→´XCoord + ´dx;;
   ´this→´YCoord :== ´this→´YCoord + ´dy"
 lemma (in Point_move_int_int_impl) Point_move_int_int_spec:
  "∀dx dy x y σ. Γ⊢\<^sub>t
    {|σ.´this→´XCoord = x ∧ ´this→´YCoord = y ∧
        ´dx = dx ∧ ´dy = dy ∧ ´this ≠ Null |}
    PROC Point_move_int_int(´this, ´dx, ´dy)
    {|´this = \<^bsup>σ\<^esup>this ∧
      ´this→´XCoord = x + dx ∧ ´this→´YCoord = y + dy|}"
 by(vcg, auto)

 procedures (imports globals_Point)
  Point_tester_Point(P::ref|result'::ref)
  "´P :== CALL Point_Point_int_int(10,11);;
    CALL Point_move_int_int(´P,1,0);;
   ´result' :== ´P"
 lemma (in Point_tester_Point_impl) Point_tester_Point_spec:
  "∀x y. Γ⊢\<^sub>t
    {| sz ≤ ´free ∧ ´P = Null |}
    ´result' :== PROC Point_tester_Point(´P)
    {| ´result' ≠ Null ∧
      ´result'→´XCoord = 11 ∧ ´result'→´YCoord = 11 |}"
 by(vcg, auto)
end
```

Memory in Simpl

- References and Heap
- Two components:
  - A list of allocated references
  - A natural number indicating the amount of available memory
- Expressed as a hoarestate

```
hoarestate  globals_memory =
   alloc = “ref list”
   free = nat
```

Fields

- Defined as maps from `ref => τ`

```
hoarestate globals_Point=globals_memory +
   XCoord :: “ref => int”
   YCoord :: “ref => int”
```

- Accessing a field:

```
   `P->`XCoord
```

Case Study—Benchmarks[1]

- Adding and Multiplying numbers
- Binary Search in an Array
- Sorting a Queue
- Layered Implementation of a Map ADT
- Linked-List Implementation of a Queue ADT
- Iterators
- Input/Output Streams
- An Integrated Application

[1] B. Weide et al., "Incremental Benchmarks for Software Verification Tools and Techniques," Verified Software: Theories, Tools, Experiments, 2008, pp. 84-98