New Geometrical Concepts in Fuzzy-ART and Fuzzy-ARTMAP: Category Regions

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Abstract

We introduce new geometric concepts regarding categories in Fuzzy ART (FA) and Fuzzy ARTMAP (FAM), which add a geometric facet to the process of node selection in the F_2 layer by patterns. Apart from providing the means to better understand the training and performance phase of these two architectures, the new concepts, namely the category regions, lead us to interesting theoretical results, when training either architecture. First, we define the Commitment Test as a novelty detection mechanism similar to the Vigilance Test. Next we define various category regions. Via those definitions and 3 derived lemmas we identify areas in the vigilance-choice parameter space, for which 4 results are stated that are applicable to both FA and the FAM classifier training phase.

1 Introduction

Fuzzy-ART (FA) [1] and *Fuzzy-ARTMAP* (FAM) [2] are two self-organizing, neural network architectures with roots in the *adaptive resonance theory* developed by Grossberg [3]. FA uses *categories* for data summarization and unsupervised learning to accomplish exemplar-based clustering. On the other hand, FAM consists of two FA networks (modules) interconnected via an inter-ART module and is intended via supervised learning to form one-to-one associations between clusters of an input and an output domain. As a special case, when the output domain is a discrete set of class labels, FAM is capable of performing classification tasks. The geometric representations of FA categories are hyper-rectangles embedded in the module's input domain and may overlap with each other or even contain each other. Input patterns contained in the same hyper-rectangle are assumed to belong to the same cluster (category). The number of categories created during learning is determined in a self-organizing manner and is influenced by the module's parameters and the evidence sampled from the module's input space. FA and FAM architectures feature some interesting properties. First, it is possible to train both in an on-line (incremental) or an off-line (batch) fashion. Under *fast learning* rule [1], when performing off-line training, both architectures feature fast, stable, finite learning, that is, training completes after a finite number of list presentations (epochs). However, using the aforementioned learning mode, the networks suffer the risk of over-fitting, since no noise removal mechanism supports the learning process [4]. Learning in a FA module occurs by forming new categories or by updating preexisting categories in light of new information about the input space. Also, FA modules feature novelty detection mechanisms that identify patterns not typical of previously experienced inputs. It is those patterns that will initiate the creation of new categories to explain the recently acquired evidence. A characteristic of FA modules is that, on occasion, a small fraction of all categories created during training may prove redundant. Also, there are some interesting features pertaining to the performance phase of FA modules. First, owing to the network architecture, it is easy to explain a module's output as a response to a stimulus, which comes in contrast to other neural architectures, like feed-forward networks, where, in general, it is hard to explain why a particular input pattern **x** generated a response **y**. Secondly, the performance phase is identical (as a computational process) to the training phase with the exception that no creation of new categories and no category updates are permitted. The interested reader will find more, detailed properties of learning and performance regarding FA modules in the

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original references [1] and [2], as well as in [5] and [6]. The structure of this paper is as follows: in Section 2 we will outline some notational conventions used in this paper and provide some limited, but necessary background about FA modules and FA categories. Next, in Section 3 we introduce the concept of the Commitment Test as another novelty detection mechanism in FA modules. Section 4 introduces the reader to the various category regions and arrives to a few useful properties. Results pertaining to FA and FAM classifier are given in Section 5. Finally, in Section 6 we provide a brief summary of our contributions.

2 Fundamentals of Fuzzy-ART modules and categories

Let $U^M \in [0,1]^M$ denote the closure of the M-dimensional unit hyper-cube that serves as an input space for any FA module (network). A block diagram of such a module is featured in Figure 1.

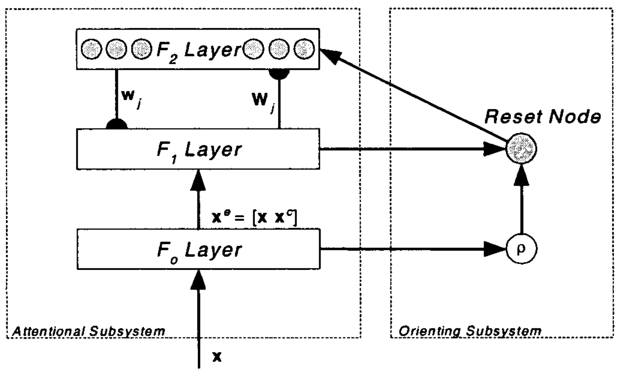

Figure 1: Block diagram of a Fuzzy ART module

Also, let $|\cdot|$ denote the L_1-norm and let $\wedge: U^{2M} \times U^{2M} \to U^{2M}$ denote the *fuzzy-min* operator; if $\mathbf{w}_1, \mathbf{w}_2 \in U^{2M}$ and $\mathbf{w}_3 = \mathbf{w}_1 \wedge \mathbf{w}_2$, then the m^{th} component w_{3m} of vector $\mathbf{w}_3$ is $w_{3m} = \min\{w_{1m}, w_{2m}\}$. If $\mathbf{x} \in U^M$ represents an input pattern, then let $\mathbf{x}^e = [\mathbf{x}\ \mathbf{x}^c] = [\mathbf{x}\ \mathbf{1}\text{-}\mathbf{x}]$ be its complement coded form, where $\mathbf{1}$ is the all-ones vector. All aforementioned and to-be-mentioned vectors are assumed to be row vectors. Complement coding of input patterns occurs in a FA module's F_o layer. The top-down weights $\mathbf{w}_j$ associated to connections emanating from a node j in the F_2 layer and terminating to the nodes of the F_1 layer are called *templates* and are of the form $\mathbf{w}_j = [\mathbf{u}_j\ \mathbf{v}_j^c] \in U^{2M}$, where $\mathbf{u}_j, \mathbf{v}_j \in U^M$. We discriminate between two types of nodes in the F_2 layer: *uncommitted* and *committed.* Uncommitted nodes correspond to the blank memory of the network, in which they participate. Before the commencement of learning, all nodes in the F_2 layer are uncommitted. Uncommitted nodes feature a template of $\mathbf{w}_u = w_u \mathbf{1}$, where $w_u \geq 1$ is a FA module parameter. On the other hand, committed nodes are the only ones that define FA categories, which constitute the system's crystallized knowledge about its input environment. During the training phase of a FA module, uncommitted nodes become committed in an incremental fashion in an effort to explain newly presented evidence. For a committed node j, its associated category is represented by a hyper-rectangle in U_M with corner points $\mathbf{u}_j$ and $\mathbf{v}_j$. An illustration of such an example for a two-dimensional input space is depicted in Figure 2.

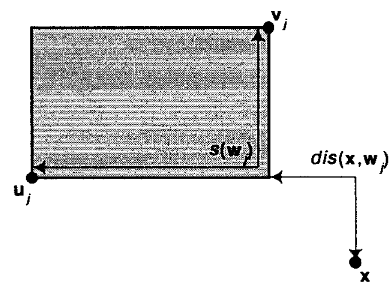

Figure 2: Representation region of a category j in 2 dimensions

The boundaries and the interior of the hyper-rectangle comprise the *representation region* of the related category. A category also corresponds to the hypothesis that all points within its representation region have been observed in the past during training. We continue by defining the *size* $s(\mathbf{w}_j)$ of a category j as

$$s(\mathbf{w}_j) = |\mathbf{v}_j - \mathbf{u}_j| = \sum_{m=1}^{M} (v_{jm} - u_{jm}) \qquad (1)$$

The distance of a pattern $\mathbf{x}$ from category j with template $\mathbf{w}_j$ is defined as

$$dis(\mathbf{x}, \mathbf{w}_j) = \sum_{m=1}^{M} \left[\left(\max\{x_m, v_{jm}\} - v_{jm} \right) + \left(u_{jm} - \min\{x_m, u_{jm}\} \right) \right] \qquad (2)$$

which is the minimum L_1 distance between the pattern and the category's representation region. Notice, that if the pattern is contained in the representation region, the distance is 0. According to the above we can define the representation region of a category j with template $\mathbf{w}_j$ the following subset of U_M:

$$R(\mathbf{w}_j) = \{\mathbf{x} \in U^M \mid \mathbf{x}^e \wedge \mathbf{w}_j = \mathbf{w}_j\} \Leftrightarrow$$
$$R(\mathbf{w}_j) = \{\mathbf{x} \in U^M \mid dis(\mathbf{x}, \mathbf{w}_j) = 0\} \qquad (3)$$

At this point, let us turn our discussion back to FA modules and roughly describe the way they process input patterns. Upon presentation of pattern $\mathbf{x}$, after being complement encoded in the F_o layer, all nodes (committed and uncommitted) in the F_2 layer will compete to select the pattern. All nodes compete in terms of *category choice function* (CCF) value $T(\mathbf{w}|\mathbf{x})$, where

$$T(\mathbf{w}_j \mid \mathbf{x}) = \frac{\left|\mathbf{x}^e \wedge \mathbf{w}_j\right|}{\left|\mathbf{w}_j\right| + a} = \frac{M - s(\mathbf{w}_j) - dis(\mathbf{x}, \mathbf{w}_j)}{M - s(\mathbf{w}_j) + a} \quad (4)$$

Here, $a>0$ is another one of the FA module's parameters, called the *choice parameter*. Uncommitted nodes, in specific, have a constant CCF value for every pattern equal to

$$T(\mathbf{w}_u \mid \mathbf{x}) = T_u = \frac{M}{2Mw_u + a} \quad (5)$$

The node, which features the maximum value of $T(\mathbf{w}_j|\mathbf{x})$, wins the competition for $\mathbf{x}$. In case of a tie, the winning node is the one of lowest index j.

Next, it is tested if $\mathbf{x}$ matches the characteristics of the category that corresponds to the winning node. The *category match function* (CMF) value is being computed as follows

$$\rho(\mathbf{w}_j \mid \mathbf{x}) = \frac{\left|\mathbf{x}^e \wedge \mathbf{w}_j\right|}{M} = \frac{M - s(\mathbf{w}_j) - dis(\mathbf{x}, \mathbf{w}_j)}{M} \quad (6)$$

and is compared to the *vigilance parameter* $\rho \in [0,1]$ as shown below.

$$\rho(\mathbf{w} \mid \mathbf{x}) \geq \rho \quad (7)$$

The above condition is called *vigilance test* (VT). Notice, that uncommitted nodes always pass the VT (inequality 7 is being satisfied), since they feature a constant CMF value equal to $\rho(\mathbf{w}_u|\mathbf{x})=1$. If the winning (committed) node fails the VT, it is being *reset* via the reset node in Figure 1, and the competition process repeats itself without the disqualified node, until a winning node is found that also passes the VT. Assume that the winning node is committed and corresponds to category j. Then, j will expand its representation region according to the following learning law:

$$\mathbf{w}_j^{new} = \gamma\left(\mathbf{x}^e \wedge \mathbf{w}_j^{old}\right) + (1-\gamma)\mathbf{w}_j^{old} \quad (8)$$

In the above equation, $\gamma \in (0,1]$ is the learning rate parameter of the FA module. If the winning node is an uncommitted node, the node becomes committed and its template is initialized to

$$\mathbf{w}_j^{new} = \mathbf{x}^e \quad (9)$$

The learning schema described in Equations 8 and 9 is called *fast-commit slow-recode*, when $\gamma<1$, and *fast learning*, when $\gamma=1$. In the latter case, during off-line learning, it can be shown that the FA module exhibits finite/stable learning, that is, after a finite number of list presentations (epochs) no new category updates will occur and no further uncommitted nodes will get committed.

3 The Commitment Test

The previously described VT simultaneously acts as a node-filtering mechanism and as novelty detector. The committed nodes are scanned to identify one that not only exhibits the highest CCF value, but also one that could explain the presence of the pattern being presented. However, the VT is not the only novelty detection device utilized by a FA module. To demonstrate this statement, assume that $\rho=0$, so all nodes in the F_2 layer pass the VT (in essence, VT's node-filtering mechanism is being disabled). There will be occasions, where the node competition process yields an uncommitted node as the winner, which means that no suitable category was found in the FA module to predict the presented pattern. Therefore, the competition of a committed node against an uncommitted can be viewed as an implicit novelty detection mechanism similar in role to the VT.

Definition 1

We define as *commitment test* (CT) the comparison of a node j's CCF value to the CCF value of an uncommitted node

$$T(\mathbf{w}_j \mid \mathbf{x}) \geq T_u \quad (10)$$

Node j passes the CT, if the above inequality is satisfied.

It becomes obvious that a pattern $\mathbf{x}$ will not choose a category j when presented, if j fails either the VT or the CT or both.

4 Category Regions

In Section 2 we already defined a category region, namely a category's representation region. However, the geometric interpretation of the VT and the CT give rise to another 2 useful category regions.

Definition 2

We define as *category match (vigilance) region* $V(\mathbf{w}_j|\rho)$ of a category j with template $\mathbf{w}_j$ for a particular value ρ of the vigilance parameter the following subset of U^M

$$V(\mathbf{w}_j \mid \rho) = \left\{\mathbf{x} \in U_M \mid \rho(\mathbf{w}_j \mid \mathbf{x}) \geq \rho\right\} \Leftrightarrow \left\{\begin{array}{l} V(\mathbf{w}_j \mid \rho) = \left\{\mathbf{x} \in U_M \mid dis(\mathbf{x}, \mathbf{w}_j) \leq d_V(\mathbf{w}_j \mid \rho)\right\} \\ d_V(\mathbf{w}_j \mid \rho) = M(1-\rho) - s(\mathbf{w}_j) \end{array}\right\} \quad (11)$$

By definition, a category's match region contains all the patterns of the input space, for which the category would pass the VT. An example of a match region for a 2-dimensional input domain is depicted in Figure 3. The quantity $d_V(\mathbf{w}_j|\rho)$ is called the *radius* of the region. All the regions introduced in the sequel feature the same shape as the match region and define similar radii. Notice, that the representation region is always going to be a subset of almost all category regions.

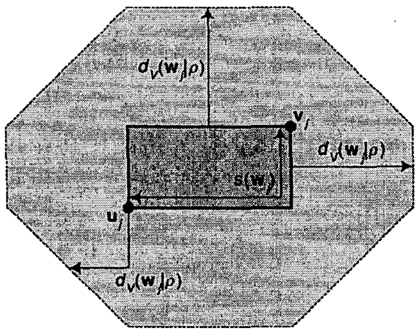

Figure 3: A typical match region in a 2-dimensional input space.

Definition 3

We define as *category choice (commitment) region* $C(\mathbf{w}_j|a,w_u)$ of a category j with template $\mathbf{w}_j$ for particular values a of the choice parameter and w_u the subset of U_M

$$C(\mathbf{w}_j \mid a, w_u) = \{\mathbf{x} \in U_M \mid T(\mathbf{w}_j \mid \mathbf{x}) \geq T_u\} \Leftrightarrow$$

$$\left\{\begin{array}{l} C(\mathbf{w}_j \mid a, w_u) = \{\mathbf{x} \in U_M \mid dis(\mathbf{x}, \mathbf{w}_j) \leq d_C(\mathbf{w}_j \mid a, w_u)\} \\ d_C(\mathbf{w}_j \mid a, w_u) = \dfrac{(2w_u - 1)M + a}{2Mw_u + a}\left[\dfrac{(2w_u - 1)M^2}{(2w_u - 1)M + a} - s(\mathbf{w}_j)\right] \end{array}\right\} \quad (12)$$

As was the case with the match region, the choice region of a category includes all input patterns, for which the category would have passed the CT, i.e. win the competition against an uncommitted node. It is interesting to observe that the similar nature of the CT and the VT as node-filtering process components is also being reflected by the similarity among the definitions of the match and choice regions in Equations 11 and 12. Next, we define two more regions that, although do not correspond to any physical test performed in a FA module, will aid towards the derivation of the results in Section 5.

Definition 4

We define as *conservative category choice (commitment) region* $C^-(\mathbf{w}_j|a,w_u)$ of a category j with template $\mathbf{w}_j$ and size

$$s(\mathbf{w}_j) \leq \frac{(2w_u - 1)M^2}{2Mw_u + a} \quad (13)$$

for particular values a of the choice parameter and w_u the subset of U_M

$$C^-(\mathbf{w}_j \mid a, w_u) = \left\{\mathbf{x} \in U_M \mid \rho(\mathbf{w}_j \mid \mathbf{x}) \geq \frac{M + a}{2Mw_u + a}\right\} \Leftrightarrow$$

$$\left\{\begin{array}{l} C^-(\mathbf{w}_j \mid a, w_u) = \{\mathbf{x} \in U_M \mid dis(\mathbf{x}, \mathbf{w}_j) \leq d_{C-}(\mathbf{w}_j \mid a, w_u)\} \\ d_{C-}(\mathbf{w}_j \mid a, w_u) = \dfrac{(2w_u - 1)M^2}{2Mw_u + a} - s(\mathbf{w}_j) \end{array}\right\} \quad (14)$$

Definition 5

We define as *optimistic category choice (commitment) region* $C^+(\mathbf{w}_j|a,w_u)$ of a category j with template $\mathbf{w}_j$ for particular values a of the choice parameter and w_u the subset of U_M

$$C^+(\mathbf{w}_j \mid a, w_u) = \left\{\mathbf{x} \in U_M \mid \rho(\mathbf{w}_j \mid \mathbf{x}) \geq \frac{a}{(2w_u - 1)M + a}\right\} \Leftrightarrow$$

$$\left\{\begin{array}{l} C^+(\mathbf{w}_j \mid a, w_u) = \{\mathbf{x} \in U_M \mid dis(\mathbf{x}, \mathbf{w}_j) \leq d_{C+}(\mathbf{w}_j \mid a, w_u)\} \\ d_{C+}(\mathbf{w}_j \mid a, w_u) = \dfrac{(2w_u - 1)M^2}{(2w_u - 1)M + a} - s(\mathbf{w}_j) \end{array}\right\} \quad (15)$$

The next definition refers to the last 3 category regions presented in this paper.

Definition 6

We define as *category claim region* $L(\mathbf{w}_j|\rho,a,w_u)$ of a category j with template $\mathbf{w}_j$ for particular values ρ of the vigilance parameter, a of the choice parameter and w_u the following subset of U^M

$$L(\mathbf{w}_j \mid \rho, a, w_u) = V(\mathbf{w}_j \mid \rho) \cap C(\mathbf{w}_j \mid a, w_u) \Leftrightarrow$$

$$\left\{\begin{array}{l} L(\mathbf{w}_j \mid \rho, a, w_u) = \{\mathbf{x} \in U_M \mid dis(\mathbf{x}, \mathbf{w}_j) \leq d_L(\mathbf{w}_j \mid \rho, a, w_u)\} \\ d_L(\mathbf{w}_j \mid \rho, a, w_u) = \min\{d_V(\mathbf{w}_j \mid \rho), d_C(\mathbf{w}_j \mid a, w_u)\} \end{array}\right\} \quad (16)$$

If we replace the choice region with the conservative (optimistic) choice region in the above definition, then we can define a category's *conservative (optimistic) claim region* $L^-(\mathbf{w}_j|\rho,a,w_u)$ ($L^+(\mathbf{w}_j|\rho,a,w_u)$).

A category's claim region stands for the set of all input patterns, for which the category would pass both the VT and the CT. From the definitions we have stated so far we can show the following 2 lemmas:

Lemma 1

Category j will pass VT and the CT with respect to an input pattern $\mathbf{x}$, if $\mathbf{x} \in L^-(\mathbf{w}_j|\rho,a,w_u)$

The previous statement follows from $L^-(\mathbf{w}_j|\rho,a,w_u) \subseteq L(\mathbf{w}_j|\rho,a,w_u)$. On the other hand, the next lemma is derived from the fact that $L(\mathbf{w}_j|\rho,a,w_u) \subseteq L^+(\mathbf{w}_j|\rho,a,w_u)$.

Lemma 2

Category j will never be chosen by a pattern $\mathbf{x} \notin L^+(\mathbf{w}_j|\rho,a,w_u)$.

Another interesting observation regarding the optimistic claim region is given in Lemma 3. Note, that a category's representation region expands, when a new pattern is learned by the category.

Lemma 3

An optimistic claim region contracts, if its associated representation region expands.

5 Results regarding Fuzzy-ART and Fuzzy-ARTMAP

Based on the definitions of category regions and the three lemmas presented so far, we can state some results that concern the FA network (a standalone FA module) and the FAM classifier. For the latter architecture, when we refer to the different network parameters ρ, a, w_u and γ, we will mean the parameters of the FA module dedicated to the input domain (ART_a module). The role of the other module (ART_b module - corresponds to the output domain) in a FAM classifier is trivial.

<u>Result 1</u>

Assume a FA network is undergoing off-line training using any $\gamma \in (0,1]$ and a training set of cardinality P. For the module to complete training by creating $N=P$ categories in only one list presentation, a sufficient condition is

$$\rho > \rho_P = 1 - \frac{d_{\min}}{M} \tag{17}$$

or

$$a > a_P = (2w_u - 1)M\left(\frac{M}{d_{\min}} - 1\right) \tag{18}$$

where d_{min} is the minimum distance between the patterns of the training set.

<u>Result 2</u>

Assume a FA network is undergoing off-line training using the fast learning rule ($\gamma=1$) and a training set of cardinality P. Further assume, that for the size $s_{max}=s(\mathbf{x}_1^e \wedge \mathbf{x}_2^e \wedge \ldots \wedge \mathbf{x}_P^e)$ of the category containing all the training patterns it holds

$$s_{\max} \leq \frac{(2w_u - 1)M^2}{2Mw_u + a} \tag{19}$$

Then, for the module to complete training by creating only one category ($N=1$) in only one list presentation, a sufficient condition is

$$\rho \leq \rho_1 = 1 - \frac{s_{\max}}{M} \tag{20}$$

and

$$a \leq a_1 = M\left[\frac{(2w_u - 1)M}{s_{\max}} - 2w_u\right] \tag{21}$$

Figure 4 identifies the areas in the (ρ,a) parameter space, which correspond to the two sufficient conditions mentioned in the above two results. The two functions plotted are defined as

$$f_1(a, w_u) = \frac{a}{(2w_u - 1)M + a} \tag{22}$$

$$f_2(a, w_u) = \frac{M + a}{2Mw_u + a} \tag{23}$$

It is worth mentioning that when $w_u \to \infty$, the only sufficient conditions for Results 1 and 2 are Equations 17 and 20 respectively; for the latter result, no constraint is necessary on s_{max}, as in Equation 19.

<u>Result 3</u>

Assume a FAM classifier is undergoing off-line training using any $\gamma \in (0,1]$ and a training set of cardinality P. For the module to complete training by creating $N=P$ categories in only one list presentation a sufficient condition is

$$\rho > \rho_P = 1 - \frac{d_{\min}^{\text{intra}}}{M} \tag{24}$$

or

$$a > a_P = (2w_u - 1)M\left(\frac{M}{d_{\min}^{\text{intra}}} - 1\right) \tag{25}$$

where $d_{\min}^{\text{intra}}$ is the minimum intra-class distance between the patterns of the training set.

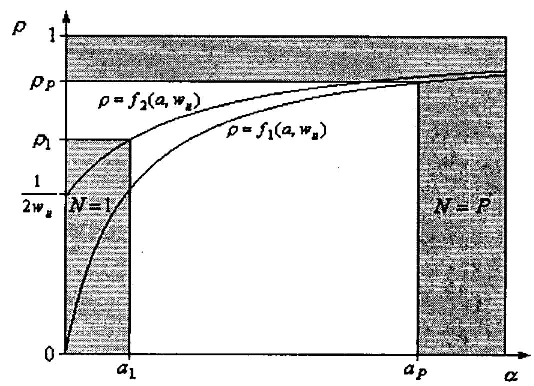

Figure 4: Depiction of sufficient conditions for a FA network in the (ρ,a) parameter space.

<u>Result 4</u>

Assume a FAM classifier is undergoing off-line training using any $\gamma \in (0,1]$. Assume also that $d_{\min}^{\text{intra}} < d_{\min}^{\text{inter}}$, where $d_{\min}^{\text{intra}}$ is the minimum intra-class and $d_{\min}^{\text{inter}}$ is the minimum inter-class distance between the patterns of the training set. For the module to complete training without match tracking ever to go into effect, a sufficient condition is

$$\rho > \rho_{MT} = 1 - \frac{d_{\min}^{\text{inter}}}{M} \tag{26}$$

or

$$a > a_{MT} = (2w_u - 1)M\left(\frac{M}{d_{\min}^{\text{inter}}} - 1\right) \tag{27}$$

Figure 5 shows the areas of the (ρ,a) parameter space that are related to the above results. Similarly to the statements that we made for Results 1 and 2, when $w_u \rightarrow \infty$, Equations 24 and 26 will reflect the sufficient conditions for Results 3 and 4 respectively. Also, notice that if $d^{intra}_{min} \geq d^{inter}_{min}$, the sufficient condition for no match tracking (MT) [2] becomes identical to the one mentioned in Result 3, which is a trivial case.

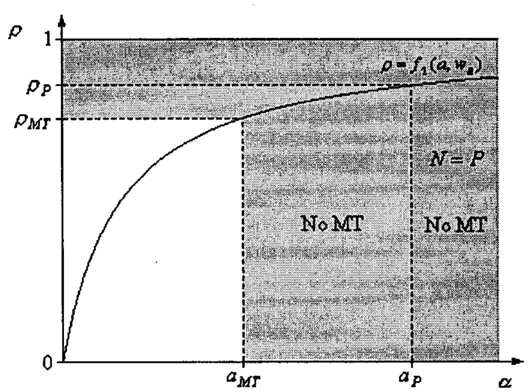

Figure 5: Depiction of sufficient conditions for a FAM classifier in the (ρ,a) parameter space.

Results 1 and 3 are based on Lemma 2. Assume that we have a training set consisting of P patterns $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_P$. During the first list presentation, pattern $\mathbf{x}_1$ will create category 1 with template $\mathbf{w}_1 = \mathbf{x}_1^c$. Under the conditions stipulated by the results, each subsequent pattern will fall outside the optimistic claim region of category 1, or in the case of Result 3, even if they fall inside, they will be of a different class label. Therefore, category 1 will not expand to encode any other patterns except $\mathbf{x}_1$. The same will happen for all $P-1$ remaining patterns. Hence, after the first list presentation, there will P categories and learning will have been completed. Result 2 can be proven via the use of Lemma 1. For the conditions mentioned in Result 2, after $\mathbf{x}_1$ creates category 1, $\mathbf{x}_2$ will fall inside it's conservative claim region (category 1 will satisfy both the VT and CT with respect to $\mathbf{x}_2$) and, thus, the category's template will be updated to $\mathbf{w}_1 = \mathbf{x}_1^c \wedge \mathbf{x}_2^c$. The same process will repeat itself upon presentation of pattern $\mathbf{x}_3$. In turns out that as category 1 expands by encoding additional patterns, all other remaining patterns will remain inside its conservative claim region and, therefore, no new, additional category will be created. The training phase will complete after all patterns have been encoded in category 1, which will occur at the end of the first list presentation. Finally, regarding Result 4, it can be shown via the use of Lemma 2 and 3 that, for the conditions expressed in Equation 26 and 27, no category will include patterns of wrong class label in its optimistic claim region even if the categories expand to encode patterns of the appropriate class label. Therefore, no match tracking will occur during the formation and expansion of categories performed by the FAM classifier.

6 Summary

In this paper we defined as commitment test the comparison of F_2-layer committed nodes against uncommitted nodes in a Fuzzy ART module in terms of category choice function values. We showed that the commitment test is an additional, implicit novelty detection mechanism, similar to the vigilance test. Stemming from the geometrical representation of those two tests, we defined various category regions, whose properties allowed us to state results in the form of sufficient conditions applicable to training with Fuzzy ART networks and Fuzzy ARTMAP classifiers.

8 References

[1] G.A. Carpenter, S. Grossberg and D.B. Rosen, "Fuzzy ART: Fast Stable Learning and Categorization of Analog Patterns by an Adaptive Resonance System", *Neural Networks*, 4(6) , 1991, pp. 759-771.

[2] G.A. Carpenter, S. Grossberg, N. Markuzon, J.H. Reynolds and D.B. Rosen, "Fuzzy ARTMAP: A Neural Network Architecture for Incremental Supervised Learning of Analog Multidimensional Maps", *IEEE Transaction on Neural Networks*, 3(5), 1992, pp. 698-713.

[3] S. Grossberg, "Adaptive Pattern Recognition and Universal Encoding II: Feedback, Expectation, Olfaction, and Illusions", *Biological Cybernetics*, 23, 1976, pp. 187-202.

[4] J.R. Williamson, "Gaussian ARTMAP: A Neural Network for Fast Incremental Learning of Noisy Multidimensional Maps", *Neural Networks*, 9(5), 1996, pp. 881-897.

[5] M. Georgiopoulos, H. Fernlund, G. Bebis, and G.L. Heileman, "Order of Search in Fuzzy ART and Fuzzy ARTMAP: Effect of the Choice Parameter", *Neural Networks*, 9(9), 1996, pp. 1541-1559.

[6] J.X. Huang, M. Georgiopoulos and G.L. Heileman, "Fuzzy ART Properties", *Neural Networks*, 8(2), 1995, pp. 203-213.