

Performance of Radial-Basis Function Networks for Direction of Arrival Estimation with Antenna Arrays

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Abstract—The problem of direction of arrival (DOA) estimation of mobile users using linear antenna arrays is addressed. To reduce the computational complexity of superresolution algorithms, e.g. multiple signal classification (MUSIC), the DOA problem is approached as a mapping which can be modeled using a suitable artificial neural network trained with input output pairs. This paper discusses the application of a three-layer radial-basis function neural network (RBFNN), which can learn multiple source-direction findings of a six-element array. The network weights are modified using the normalized cumulative delta rule. The performance of this network is compared to that of the MUSIC algorithm for both uncorrelated and correlated signals. It is also shown that the RBFNN substantially reduced the CPU time for the DOA estimation computations.

Index Terms—Antenna arrays, direction of arrival estimation.

I. INTRODUCTION

MOBILE satellite communication systems using frequency division multiple access (FDMA) are facing an increasing number of potential users to be served in the same allocated bandwidth. Multiple reuse of each channel, accomplished by the spatial separation of channels assigned the same narrow frequency band, is used to avoid co-channel interference. Cells with the same frequency are separated by the reuse distance D which is directly related to the cluster size C . Increasing C allows more users to be served in the same geographic area, increases the carrier to interference ratio but also yields larger reuse distances. Closer proximity of cofrequency cells or beams allows additional frequency reuse [1]–[3]. This can be accomplished through two steps. First, a superresolution angle of arrival (DOA) algorithm, multiple signal classification (MUSIC) [4], is used to locate desired as well as cochannel mobile users. This algorithm has the advantage of high resolution for signals with small angular separation (few degrees to few tenths of a degree in many mobile satellite systems) and is known to perform well under low signal-to-noise ratios (SNR's). Once the direction of the users are specified, this information can be used in conjunction with any adaptive array technique [5] so that the radiation pattern of the array is adapted to allocate the maximum toward the mobiles of interest while other sources of interference in the same frequency slot are nulled and the system is able to track these mobiles in real time.

Superresolution algorithms have been successfully applied to the problem of DOA estimation to locate radiating sources with additive noise, uncorrelated, and correlated signals. One of the main disadvantages of the superresolution algorithms is that they require extensive computation and as a result they are difficult to implement in real-time. Recently, neural networks have been proposed as successful candidates to carry on the computational tasks required in several array processing applications [6], [7]. Also, in the DOA estimation problem [8], [9], neural network are used in the estimation of the noise subspace necessary for the computation of the MUSIC spectrum by mapping the problem to the quadratic energy function of the network. In this paper, the application of neural networks to handle the computational problem of the DOA estimation step is treated from a different point of view. The DOA problem is approached as a mapping which can be modeled using a suitable artificial neural network trained with input output pairs [10]. The network is then capable of estimating or predicting outputs not included in the learning phase through generalization. Moreover, one of the main advantages of neural networks is that they can be implemented in analog circuits with time constants in the order of nanoseconds [6], [14] and consequently they have fast convergence rates. In Section II, the architecture of a radial-basis function neural network (RBFNN) is presented as well as the input preprocessing and output post-processing. The MUSIC algorithm is briefly described in Section III. In Section IV the training algorithm used in this paper is discussed. Section V presents results obtained from the application of the RBFNN to the DOA estimation for multiple sources with comparisons to the performance of the MUSIC algorithm for uncorrelated and correlated signals.

II. RADIAL-BASIS FUNCTION NEURAL NETWORK

RBFNN's [11], [12] are a member of a class of general-purpose method for approximating nonlinear mappings since the DOA problem is of nonlinear nature. Unlike the backpropagation networks which can be viewed as an application of an optimization problem, RBFNN can be considered as designing neural networks as a curve fitting (or interpolation) problem in a high-dimensional space. The mapping from the input space to the output space may be thought of as a hypersurface Γ representing a multidimensional function of the input. During the training phase, the input-output patterns presented to the network are used to perform a fitting for Γ . The generalization phase represents an interpolation of the input data points along

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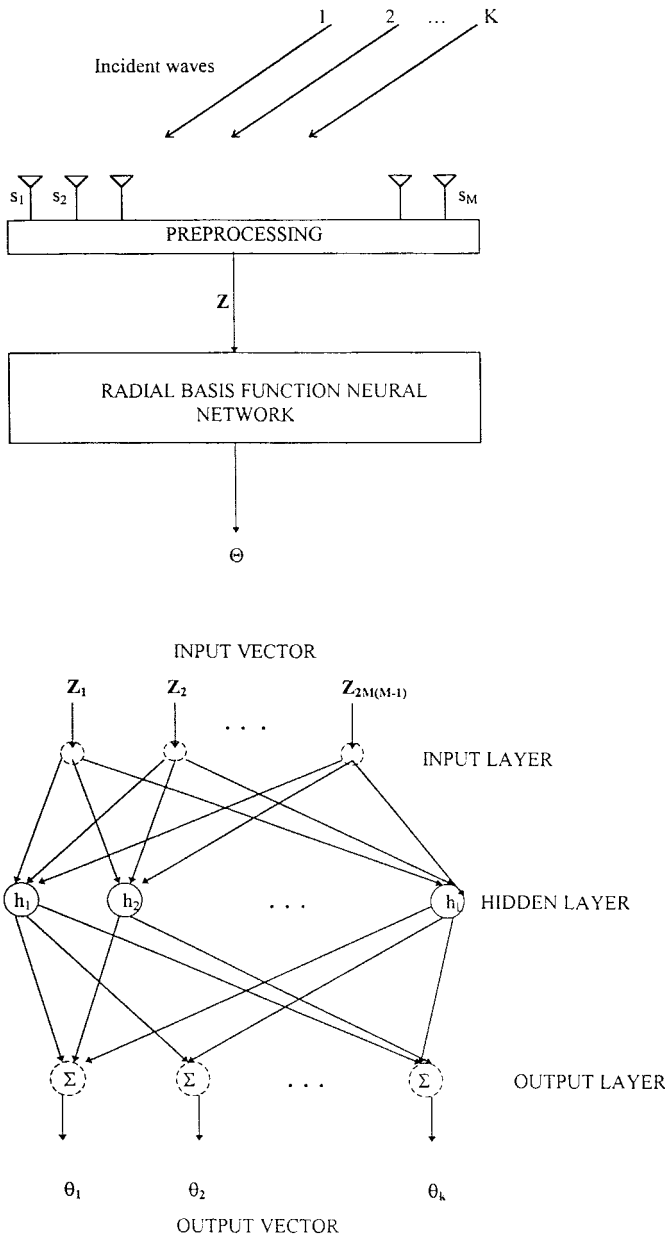


Fig. 1. Architecture of a three-layered radial-basis function network.

the surface built as an approximation for Γ . The architecture considered in this paper involves three layers, the input layer (sensory nodes), a hidden layer of high dimension, and an output layer, as shown in Fig. 1. The transformation from the input space to the hidden-unit space is nonlinear, whereas the transformation from the hidden layer to the output space is linear.

The array performs the mapping $G: \mathbf{R}^K \rightarrow \mathbf{C}^M$ from the space of DOA, $\{\theta = [\theta_1, \theta_2, \dots, \theta_k]\}$ to the space of sensor output $\{\mathbf{s} = [s_1, s_2, \dots, s_M]\}$, namely

$$S_m = \sum_{k=1}^K a_k e^{j(m(\omega_0/c)d \sin \theta_k + \alpha_k)} \quad (1)$$

where K is the number of signals, M is the number of elements of a linear array, a_k represents the complex amplitude of the k th signal, α_k the initial phase and ω_0 is the center

frequency. Based on the information theoretic criteria for model selection [13], one can estimate the number of signals K *a priori*. A neural network approach to this problem may be the subject of further investigation. A neural network is used to perform the inverse mapping $F: \mathbf{C}^M \rightarrow \mathbf{R}^K$. The network is to be trained by N patterns generated from (1) so that it can associate the output vectors $\mathbf{s}(1), \mathbf{s}(2), \dots, \mathbf{s}(N)$ with the corresponding DOA vectors $\theta(1), \theta(2), \dots, \theta(N)$. Input vectors \mathbf{s} are mapped through the hidden layer then each output node computes a weighted sum of the hidden layer outputs. Thus, we can write for a set of data $\{(s(i), \theta(i)), i = 1, 2, \dots, N\}$

$$\theta_k(j) = \sum_{i=1}^N w_i^k h(\|s(j) - s(i)\|^2) \quad k = 1, \dots, K, \quad j = 1, \dots, N \quad (2)$$

where w_i^k represents the i th weight of the network. Using the Gaussian function for h we can rewrite (2) as

$$\theta_k(j) = \sum_{i=1}^N w_i^k e^{-\|s(j) - s(i)\|^2 / \sigma_g^2} \quad (3)$$

The parameter σ_g controls the influence of each basis function. Using matrix notation (3) becomes

$$\Theta = \mathbf{W}\mathbf{H} \quad (4)$$

where Θ and \mathbf{W} (the weight matrix) are $K \times N$ matrices and \mathbf{H} is $N \times N$ matrix.

Since a large matrix is highly likely to be ill-conditioned, the dimension of \mathbf{H} may be reduced by selecting the number of centers $s(i)$ to be lower than the number of data points. Let the number of centers be L where $L < N$; it follows that \mathbf{W} and \mathbf{H} are now $K \times L$ and $L \times N$ matrices. To derive the optimal solution for the network weights the least squares (LS) approach can be used [10] to obtain

$$\hat{\mathbf{W}} = \Theta \mathbf{H}^+ \quad (5)$$

where \mathbf{H}^+ is the pseudo-inverse given by

$$\mathbf{H}^+ = \mathbf{H}^T (\mathbf{H}\mathbf{H}^T)^{-1} \quad (6)$$

The estimate of the DOA can thus be given as

$$\hat{\theta} = \hat{\mathbf{W}}\mathbf{H} = \theta^T (\mathbf{H}\mathbf{H}^T)^{-1} \mathbf{H} \quad (7)$$

A. Data Preprocessing

First, the array output vectors are generated then transformed into appropriate input vectors to be presented to the network. The estimation phase consists of transforming the sensor output vector into an input vector and producing the DOA estimate. Since in the DOA problem, the initial phase α contains no information about the direction of the incoming signals, it is eliminated from the training data by forming the spatial correlation matrix \mathbf{R}

$$R_{mm'} = \sum_{k=1}^K p_k e^{j(m-m')\omega_0 d \sin \theta_k / c} + \delta R_{mm'} \quad (8)$$

The last term of the right-hand side of this equation contains all the cross-correlated terms between signals. Since for $m = m'$ R_{mm} does not carry any information on the DOA ($R_{mm} = \sum_{k=1}^K p_k$), we can rearrange the rest of the elements into a new input vector b given as

$$b = [R_{21}, \dots, R_{M2}, R_{12}, \dots, R_{M2}, R_{1M} \dots, R_{M(M-1)}]^T. \quad (9)$$

It follows that the number of input units is given by $M(M-1)$. Note that we need twice as many input nodes for the neural network since it does not deal with complex numbers. Hence, the total number of input nodes needed is $2M(M-1)$. The dimension of the hidden layer is equal to the number of the Gaussian functions L that can be chosen to be equal to N if perfect recall is desired. Obviously, the number of output node is equal to the number of signals K . In the simulations performed later, the relative signal power is taken as unity though different power levels do not affect the procedure of detecting the DOA. The input vector is then normalized by its norm in the training, testing, and estimation phases, i.e.,

$$z = \frac{b}{\|b\|}. \quad (10)$$

B. Network Training

- 1) Generate array output vectors $\{s(n), n = 1, 2, \dots, N\}$.
- 2) Evaluate the correlation matrix of the n th array output vector $\{R(n), n = 1, 2, \dots, N\}$.
- 3) Form the vectors $\{b(n), n = 1, 2, \dots, N\}$.
- 4) Normalize the input vectors using (4).
- 5) Generate the training set $\{b(n), (n), n = 1, 2, \dots, N\}$.
- 6) Employ an appropriate RBFNN training procedure to learn the training set generated in step 5).

The main advantage of using an RBFNN over other approaches is that it does not require training the network with all possible combinations of input vectors. For the network to generalize it is sufficient to perform the training with vectors that span the expected range of input data, e.g., uniformly distributed from -90 to $+90$ in the simulations reported in this paper.

C. DOA Estimation or Generalization Phase

- 1) Evaluate the sample correlation matrix using the collected array output measurements.
- 2) Form the vectors.
- 3) Produce the normalized input vectors.
- 4) Present input vectors to the RBFNN and obtain the estimate of DOA.

III. MUSIC ALGORITHM

Assuming that the signals received at the different sensors are contaminated with statistically independent white noise of variance σ^2 , it follows that the received spatial correlation matrix \mathbf{R} of the noisy signals can be rewritten as

$$\mathbf{R} = \mathbf{A}\mathbf{P}\mathbf{A}^H + \sigma^2\mathbf{I} = \sum_{i=1}^M \lambda_i e_i e_i^H \quad (11)$$

with $\mathbf{P} = E\{\mathbf{s}\mathbf{s}^H\}$ is the signal covariance matrix, the superscript “ H ” denotes the conjugate transpose, and \mathbf{I} is the unit matrix. Note that \mathbf{P} has dimension $K \times K$, while \mathbf{R} has dimension $M \times M$, $\lambda_1 \geq \lambda_2 \geq \lambda_k > \lambda_{k+1} = \dots = \lambda_M = \sigma^2$ are the eigenvalues of \mathbf{R} and e_i are its orthonormal eigenvectors. The eigenvectors corresponding to the first K largest eigenvalues are referred to as the signal eigenvectors and those corresponding to the minimum eigenvalues are referred to as the noise eigenvectors. The subspace spanned by the signal eigenvectors is called the *signal subspace*, and its orthogonal complement spanned by the noise eigenvectors is called the *noise subspace*. The matrix $\mathbf{R} - \sigma^2\mathbf{I} = \mathbf{A}\mathbf{P}\mathbf{A}^H$ has the same eigenvectors as \mathbf{R} with eigenvalues $\lambda_i - \sigma^2$ for $i = 1, 2, \dots, K$, and $\lambda_i = 0$ for $i > K$. It follows that

$$\mathbf{A}\mathbf{P}\mathbf{A}^H = \sum_{i=1}^K (\lambda_i - \sigma^2) e_i e_i^H. \quad (12)$$

Therefore, the signal direction vectors and the signal eigenvectors span the same subspace. This implies that all signal direction vectors are orthogonal to the noise subspace. The MUSIC algorithm estimates the DOA of the K signals by finding the values of θ corresponding to the K maxima of the function

$$S_{\text{MUSIC}} = \frac{1}{\mathbf{A}^H \mathbf{V} \mathbf{V}^H \mathbf{A}} \quad (13)$$

where \mathbf{V} is the $M \times M - K$ matrix whose columns are the $M - K$ eigenvectors spanning the noise subspace of \mathbf{R} , i.e.,

$$\mathbf{V} = [e_{K+1} \quad e_{K+2} \quad \dots \quad e_M]. \quad (14)$$

IV. NORMALIZED CUMULATIVE DELTA RULE

After experimenting with various learning algorithms, the Norm-Cum [11] was used to perform the training. In the standard delta rule the error is backpropagated to prior layers where it is accumulated until the first layer is reached and then the weights are updated after each training presentation. A momentum term is used to smooth out the weight changes. In the Norm-Cum rule, the weight changes are accumulated over several training presentations (specified by the Epoch) and the application of the weight updates is made all at once. When a learning counter reaches an integer multiple of the accumulation period in the epoch, the accumulated weight changes are applied to the connecting weight. The learning rate is normalized (divided by the square root of the epoch size).

V. SIMULATION RESULTS

A. Uncorrelated Signals

In the simulations performed, an array of $M = 6$ elements is used, therefore, the dimension of the input layer was set to 60 nodes. A hidden layer of 50 nodes was chosen. In Fig. 2, the array receives two uncorrelated signals with different angular separations ($\Delta\theta = 2^\circ$ and 5°) where the DOA were assumed to be uniformly distributed from -90° to $+90^\circ$ in both the training and testing phases. Two hundred input vectors were used for training. For the testing phase 50 input vectors were

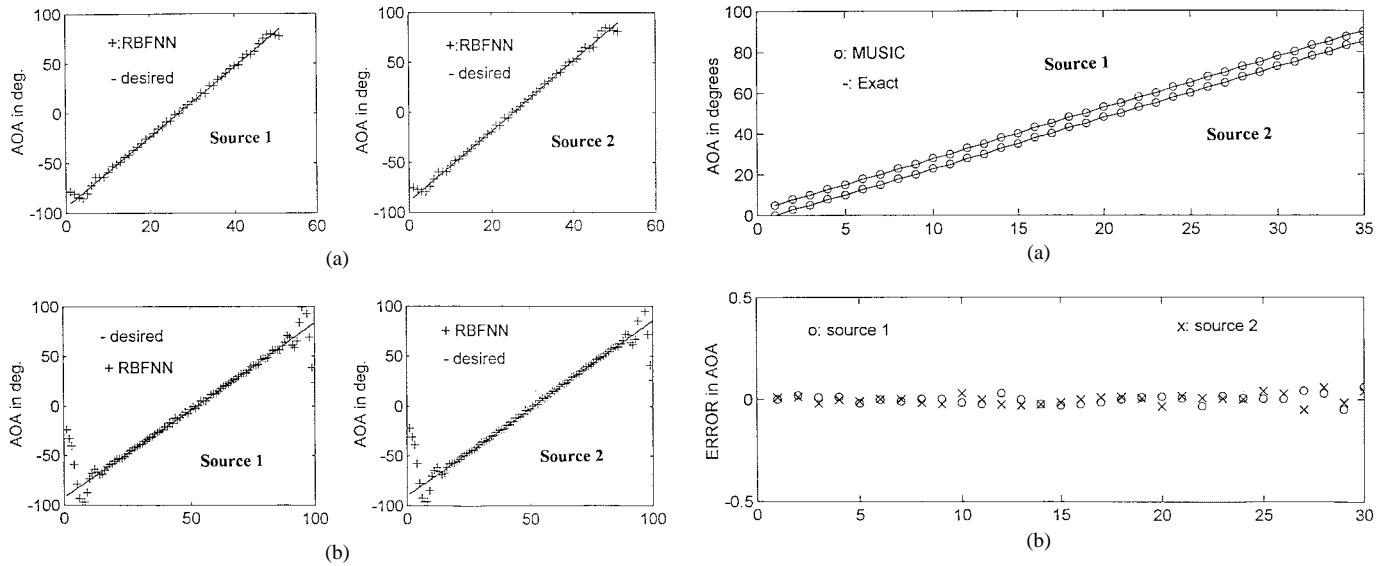


Fig. 2. DOA estimate versus number of samples N using RBFNN. Source 1 is varied from -90 to 90 , while source 2 is 5° and 2° separated from source 1.

used for the network simulated with $\Delta\theta = 5^\circ$ and 100 input vectors for all the rest of the networks. For all networks a learning coefficient of 0.3 was used for the hidden layer and 0.15 for the output layer while the epoch size was set to 16. The width σ_g of the Gaussian transfer function is set as the root mean square (rms) distance of a specific cluster center to the nearest neighbor cluster center(s). The results show that the network successfully produced actual outputs (+) very close to the desired DOA (dotted). DOA obtained from the MUSIC algorithm are shown in Fig. 3 and compared to those obtained from the RBFNN method for $\Delta\theta = 5^\circ$. Also, the error in the DOA estimate of the two incoming signals (sources) is plotted. Fig. 4 shows the results obtained from MUSIC in the case of 2° angular separation. It can be concluded from Fig. 3 that the performance of the RBFNN method approaches that of the MUSIC algorithm. Fig. 5 shows a network trained with input vectors generated from two signals with angular separation of 3° and tested with a set of data generated from signals with $\Delta\theta = 1.5^\circ$. This shows that the network improved its performance through generalization and yielded satisfactory results. Since the maximum number of signals that an array can resolve is bounded by the number of its elements, a network with six output nodes was trained and tested with six signals incoming from sources at different angular separations. The performance of this network is shown in Fig. 6.

B. Correlated and Coherent Sources

In many applications, the signals received by the array are correlated or coherent (perfectly correlated). To study the effect of such cases on the performance of the neural network, the training data was generated assuming the array receives two signals with angular separation of 10° . A correlation coefficient γ was assumed with a signal covariance matrix (or the power matrix) in case of two sources given by

$$P = \begin{pmatrix} p & \gamma^* p \\ \gamma p & |\gamma|^2 p \end{pmatrix}. \quad (15)$$

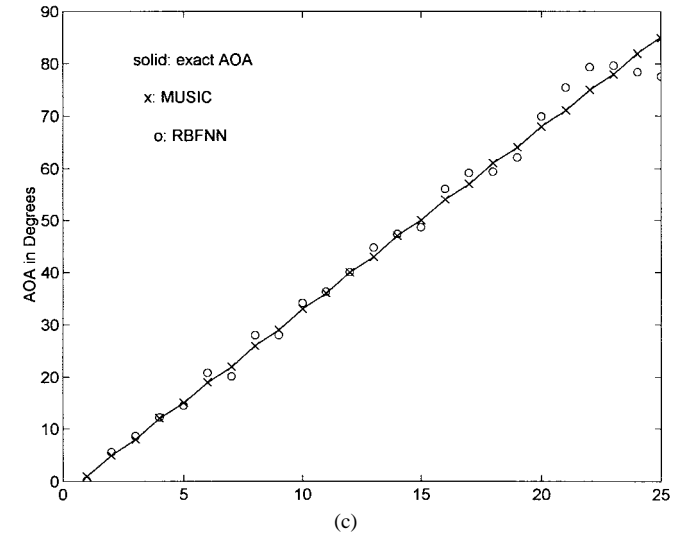


Fig. 3. (a) DOA estimates versus number of samples N using MUSIC for an array of six elements and $\Delta\theta = 5^\circ$. (b) Error in the MUSIC estimates for the two signals versus N . (c) Comparison between MUSIC and RBFNN estimates for an array of six elements and $\Delta\theta = 5^\circ$.

Moreover, the training was performed with data derived from ideal signals (assuming the absence of noise) whereas the testing was performed with data contaminated with additive Gaussian noise to simulate real measurements. For comparison, DOA obtained from MUSIC and RBFNN as well as the error in DOA estimation for correlated signals are plotted in Figs. 7 and 8, respectively. The RBFNN outperformed the conventional MUSIC yielding smaller error. In this case, the correlation matrix approaches a singular matrix. Although the performance of the MUSIC algorithm under correlated signal environment can be improved using preprocessing scheme such as spatial smoothing, this technique involves additional computational complexity to the algorithm, whereas the RBFNN approach dealt with this situation simply by taking into consideration the correlation between incoming signals when the correlation matrix R was generated for training. The case of coherent signals is shown in Fig. 9 with $\gamma = 1e^{j\pi/4}$. To investigate the effect of the number of nodes

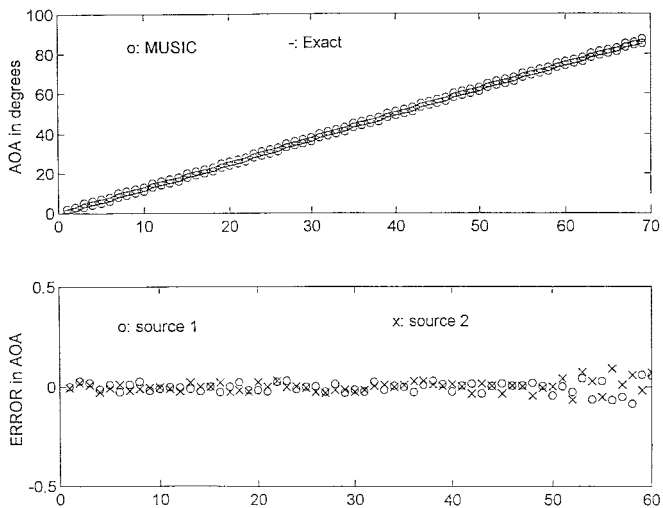


Fig. 4. DOA estimates and respective errors versus number of samples N with MUSIC algorithm for $\Delta\theta = 2^\circ$.

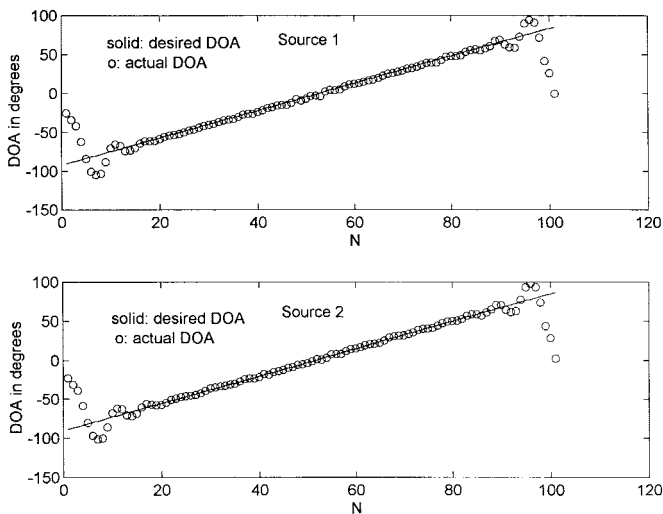


Fig. 5. RBFNN estimates for two sources with $\Delta\theta = 3^\circ$ for training and 1.5° for testing versus number of samples N .

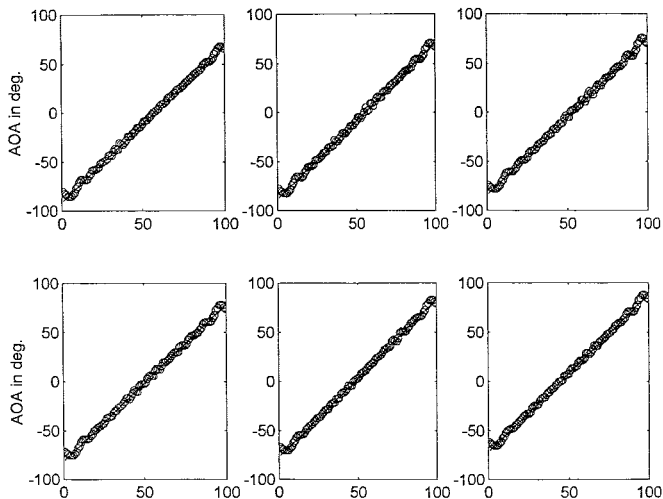


Fig. 6. RBFNN DOA estimates for an array of six elements with six uncorrelated sources. --- : Exact DOA; \circ : RBFNN.

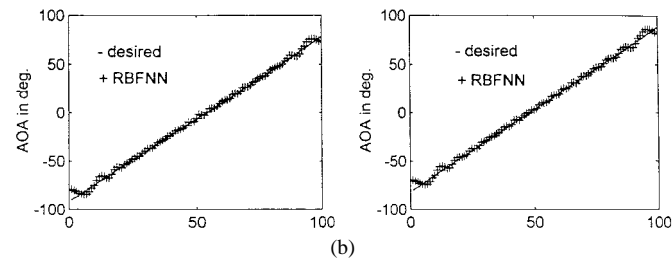
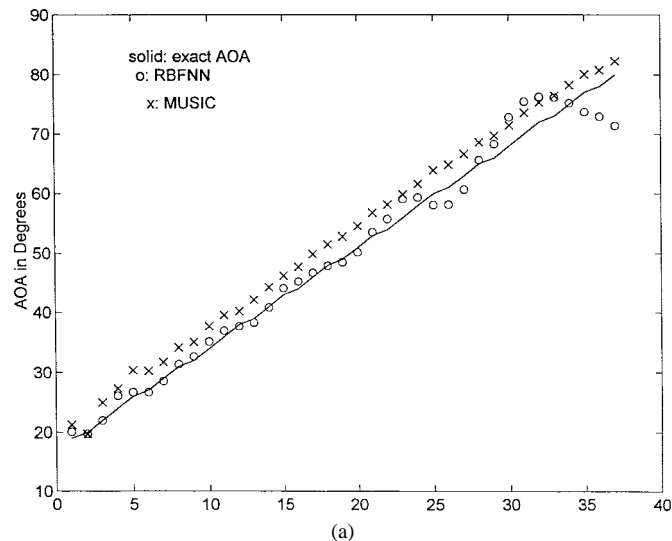


Fig. 7. (a) DOA estimate for an array of six elements with two correlated sources with $\gamma = 0.8e^{j\pi/3}$, \times : MUSIC; \circ : RBFNN --- : Exact DOA. $\Delta\theta = 10^\circ$. (b) DOA estimate for an array of six elements with two correlated sources with $\gamma = 0.8e^{j\pi/3}$, $+$: RBFNN --- : Exact DOA $\Delta\theta = 10^\circ$.

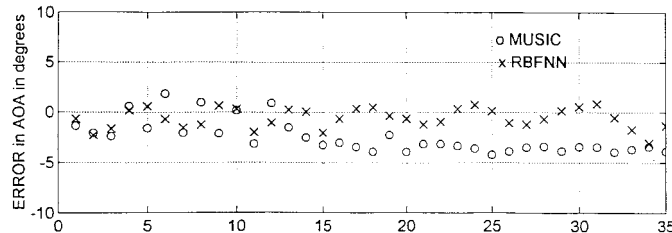


Fig. 8. Comparison between the error in MUSIC and RBFNN DOA estimates for two correlated signals $\gamma = 0.8e^{j\pi/3}$, $\Delta\theta = 10^\circ$.

of the hidden layer the network was trained using 50 and 100 nodes. It was expected that increasing the dimension of the hidden layer may improve the interpolation performed by the RBFNN by moving to higher dimensional spaces, however the ability of the network to produce estimates closer to the desired DOA was not improved dramatically when the number of units was increased from 50 to 100 as shown in Fig. 10. In Fig. 11, the CPU time taken by the MUSIC algorithm to perform the eigendecomposition and obtain the spectrum is plotted as a function of N —the number of different pairs of sources. For $N = 50$ and 100, the RBFNN needed less than a second to estimate the DOA.

VI. CONCLUSION

The problem of DOA estimation is dealt with as a nonlinear mapping from the space of sensor output to that of the angles

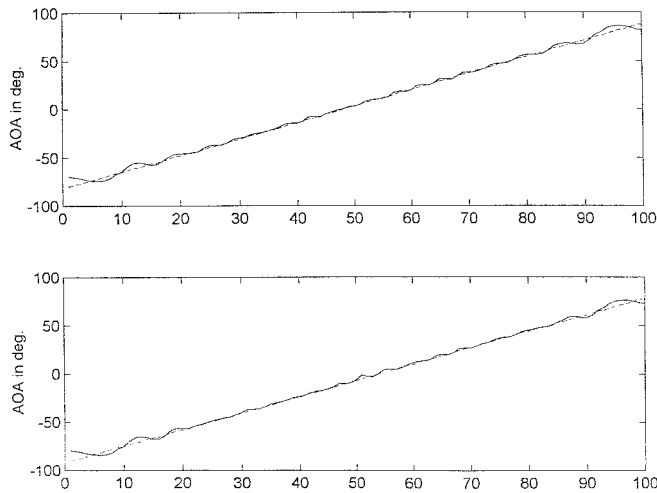


Fig. 9. RBFNN DOA estimate for two coherent signals $\Delta\theta = 10^\circ$
 $\gamma = 1e^{j\pi/4}$.

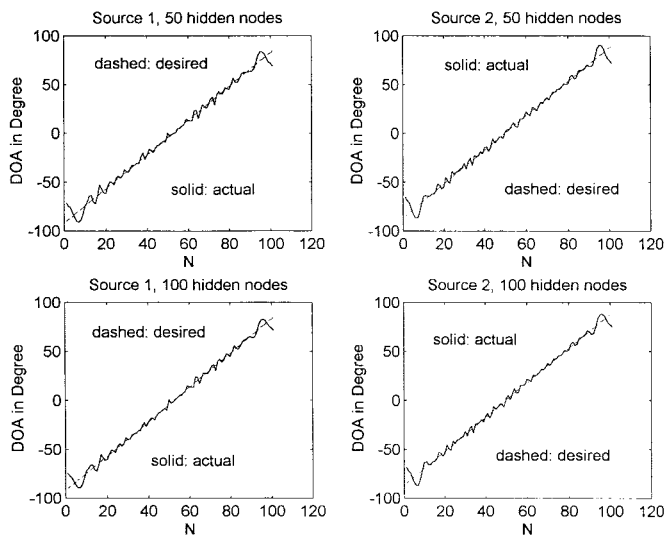


Fig. 10. Effect of the dimension of the hidden layer on the performance of the RBFNN.

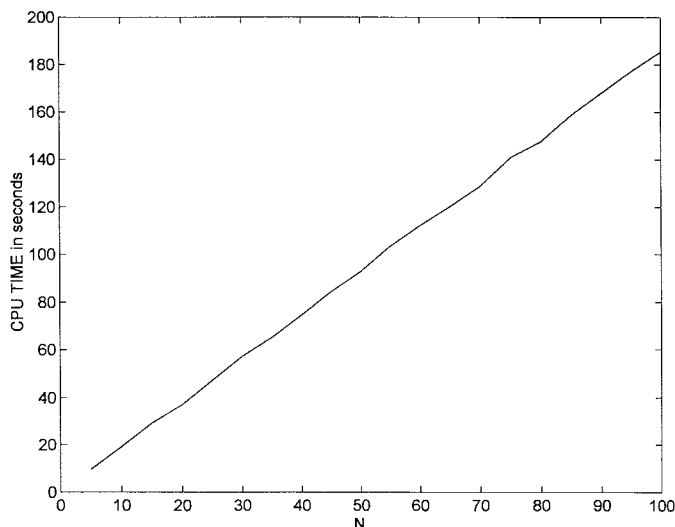


Fig. 11. CPU time required by the MUSIC algorithm as function of number of samples N .

θ . In this paper, the neural network approach was chosen to solve this problem. In particular, RBFNN were used due to their ability for data interpolation in higher dimensions. It was found that networks implementing these functions were indeed successful in performing the required task and yielded good performance in the sense that the network produced actual output very close to the desired DOA. Also it was demonstrated that these networks are able to generalize, by training and testing using data sets derived from different signal conditions mainly with the effect of noise added to the data used for testing. The main advantage of the RBFNN is the substantial reduction in the CPU time needed to estimate the DOA.

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