Integrating Math Units and Proof Checking for Specification and Verification

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Overview

- RESOLVE Verification System
- Role of Proof Checker in Verification System

- Requirements of a Proof Checker in such a system

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- RESOLVE Verification System
- Role of Proof Checker in Verification System
  - Issues
  - Solutions
- Requirements of a Proof Checker in such a system
  - Issues
  - Solutions

RESOLVE Verification System

RESOLVE

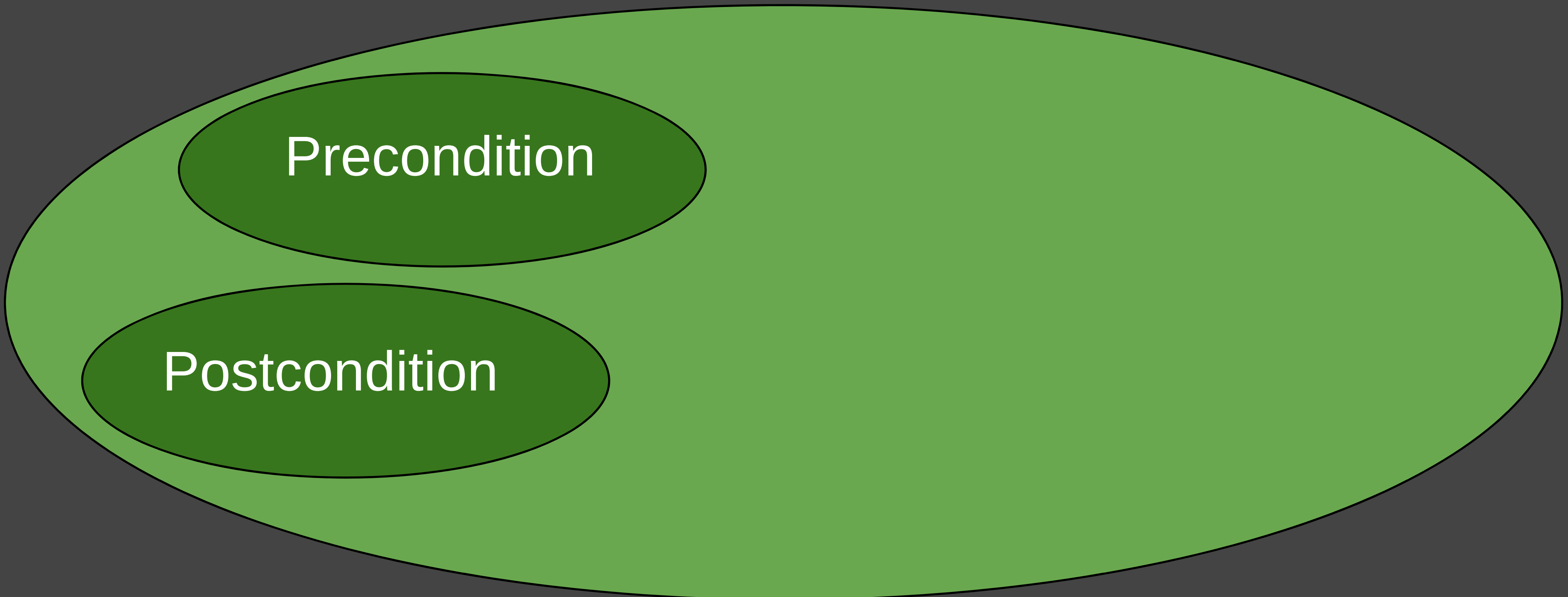
- Reusable Software Research Group at Clemson
- Integrated Programming, Specification, and Proof Language
- Full end-to-end verification
    - Scalability
    - Performance
- Isabelle Backend

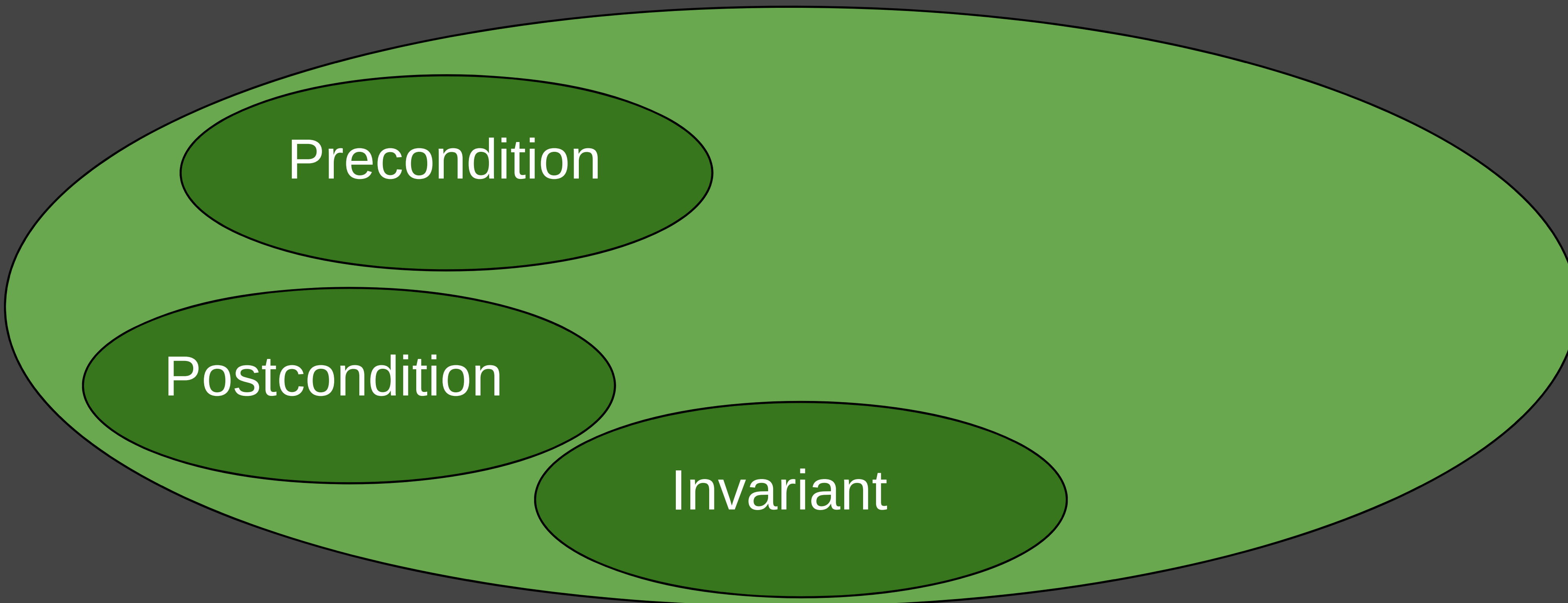
cs.clemson.edu/~resolve

Proof Checkers in a Verification System

PROOF OBLIGATIONS


Precondition


Precondition
Postcondition


Precondition
Postcondition
Invariant

Enhancement for Stacks

```
Enhancement Flipping_Capability for Stack_Template;

    Operation Flip( updates S : Stack );
        ensures S = Rev( #S );

end Flipping_Capability;
```

Implementation of Flipping

```
Realization Obvious_Flipping_Realization for
      Flipping_Capability of Stack_Template;

   Procedure Flip ( updates S : Stack );
      Var Next_Entry : Entry;
      Var S_Flipped : Stack;

      While ( Depth( S ) /= 0 )
         changing S, Next_Entry, S_Flipped;
         maintaining #S = Rev( S_Flipped ) o S;
         decreasing |S|;
      do
         Pop( Next_Entry, S );
         Push( Next_Entry, S_Flipped);
      end;

      S :=: S_Flipped;
   end Flip;
end Obvious_Flipping_Realization;
```

Verification Condition

((|S| <= Max_Depth) and (S = (Rev(?S_Flipped) o ??S) and
(|??S| /= 0 and ??S = (<?Next_Entry> o ?S))))

============================>

(Rev(?S_Flipped) o ??S) =
(Rev(<?Next_Entry> o ?S_Flipped) o ?S)

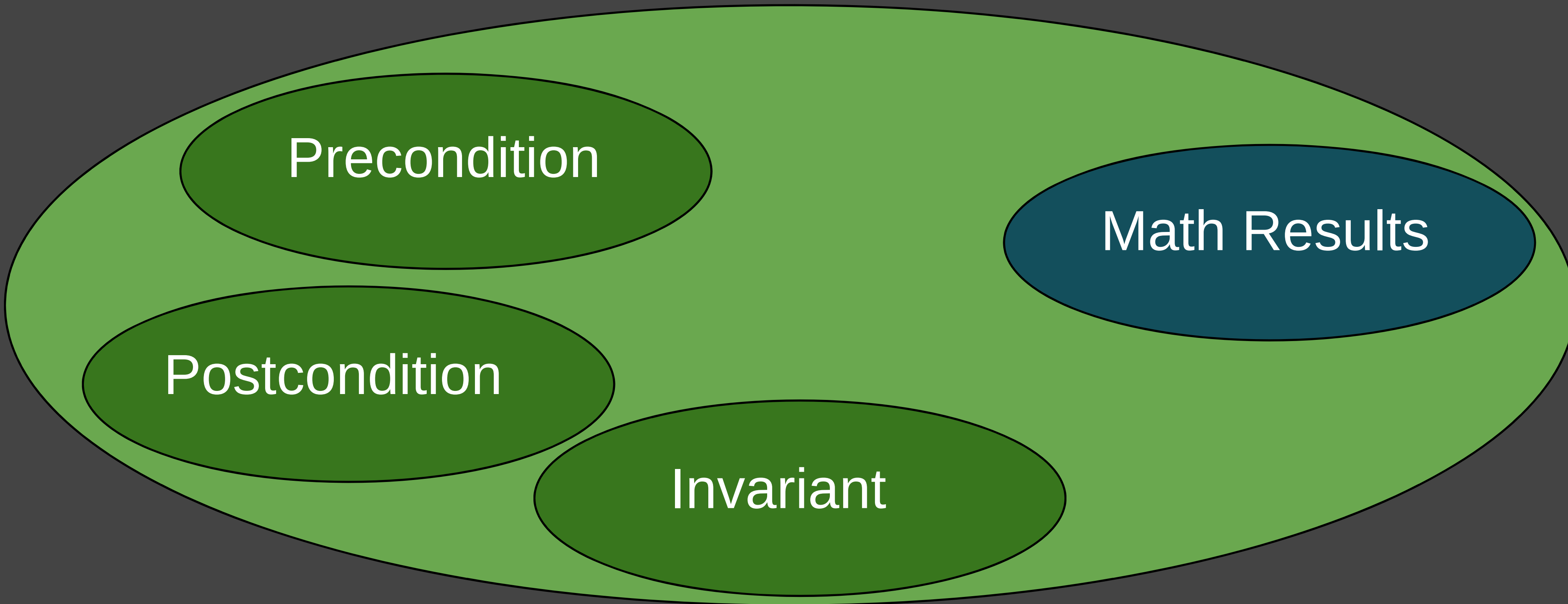
A little help

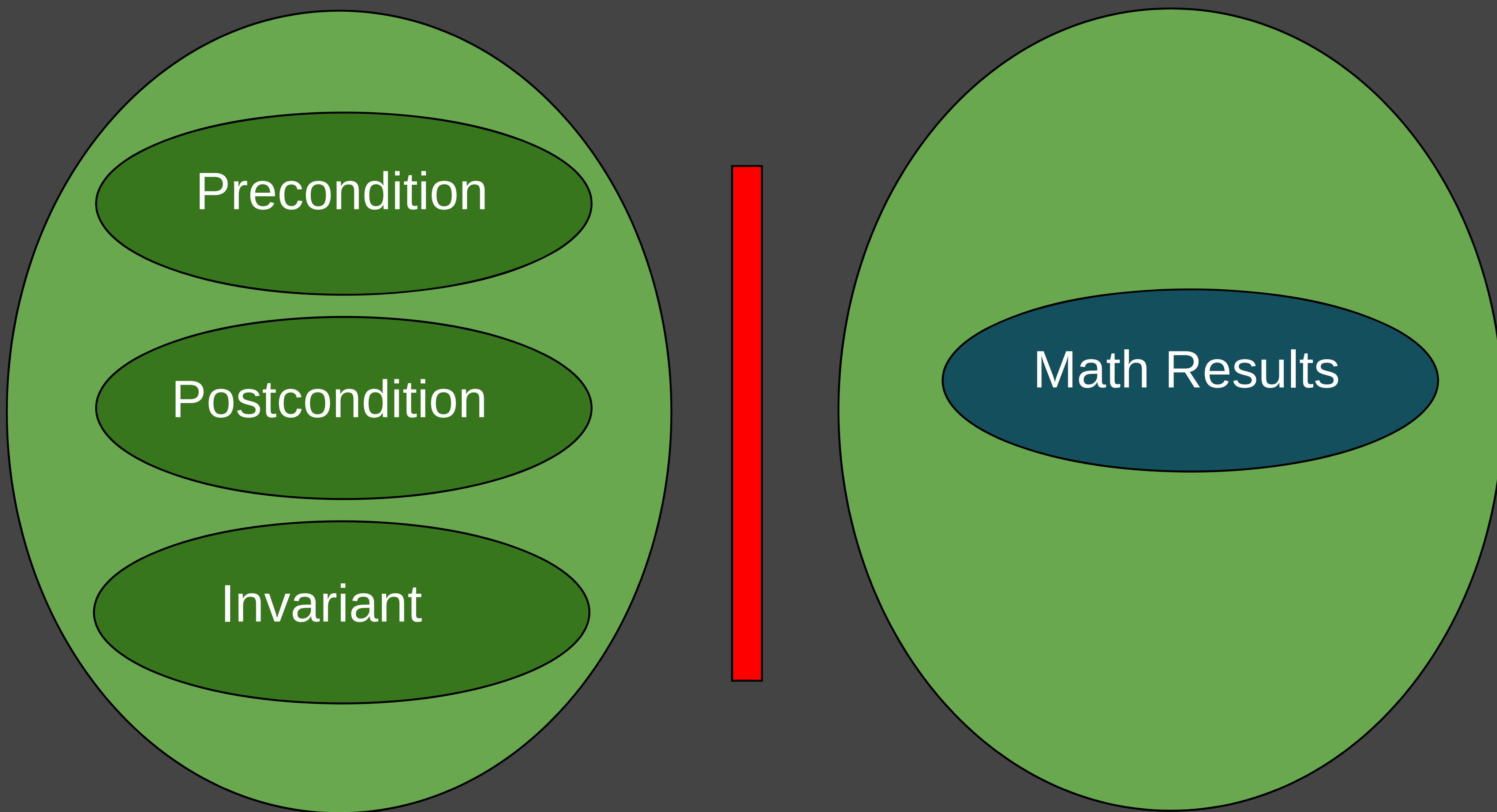
Theorem 1:

$\forall \alpha$: Str(E), $\forall x$: E, $(\alpha \circ \langle x \rangle)^{Rev} = (\langle x \rangle \circ \alpha^{Rev})$

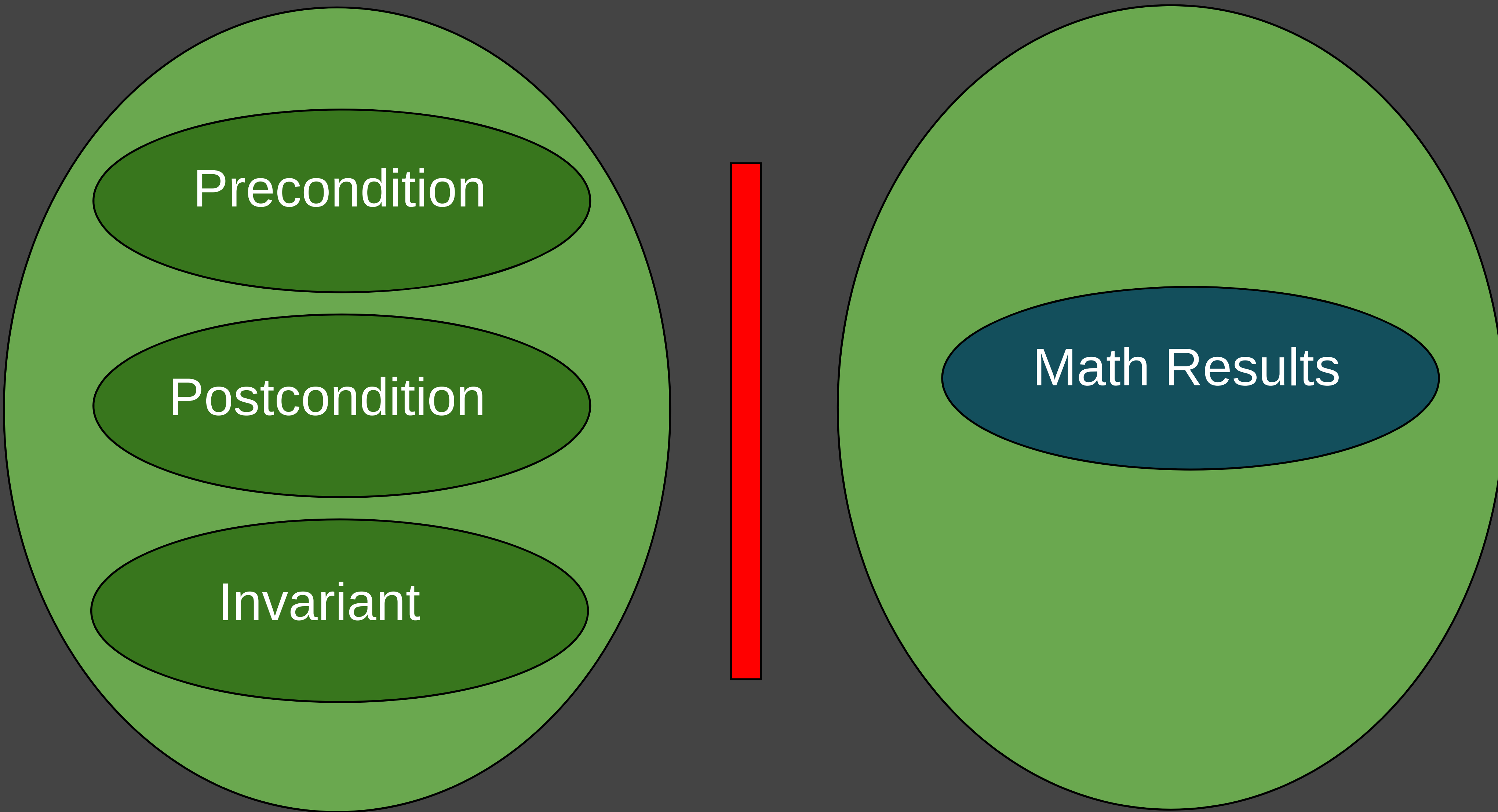
Theorem 2:

Is_Associative($\circ$)

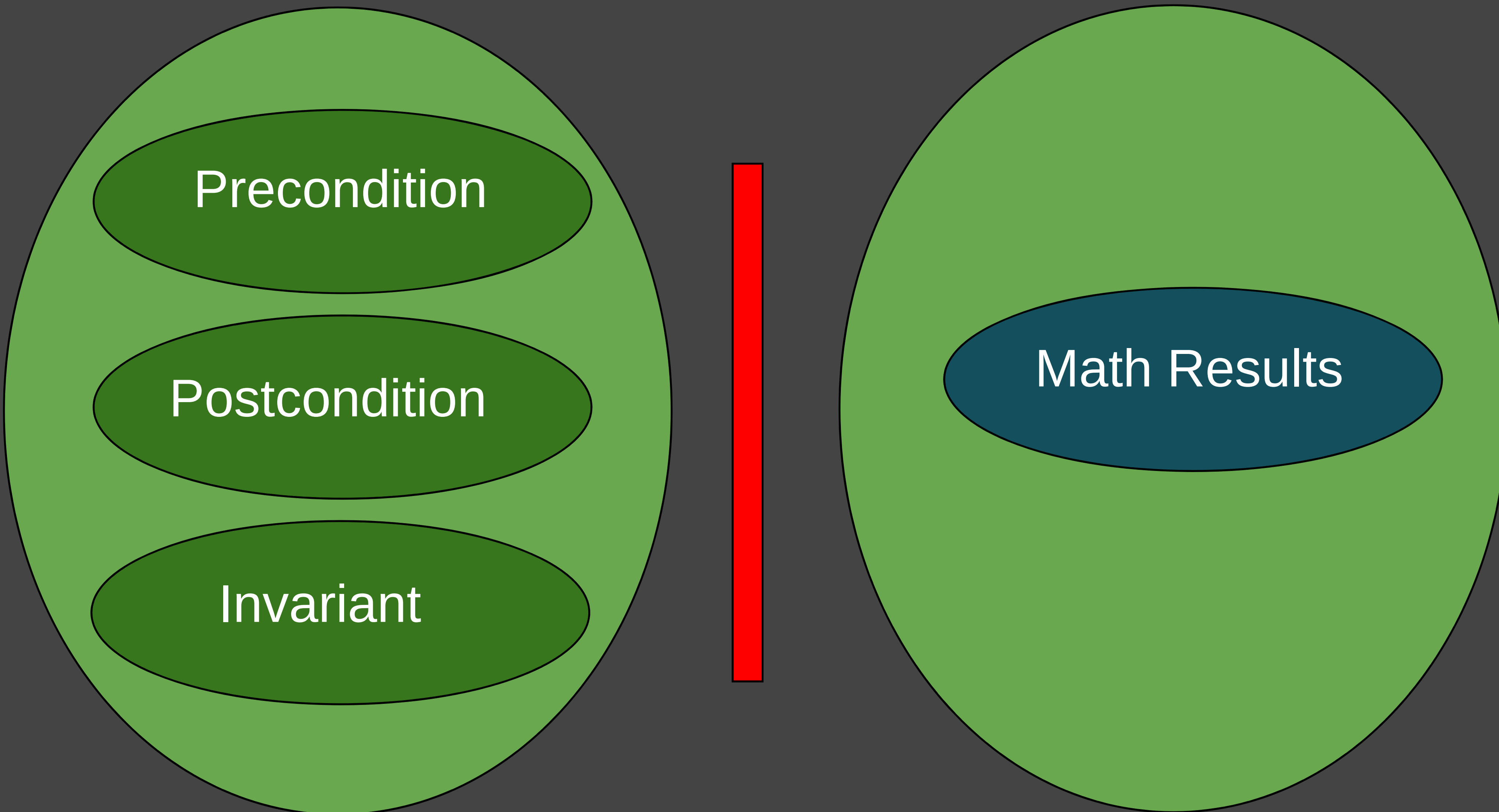

Precondition
Math Results
Postcondition
Invariant


Precondition
Postcondition
Invariant
Math Results

Automated Prover



Automated Prover



User Provided Proof +
Proof Checker

Verification System

"Requiring programmers to engage in a fine level of proof activity is unlikely to lead to wide-spread verification .... [T]he limitations of automated theorem proving often require substantial human intervention."

Verification System

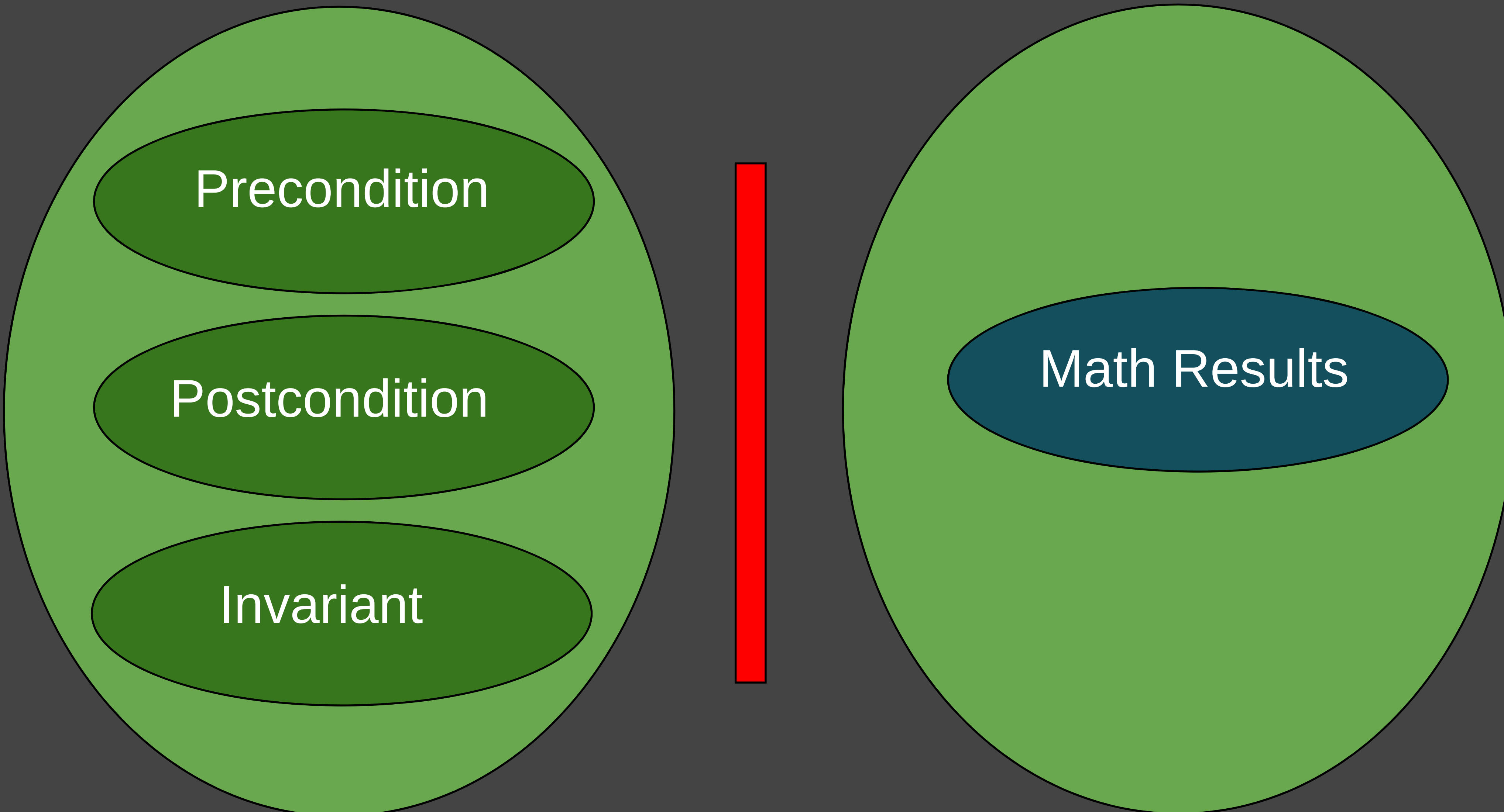
"Requiring programmers to engage in a fine level of proof activity is unlikely to lead to wide-spread verification .... [T]he limitations of automated theorem proving often require substantial human intervention."

Clear division between verification conditions and math results.

Rethink the latter as a job for trained mathematicians.

Requirements for such a Proof Checker

Automated Prover



User Provided Proof +
Proof Checker

Reusability

Programming Language

- Abstraction
- Modules
- Interfaces
- Readability

Proof Language

Reusability

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Proof Language

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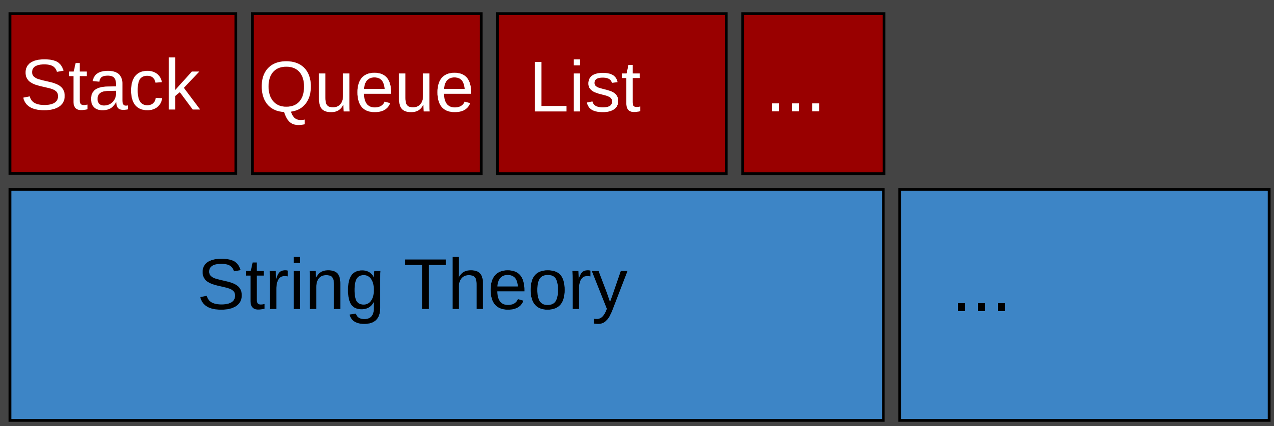
Abstraction and Modules



Consumers of Theories

- Proof Checker
- Automated Prover
- Mathematicians
- Programmers

Précis vs. Proof Units

Header file for theories.

Précis vs. Proof Units

```
Précis Natural_Number_Theory;
uses Basic_Function_Properties,
    Monogenerator_Theory...

    Inductive Definition on i : N of
            (a : N) + (b) : N is
        (i) a + 0 = a;
        (ii) a + suc(b) = suc(a + b);

    Theorem N1:
        Is_Associative( + );

    ...

end Natural_Number_Theory;
```

Précis vs. Proof Units

```
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        (i) a + 0 = a;
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    Theorem N1:
        Is_Associative( + );

    ...

end Natural_Number_Theory;
```
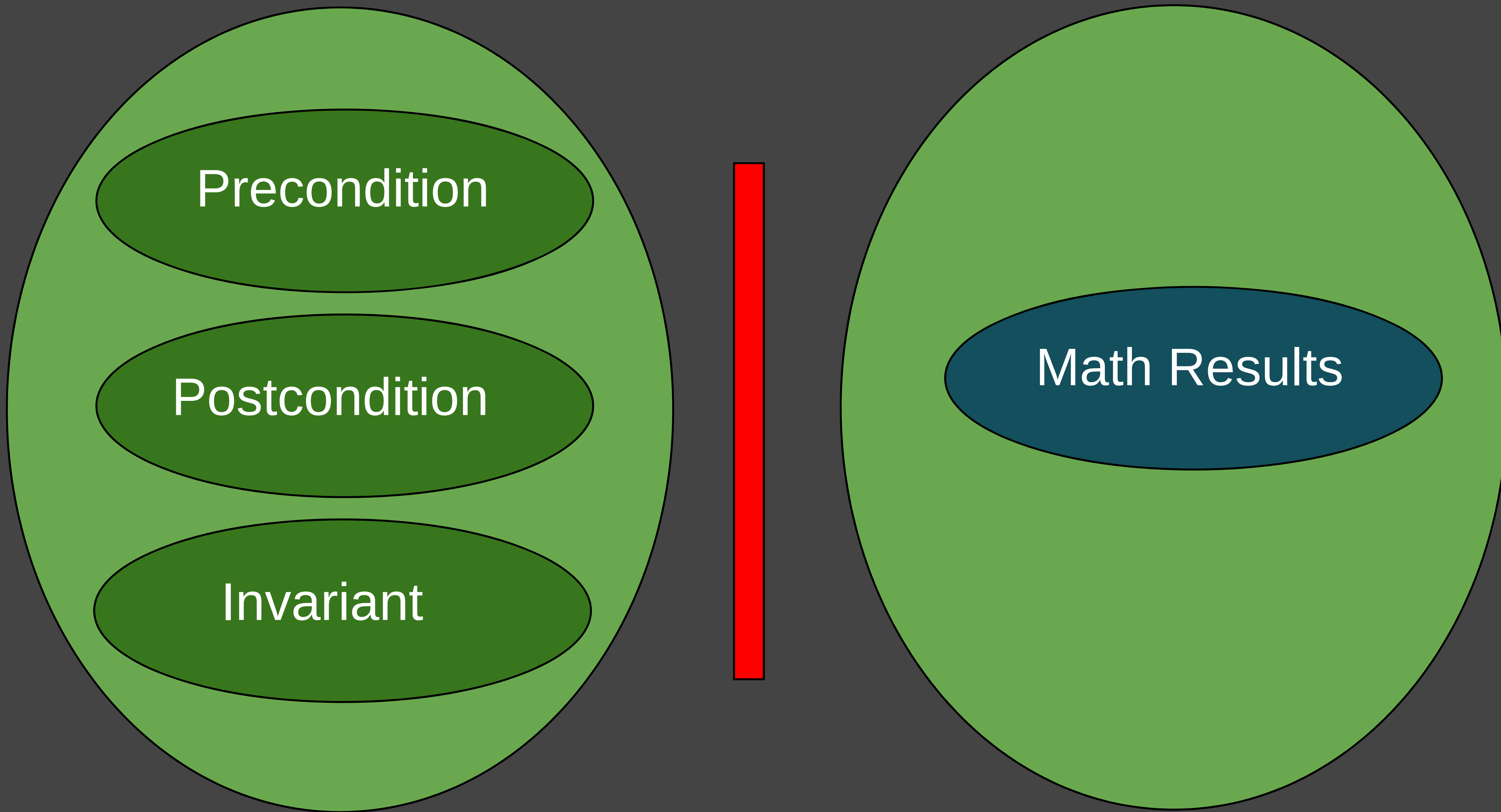

```
Proof unit
Natural_Number_Theory_Proofs
    for Natural_Number_Theory;
    Uses ...

    Proof of Theorem N1:
        Goal for all k, m, n: N,
            k + (m + n) = (k + m) + n;
        Definition S1: Powerset(N) =
            {n : N, for all k, m : N,
            k + (m + n) = (k + m) + n};

    ...
```

Automated Prover



User Provided Proof + Proof Checker

Popular Proof Checkers

Isabelle [2]

```
lemma assumes AB:
    "large_A /\ large_B"
    shows
        "large_B /\ large_A"
        ( is "?B /\ ?A" )
    using AB
proof
    assume "?A" "?B"
    show ?thesis ..
qed
```

Coq [1]

```
Variables A B C : Prop.

Lemma and_commutative :
        (A /\ B) -> (B /\ A).
    intro.
    elim H.
    split.
    exact H1.
    exact H0.
Save.
```

Mathematical Proof

Supposition k, m: N

Goal k + (m + 0) = (k + m) + 0

k + (m + 0) = k + m

by (i) of Definition +

k + m = (k + m) + 0

by (i) of Definition +

Deduction if k $\in \mathrm{N}$ and $\mathrm{m} \in \mathrm{N}$ then

k + (m + 0) = (k + m) + 0

[ZeroAssociativity] For all k: N, for all m: N,

k + (m + 0) = (k + m) + 0

by universal generalization

RESOLVE Proof Language

```
Supposition k, m: N;
    Goal k + (m + 0) = (k + m) + 0;
    k + (m + 0) = k + m
                                        by (i) of Definition +;
    k + m = (k + m) + 0
                                        by (i) of Definition +;
  Deduction if k is_in N and m is_in N then
      k + (m + 0) = (k + m) + 0;

  [ZeroAssociativity] For all k: N, for all m: N,
      k + (m + 0) = (k + m) + 0
                                 by universal generalization;
```

Demo

```
Corollary Identity: a : N and
    a + 0 = a;

Proof of Theorem Nothing:
    Supposition k, m: N;
        (k + m) + 0 = k + m
            by Corollary Identity & equality;
    Deduction if k is_in N and
        m is_in N then
        (k + m) + 0 = k + m;
QED
```

Demo

```
Corollary Identity: a : N and
    a + 0 = a;

Proof of Theorem Nothing:
    Supposition k, m: N;
        (k + m) + 0 = m + 0
            by Corollary Identity & equality;
    Deduction if k is_in N and
        m is_in N then
        (k + m) + 0 = k + m;
QED
```

```
Error: Simple.mt(10):
    Could not apply substitution to the
justified expression.
    (k + m) + 0 = m + 0
            by Corollary Identity & equality;
```

Demo

```
Corollary Identity: a : N and
    a + 0 = a;

Proof of Theorem Nothing:
    Supposition k, m: N;
        (k + m) + 0 = k + m
              by Corollary Identity & or rule;
    Deduction if k is_in N and
        m is_in N then
        (k + m) + 0 = k + m;
QED
```

```
Error: Simple.mt(10):
    Could not apply the rule Or Rule to
the proof expression.
    (k + m) + 0 = k + m
              by Corollary Identity & or rule;
```

Conclusions

- A clearer distinction is required between those proof obligations that we expect to be dispatched by an automated prover, and those for which we intend to furnish a proof.

- Programmers should not be required to provide proofs.

- Robust mathematical library of theories is required.

- Techniques from programming languages should be applied to mitigate the complexity of such theories.

References

[1] G. Huet, G. Kahn, and C. Paulin-Mohring, “The Coq Proof Assistant: A Tutorial.” INRIA, 2004, pp. 3-18; 45-47.

[2] T. Nipkow. “A Tutorial Introduction to Structured Isar Proofs,” http://www.cl.cam.ac.uk/research/hvg/Isabelle/dist/Isabelle/doc/isar-overview.pdf.