

# **A novel approach to adaptive nulling with neural networks**

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## **Abstract**

A neural network approach to the problem of finding the weights of one and two-dimensional adaptive arrays is presented. In modern cellular, satellite mobile communications systems, and in GPS systems, both desired and interfering signals change their directions continuously. Therefore, a fast tracking system is needed to constantly track the users, and then adapt the radiation pattern of the antenna to direct multiple narrow beams to desired users and nulls to sources of interference. In the approach suggested in this paper, the computation of the optimum weights is viewed as a mapping problem which can be modeled using a suitable artificial neural network trained with input output pairs. A three-layer Radial Basis Function Neural Networks (RBFNN) are used in the design of one and two-dimensional array antennas. The results obtained from this network are in excellent agreement with the Wiener solution. It was found that networks implementing these functions are successful in tracking mobile users as they move across the antenna's field of view.

## **I. Introduction**

Interference rejection often represents an inexpensive way to increase the system capacity by allowing closer proximity of cofrequency cells or beams providing additional frequency reuse [1] in a cellular system. This paper presents the development of a neural network-based algorithm to compute the weights of an adaptive array antenna. In this new approach, the adaptive array can detect and estimate mobile users' locations [2],[3]), track these mobiles as they move within or between cells, and allocate narrow beams in the directions of the desired users while simultaneously nulling unwanted sources of interference. This adaptive antenna results in an increased system capacity for the existing cellular and mobile communications systems as well as improved interference rejection capabilities for satellite-based Personal Communication Systems (PCS), geosynchronous satellites, Low Earth Orbit satellites, and Global Positioning Systems (GPS). The organization of the paper is as follows: In section II a brief derivation of the optimum array weights in 1-D adaptive beamforming is presented. The RBFNN approach for the computation of the adaptive array weights is introduced in section III. Finally, Section IV presents the simulation results and Section V offers conclusive remarks.

## **II. 1-D Adaptive beamforming**

Using vector notation we can write the output of an M-element linear array or an M x N planar array on the matrix form:

$$\mathbf{X}(t) = \mathbf{A} \mathbf{S}(t) + \mathbf{N}(t) \quad (1)$$

And the received spatial correlation matrix,  $\mathbf{R}$ , of the received noisy signals can be expressed as:

$$\begin{aligned}\mathbf{R} &= E\{\mathbf{X}(t)\mathbf{X}(t)^H\} = A E[\mathbf{S}(t)\mathbf{S}^H(t)]A^H + E[\mathbf{N}(t)\mathbf{N}^H(t)] \\ &= APA^H + \sigma^2 I = \sum_{i=1}^M \lambda_i \mathbf{e}_i \mathbf{e}_i^H\end{aligned}\quad (2)$$

To derive the optimal weight vector, the array output is minimized so that the desired signals are received with specific gain, while the contributions due to noise and interference are minimized [4]. In other words:

$$\min \mathbf{W}^H \mathbf{R} \mathbf{W} \quad \text{subject to } \mathbf{W}^H \mathbf{S}_d = \mathbf{r} \quad (3)$$

In the above equation,  $\mathbf{W}$  is the vector of the weights of the array element outputs,  $\mathbf{r}$  is the  $V \times 1$  constraint vector, where  $V$  is the number of desired signals, and  $\mathbf{S}_d$  is the steering vector associated with the look direction. It can be shown that the optimum weight vector is given by the following equation:

$$\hat{\mathbf{W}}_{opt} = \mathbf{R}^{-1} \mathbf{S}_d \left[ \mathbf{S}_d^H \mathbf{R}^{-1} \mathbf{S}_d \right]^{-1} \mathbf{r} \quad (4)$$

Since the above equation is not practical for real time implementation, an adaptive algorithm must be used to adapt the weights of the array in order to track the desired signal and to place nulls in the direction of the interfering signals.

### III. Neural Network –based interference cancellation:

This section describes a new implementation for the problem of beamforming using neural networks. The optimum weight vector is a nonlinear function of the correlation matrix and the constraint matrix. Therefore it can be approximated using a suitable architecture such as a Radial Basis Function Neural Network. Note that a Radial Basis Function Neural Network can approximate an arbitrary function from an input space of arbitrary dimensionality to an output space of arbitrary dimensionality. The RBFNN consists of three layers of nodes; the input layer, the output layer and the hidden layer as shown in figure 1. The input layer is the layer where the inputs are applied; the output layer is the layer where the outputs are produced. In our application the input layer consists of  $J=2M$  nodes for an  $M$  element linear array, to accommodate both the real and the imaginary part of the input vector (i.e.,  $\mathbf{X}(t)$ ). The output layer consists of  $2M$  nodes to accommodate the output vector (i.e.,  $\mathbf{W}_{opt}$ ). As it is the case, with most neural networks the RBFNN is designed to perform an input-output mapping trained with examples  $(\mathbf{X}^l(t); \mathbf{W}_{opt}^l); l = 1, 2, \dots, N_T$ , where  $N_T$  stands for the number of examples contained in the training set. The purpose of the hidden layer in a RBFNN is to transform the input data  $\mathbf{X}(t)$  from an input space of dimensionality  $J$  to a space of higher dimensionality  $L$ . The rationale behind this transformation is based on Cover's theorem which states that an input/output mapping problem cast in a high-dimensionality space nonlinearly is easier to solve. The nonlinear functions that perform this transformation are usually taken to be Gaussian functions of appropriately chosen means and variances.

#### Generation of Training Data

1. Generate array output vectors  $\{\mathbf{X}^l(t); \mathbf{W}_{opt}^l; l = 1, 2, \dots, N_T\}$  using equation (1).
2. Normalize each one of the above array output vectors by its norm. For simplicity of notation we still refer to these vectors by  $\mathbf{X}(t)$ 's.
3. Evaluate the correlation matrix  $\mathbf{R}^l (l = 1, 2, \dots, N_T)$  for each of the array output vectors generated in Step 1; to do so use equation (2). Using the calculated  $\mathbf{R}^l$ 's calculate the vectors  $\{\mathbf{W}_{opt}^l; l = 1, 2, \dots, N_T\}$  from equation (4).

4. Produce the required training input/output pairs of the training set, that is  $\{(X^l(t); \mathbf{W}_{opt}^l) ; l = 1, 2, \dots, N_T\}$

As we have emphasized before, once the RBFNN is trained with a representative set of training input/output pairs it is ready to function in the performance phase. In the performance phase, the RBFNN produces estimates of the optimum weights for the array outputs, through a simple, computationally inexpensive, two-step process, described below.

#### Performance Phase of the RBFNN

1. Generate the array output vector  $\hat{\mathbf{X}}(t)$ . Normalize this array output vector by its norm.
2. Present the normalized array output vector at the input layer of the trained RBFNN. The output layer of the trained RBFNN will produce as an output the estimates of optimum weights for the array outputs (i.e.,  $\hat{\mathbf{W}}_{opt}$ ).

### V. Simulation results

Figure 2 shows the adapted pattern of a 10 element linear array obtained from the RBFNN and how it compares with the optimum Wiener solution for angular signal separations of  $15^\circ$  and  $10^\circ$ , respectively. It can be concluded from these figures that the RBFNN produced a solution for the beamforming weight vector that is very close to the optimum solution.

### VI. Conclusion

A new approach to the problem of adaptive beamforming was introduced. The weights were computed using an RBFNN that approximates the Wiener solution. The network was successful in tracking multiple users while simultaneously nulling interference caused by cochannel users.

### VII. References

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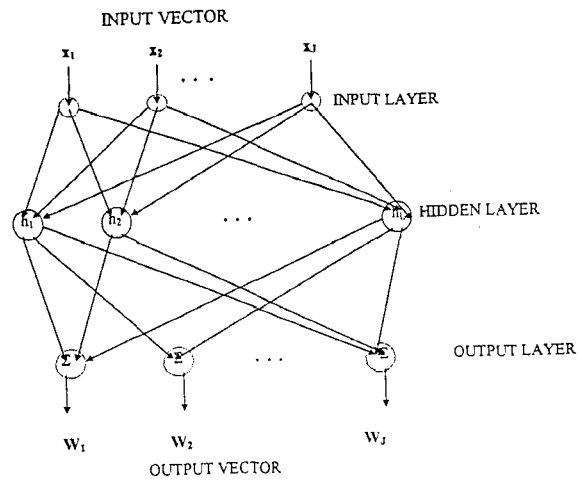


Figure 1 RBFNN architecture

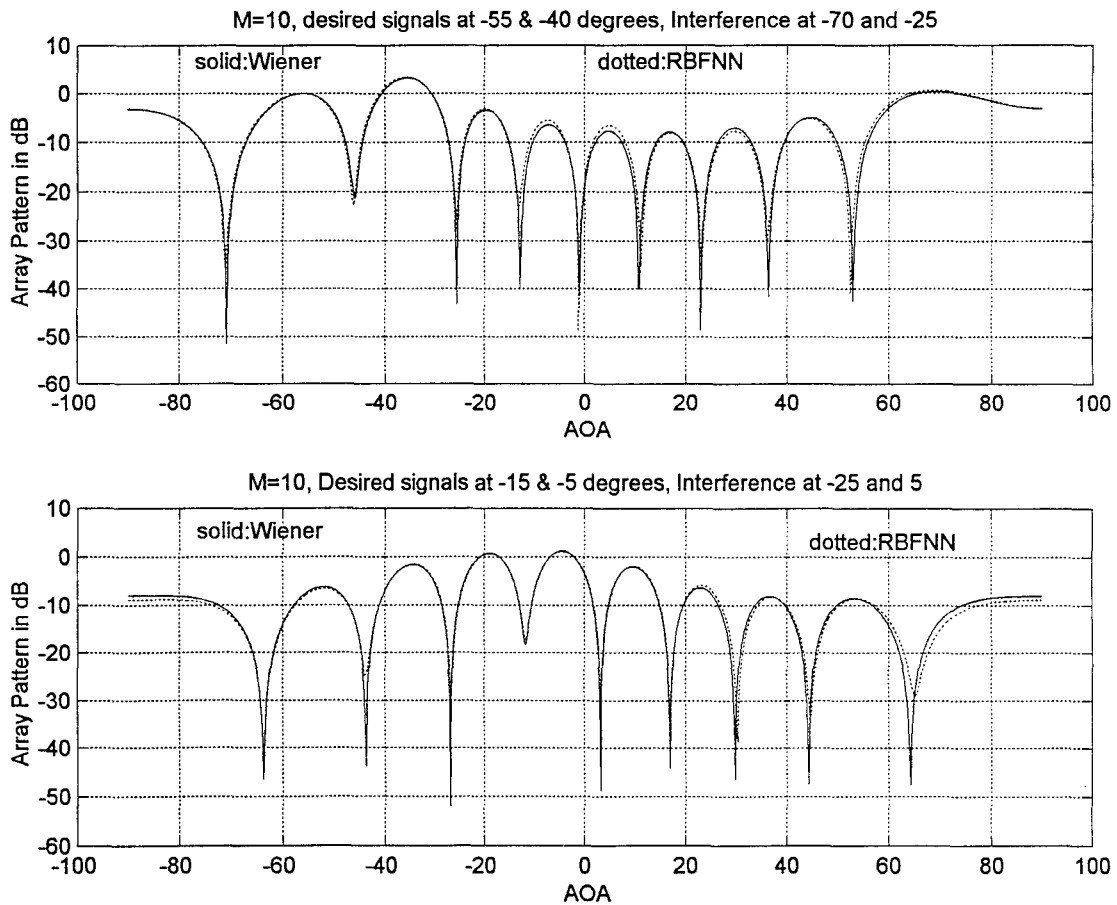


Figure 2 10 element linear array with four sources