Neural Network approach for Direction Of Arrival estimation

A.H.EL Zooghby, C. G. Christodoulou, and M. Georgiopoulos

Electrical and Computer Engineering Department
University of Central Florida
Orlando , Florida 32816

ABSTRACT

The problem of Direction Of Arrival (DOA) estimation of users in mobile communication systems using linear antenna arrays is addressed. Superresolution algorithms , such as Multiple Signal Classification (MUSIC), are used to locate desired as well as cochannel mobile users. However these algorithms require extensive computation and are difficult to implement in real-time. In this paper, the DOA problem is approached as a mapping problem which can be modeled using a suitable artificial neural network trained with input output pairs. A study of a three-layer Radial Basis Function Neural Network (RBFNN) which can learn multiple source direction finding with a six-element array is conducted. RBFNNs were used due to their ability for data interpolation in higher dimensions. The network weights are modified using the normalized cumulative delta rule. The performance of this network is compared to that of the MUSIC algorithm for both uncorrelated and correlated signals. It was found that networks implementing these functions were indeed successful in performing the required task and their performance approached that of the MUSIC algorithm. It is also shown that the RBFNN substantially reduced the CPU time for the DOA estimation computations.

Keywords: Artificial neural networks, array signal processing , Angle of Arrival estimation.

I. INTRODUCTION

Superresolution algorithms have been successfully applied to the problem of Direction Of Arrival (DOA) estimation to locate radiating sources with additive noise for uncorrelated and correlated signals. One of the widely used algorithms is the MUSIC (MUltiple Signal Classification)[1] ,which has the advantage of high resolution for signals with small angular separation (few degrees to few tenths of a degree). The MUSIC algorithm has been useful to mobile satellite communication systems using Frequency Division Multiple Access (FDMA) which are faced with an increasing number of potential users to be served in the same allocated bandwidth [2] . Multiple reuse of each channel, accomplished by the spatial separation of channels assigned the same narrow frequency band, is used to avoid cochannel interference [3] . Further interference reduction can be accomplished by the combined use of DOA estimation algorithms and adaptive arrays [4] . One of the main disadvantages of the superresolution algorithms is that they require extensive computation and as a result they are difficult to implement in real-time. Recently, neural networks have been proposed as successful candidates to carry on the computational tasks required in several array processing applications [5, 6]. Also, in the DOA estimation problem, [7, 8] neural networks have been used in the estimation of the noise subspace necessary for the computation of the MUSIC spectrum by mapping the problem to the quadratic energy function of the network. In this paper , the application of neural networks to handle the computational problem of the DOA estimation step is approached as a mapping problem and solved by using a Radial Basis Function Neural Network or (RBFNN) that can be trained with input output pairs . The trained network is then capable of estimating or predicting outputs not included in the learning phase through generalization. Moreover. one of the main advantages of neural networks is that they can be implemented in analog circuits with time constants in the order of nanoseconds and consequently they have fast convergence rates. In section II, the architecture of a radial basis function neural network (RBFNN) is presented as well as the input preprocessing and output post -processing. The MUSIC algorithm is briefly described in section III. In Section IV the training algorithm used in this paper is discussed. Section V presents results obtained from the application of the RBFNN to the DOA estimation for multiple sources. Also in section V,comparisons of the RBFNN and MUSIC algorithm performances are conducted for uncorrelated and correlated signals.

II. RADIAL BASIS FUNCTION NEURAL NETWORK

Radial Basis Function Neural Networks (RBFNN) [9,10] are members of a class of a general purpose method for approximating nonlinear mappings . Unlike the back-propagation networks which can be viewed as an application of an optimization problem, RBFNN can be considered as designing neural networks to solve curve fitting (or interpolation) problem in a high-dimensional space. The mapping from the input space to the output space may be thought of as a hypersurface Γ representing a multidimensional function of the input . During the training phase, the input-output patterns presented to the network are used to perform a fitting for Γ. The generalization phase represents an interpolation of the input data points along the surface built as an approximation for Γ. The architecture considered in this paper involves three layers , the input layer (sensory nodes), a hidden layer of high dimension and an output layer as shown in Figure 1. The transformation from the input layer to the hidden-unit layer is nonlinear, whereas the transformation from the hidden layer to the output layer is linear. The array performs the mapping G: $\mathbf{R}^K \to \mathbf{C}^M$ from the space of DOA , { $\Theta = [\theta_1, \theta_2, \cdots, \theta_K]$ } to the space of sensor output { $\mathbf{s} = [s_1, s_2, \cdots s_M]$ }. The m^{th} component of s is equal to

$$\mathbf{s}_m = \sum_{k=1}^{K} a_k e^{j(m\frac{\omega_0}{c} d \sin\theta_k + \alpha_k)} \tag{1}$$

where K is the number of signals , M is the number of elements of a linear array , a_k represents the complex amplitude of the k^{th} signal, α_k the initial phase and ω_0 is the center frequency. A neural network is used to perform the inverse mapping F :$C^M \to R^K$. The network is to be trained by patterns generated from equation(1) so that it can associate the output vectors **s**(1), **s**(2), ..., **s**(N) with the corresponding DOA vectors $\Theta(1)$, $\Theta(2)$, ..., $\Theta(N)$. Input vectors are transformed through the hidden layer outputs. Then, each output node computes a weighted sum of the hidden layer outputs [11]. The estimation phase consists of transforming the sensor output vector into an input vector and producing the DOA estimate. The training data is obtained by forming the spatial correlation matrix **R**

$$R_{mm'} = \sum_{k=1}^{K} p_k e^{\frac{j(m-m')\omega_0 d \sin\theta_k}{c}} + \delta R_{mm'} \tag{2}$$

where p_k is power of the K^{th} signal. The last term of the right hand side of this equation contains all the cross-correlated terms between signals. Since for m=m', R_{mm} does not carry any information on the DOA ($R_{mm} = \sum_{k=1}^{K} p_k$) , we can rearrange the rest of the elements into a new input vector , **b** , given as

$$\mathbf{b} = \left[\mathbf{R}_{21}, \cdots, R_{M2}, R_{12}, \cdots, R_{M2}, \cdots, R_{M(M-1)}\right]^T \tag{3}$$

It follows that the number of input units is given by M(M-1). Note that we need twice as many input nodes for the neural network since it does not deal with complex numbers. Hence the total number of input nodes needed is 2M(M-1). The dimension of the hidden layer is equal to the number of the Gaussian functions L which can be chosen to be equal to or less than N (number of training examples) . Obviously , the number of output nodes is equal to the number of signals K. In the simulations performed later, the relative signal power is taken as unity. The input vector is then normalized by its norm in the training, testing and estimation phases , i.e.

$$\mathbf{z} = \frac{\mathbf{b}}{\|\mathbf{b}\|} \tag{4}$$

II.1. NETWORK TRAINING

1. Generate array output vectors { $\mathbf{s}(n)$, n = 1,2,..., N }.
2. Evaluate the correlation matrix of the n^{th} array output vector { $\mathbf{R}(n)$, n = 1,2,..., N }.
3. Form the vectors { $\mathbf{b}(n)$, n = 1,2,..., N }.
4. Normalize the input vectors using equation 4 .
5. Generate the training set { $\mathbf{b}(n),\Theta(n)$, n = 1,2,..., N }.
6. Employ an appropriate RBFNN training procedure [10] to learn the training set generated in step 5.

II.2. DOA ESTIMATION OR GENERALIZATION PHASE

1. Evaluate the sample correlation matrix using the collected array output measurements.
2. Form the vectors $\hat{\mathbf{b}}$.
3. Produce the normalized input vectors $\hat{\mathbf{z}}$
4. Present input vectors $\hat{\mathbf{z}}$ to the RBFNN and obtain the estimate of DOA $\hat{\Theta}$.

III. MUSIC ALGORITHM

Assuming that the signals received at the different sensors are contaminated with statistically independent white noise of variance σ^2, it follows that the received spatial correlation matrix $\mathbf{R}$ of the noisy signals can be rewritten as

$$\mathbf{R} = \mathbf{APA}^H + \sigma^2\mathbf{I} = \sum_{i=1}^{M} \lambda_i e_i e_i^H \tag{5}$$

with $\mathbf{P} = E\{\mathbf{ss}^H\}$ is the signal covariance matrix, the superscript "H" denotes the conjugate transpose and $\mathbf{I}$ is the unit matrix. Note that $\mathbf{P}$ has dimension NxN while $\mathbf{R}$ has dimension MxM , $\lambda_1 \geq \lambda_2 \geq \lambda_k > \lambda_{k+1} = \ldots \lambda = \sigma^2$ are the eigenvalues of $\mathbf{R}$, and e_i are its orthonormal eigenvectors. The eigenvectors corresponding to the first K largest eigenvalues are referred to as the signal eigenvectors , and those corresponding to the minimum eigenvalues are referred to as the noise eigenvectors. The subspace spanned by the signal eigenvectors is called the ***signal subspace*** , and its orthogonal complement spanned by the noise eigenvectors is called the ***noise subspace***. The matrix $\mathbf{R}\text{-}\sigma^2\ \mathbf{I} = \mathbf{APA}^H$ has the same eigenvectors as $\mathbf{R}$, with eigenvalues $\lambda_i - \sigma^2$ for i = 1,2,...,K and $\lambda_i = 0$ for i > K. It follows that :

$$\mathbf{APA}^H = \sum_{i=1}^{K} (\lambda_i - \sigma^2) e_i e_i^H \tag{6}$$

Therefore the signal direction vectors and the signal eigenvectors span the same subspace. This implies that all signal direction vectors are orthogonal to the noise subspace. The MUSIC algorithm estimates the DOA of the K signals by finding the values of θ corresponding to the K maxima of the function

$$S_{MUSIC} = \frac{1}{\mathbf{A}^H \mathbf{N}\mathbf{N}^H \mathbf{A}} \tag{7}$$

where N is the M x M-K matrix whose columns are the M-K eigenvectors spanning the noise subspace of $\mathbf{R}$,.i.e.

$$\mathbf{N} = \begin{bmatrix} e_{K+1} & e_{K+2} & \cdots & e_M \end{bmatrix} \tag{8}$$

IV. SIMULATION RESULTS

IV.1. UNCORRELATED SIGNALS

In the simulations performed an array of M=6 elements is used , therefore the dimension of the input layer was set to 60 nodes. A hidden layer of 50 nodes was chosen . In Figure 2 the array receives two uncorrelated signals with different angular separations ($\Delta\theta = 2^0$ and 5^0). 200 input vectors were used for training . For the testing phase 50 input vectors were used for the network simulated in Figure 2 and 100 input vectors for all the rest of the networks. The results show that the network successfully produced actual outputs (solid) very close to the desired DOA (dotted). In Figure 3 DOA obtained from the MUSIC algorithm are compared to those obtained from the RBFNN method for $\Delta\theta = 5^0$. It can be concluded that the performance of the RBFNN method approaches that of the MUSIC algorithm. Figure 4 shows a network trained with input vectors generated from 2 signals with angular separation of 3^0 and tested with a set of data generated from signals with $\Delta\theta = 1.5^0$. The network was able to generalize and give satisfactory results.

IV.2. CORRELATED SOURCES

In many applications, the signals received by the array are correlated or coherent (perfectly correlated). To study the effect of such cases on the performance of the neural network, the training data was generated assuming the array receives two signals with angular separation of 10^0 . A correlation coefficient γ was assumed with a signal covariance matrix (or the power matrix) in the case of two sources given by :

$$\mathbf{P} = \begin{pmatrix} p & \gamma^* p \\ \gamma\, p & |\gamma|^2 p \end{pmatrix} \tag{9}$$

Moreover, the training was performed with data derived from ideal signals (assuming the absence of noise) whereas the testing was performed with data contaminated with additive gaussian noise to simulate real measurements. For comparison, DOA obtained from MUSIC and RBFNN for correlated signals are plotted in Figure 5 . The RBFNN outperformed the conventional MUSIC yielding smaller error as shown in Figure 6. In this case, the correlation matrix approaches a singular matrix. Although the performance of the MUSIC algorithm under correlated signal environment can be improved using a preprocessing scheme such as spatial smoothing, this technique involves additional computational complexity to the algorithm,whereas the RBFNN approach dealt with this situation simply by taking into consideration the correlation between incoming signals when the correlation matrix R was generated for training. In practice the network can be trained with real data collected from actual measurements without any additional computational requirements. In Figure 7 , the CPU time taken by the MUSIC algorithm to perform the eigendecomposition and obtain the spectrum is plotted as a function of N, the number of different pairs of sources. For N=50 and 100 , the RBFNN needed less than a second to estimate the DOA.

V. CONCLUSION

The problem of DOA estimation is dealt with as a nonlinear mapping from the space of sensor output to that of the angles θ . In this paper the neural network approach was chosen to solve this problem. In particular, RBFNN were used due to their ability for data interpolation in higher dimensions. It was found that RBFNNs implementing these functions were indeed successful in performing the required task and yielded good performance in the sense that the network produced actual output very close to the desired DOA . Also, it was demonstrated that these networks were able to generalize, since testing was performed with data sets derived from different signal conditions than the ones used for training . In review, one of the main advantages of the RBFNN is the substantial reduction in the CPU time needed to estimate the DOA .

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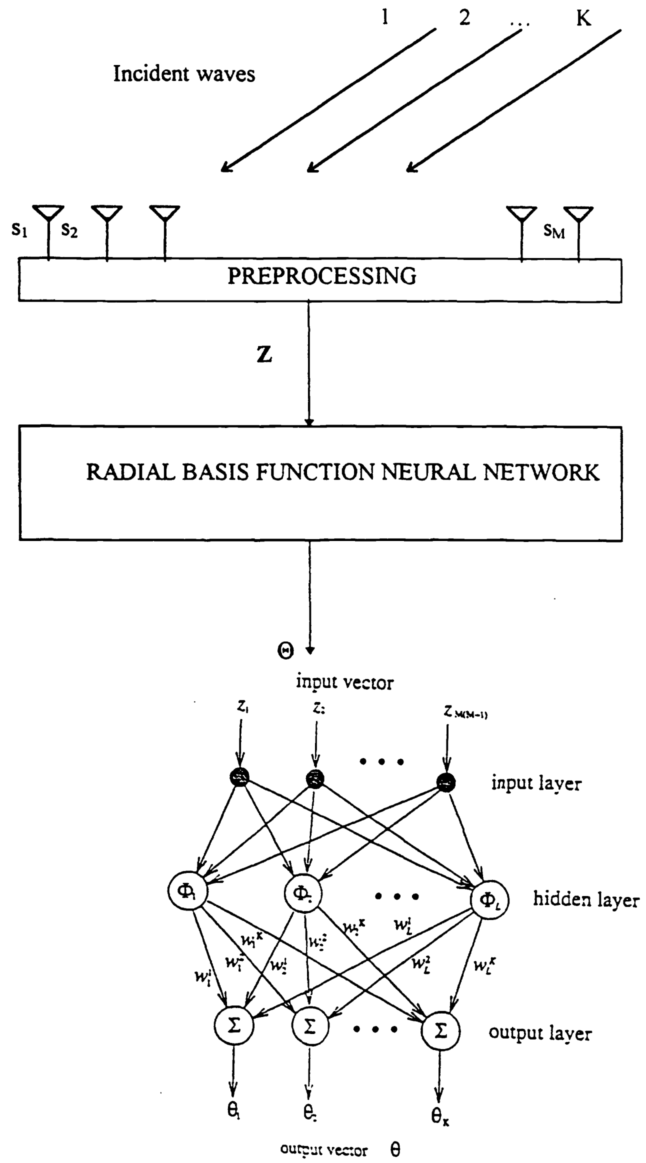
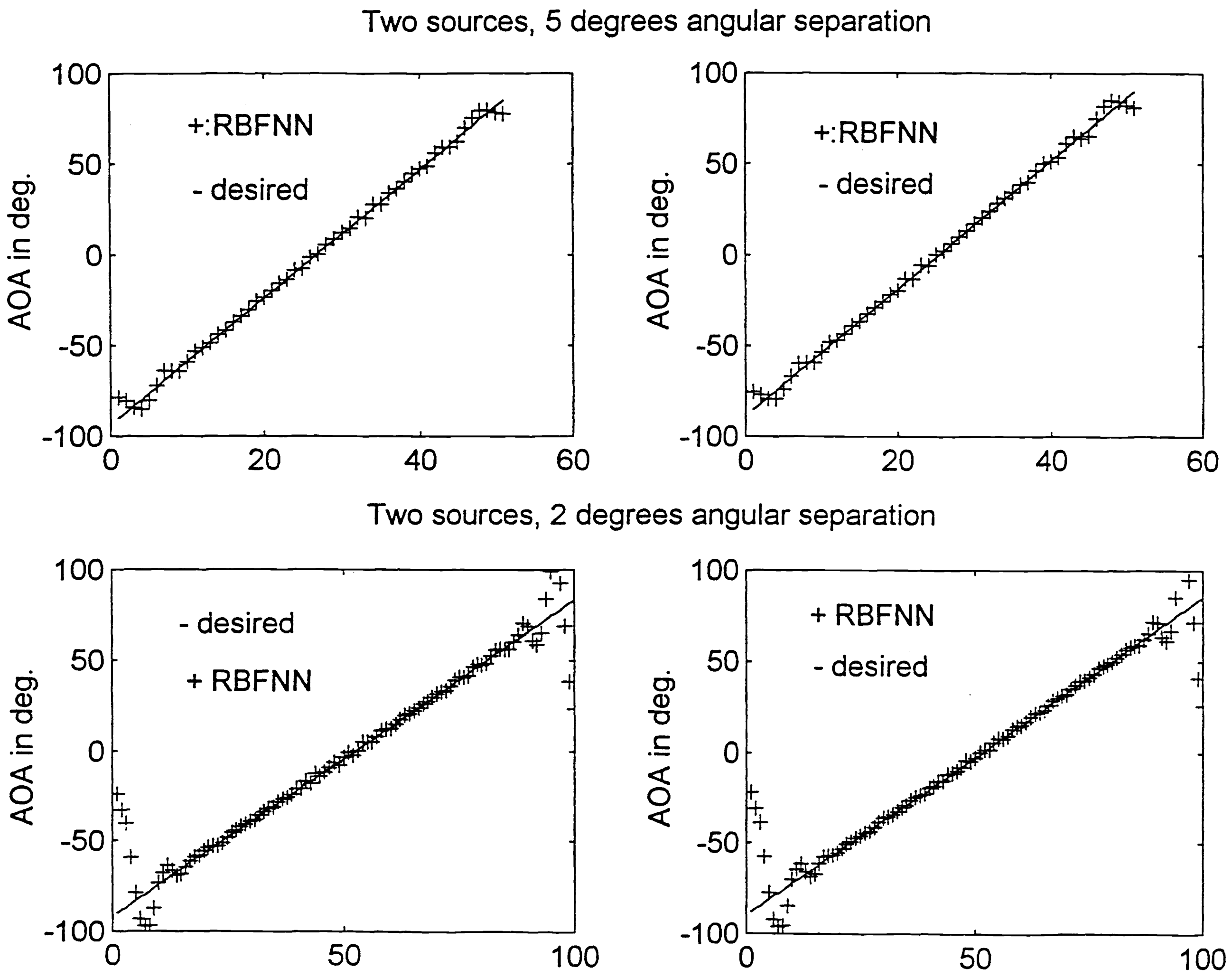


Figure(1)



Two sources, 5 degrees angular separation
+:RBFNN
- desired
AOA in deg.
100
50
0
-50
-100
0
20
40
60
Two sources, 2 degrees angular separation
- desired
+ RBFNN
+ RBFNN
- desired
50
100

Figure (2)

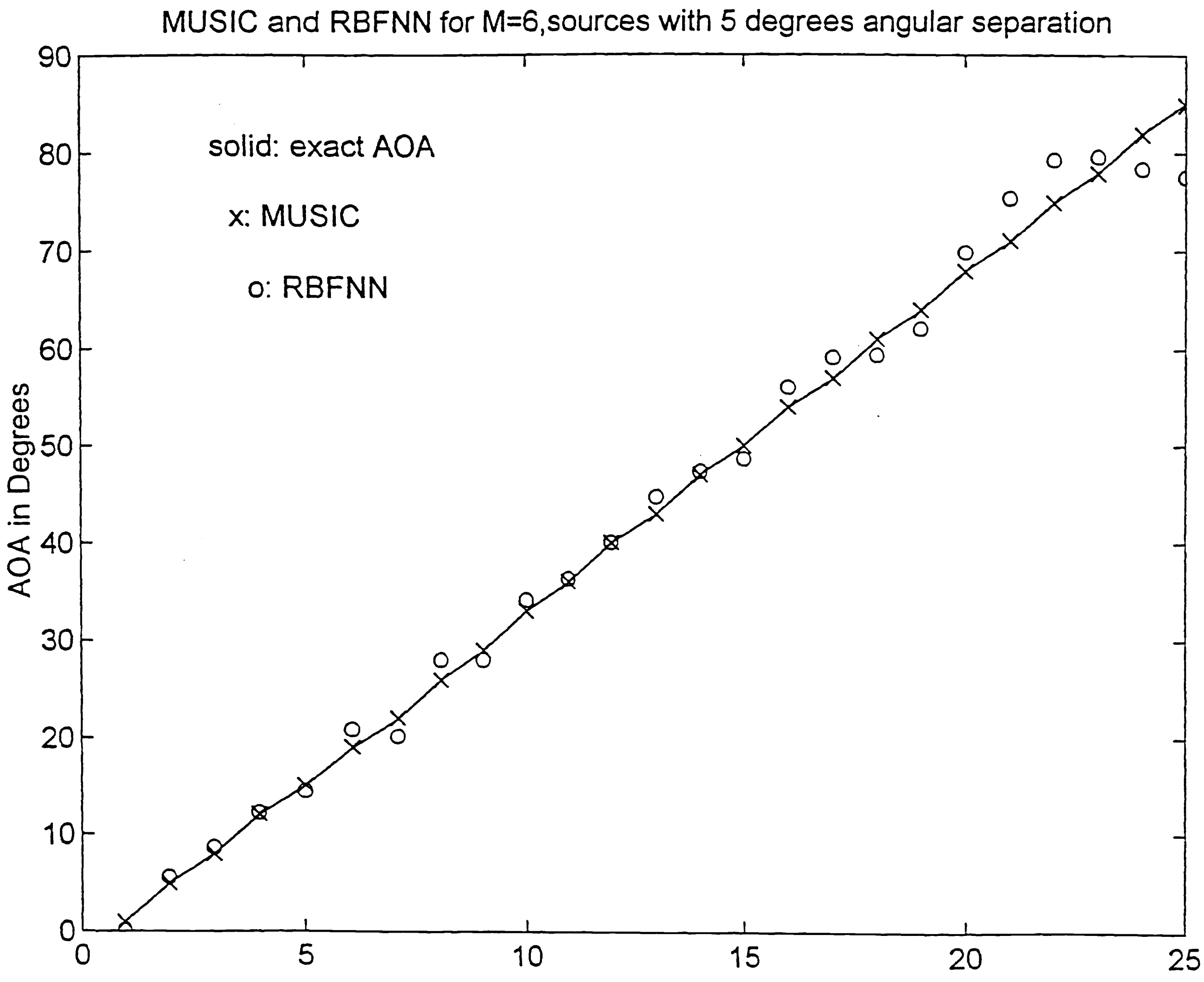
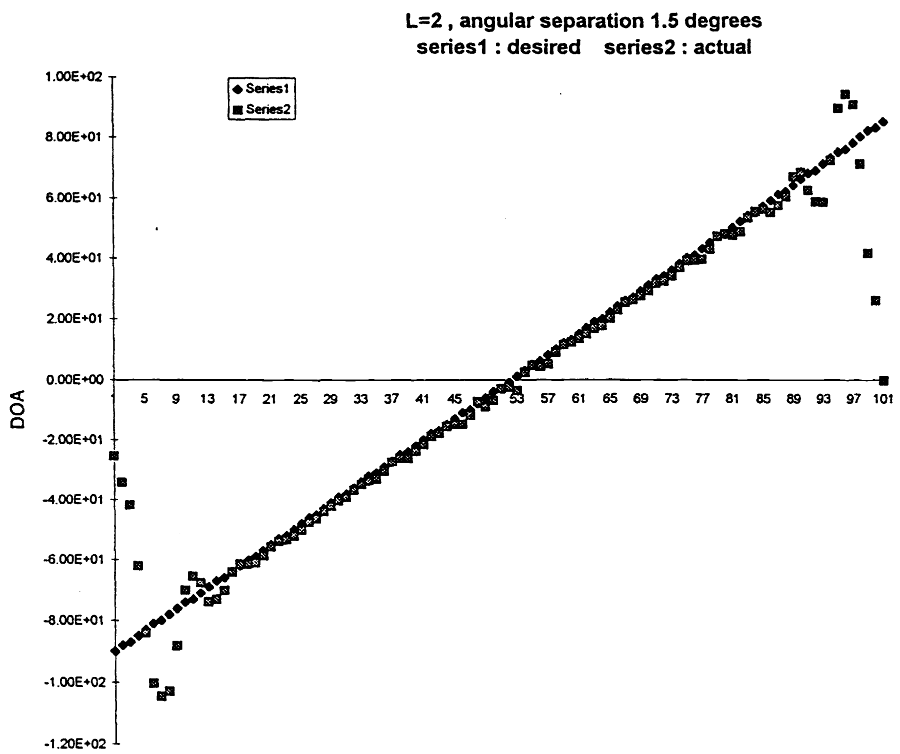
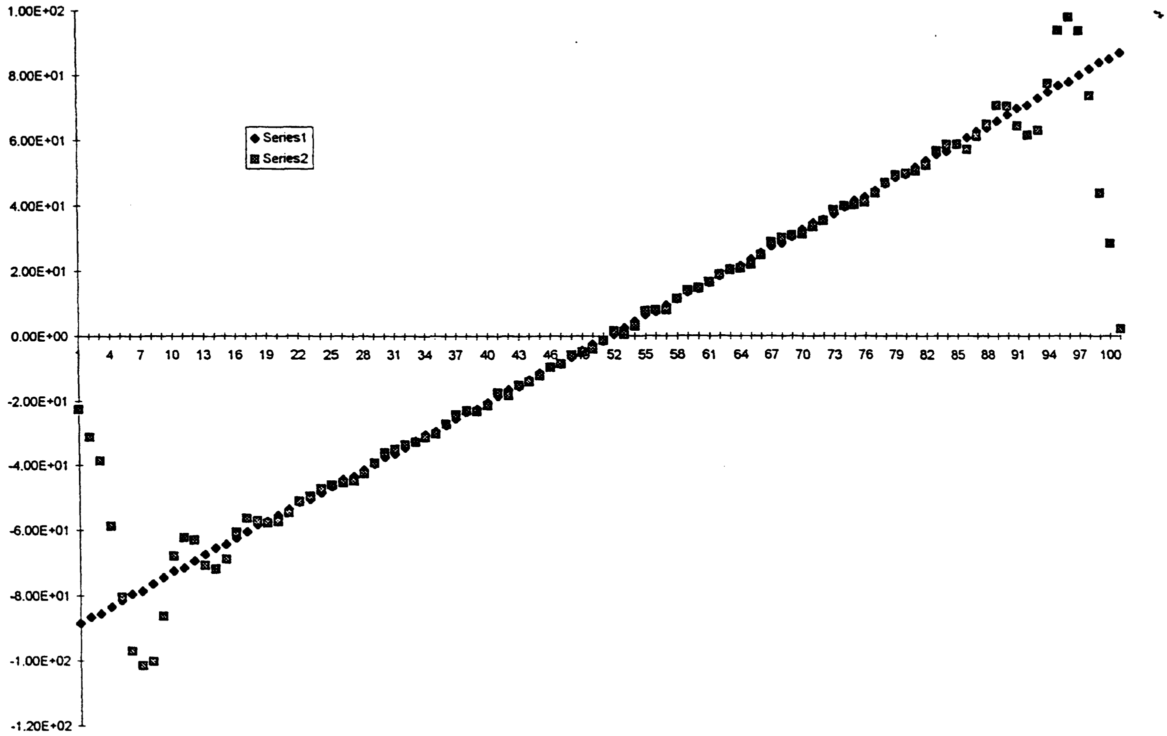


Figure (3) Comparison between MUSIC and RBFNN



(a)



(b)

Figure (4) RBFNN trained with $\Delta\theta = 3^0$, tested with $\Delta\theta = 1.5^0$.

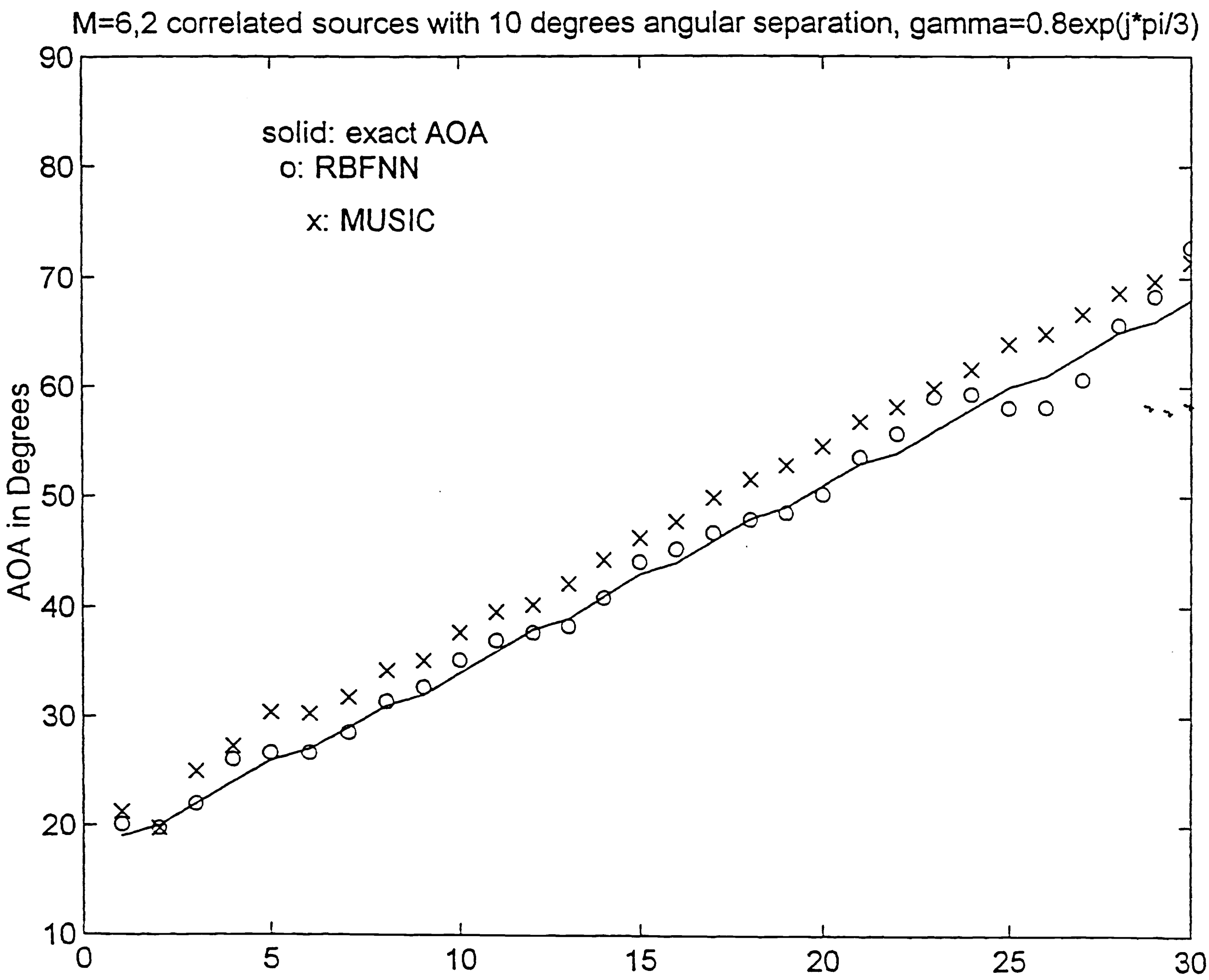
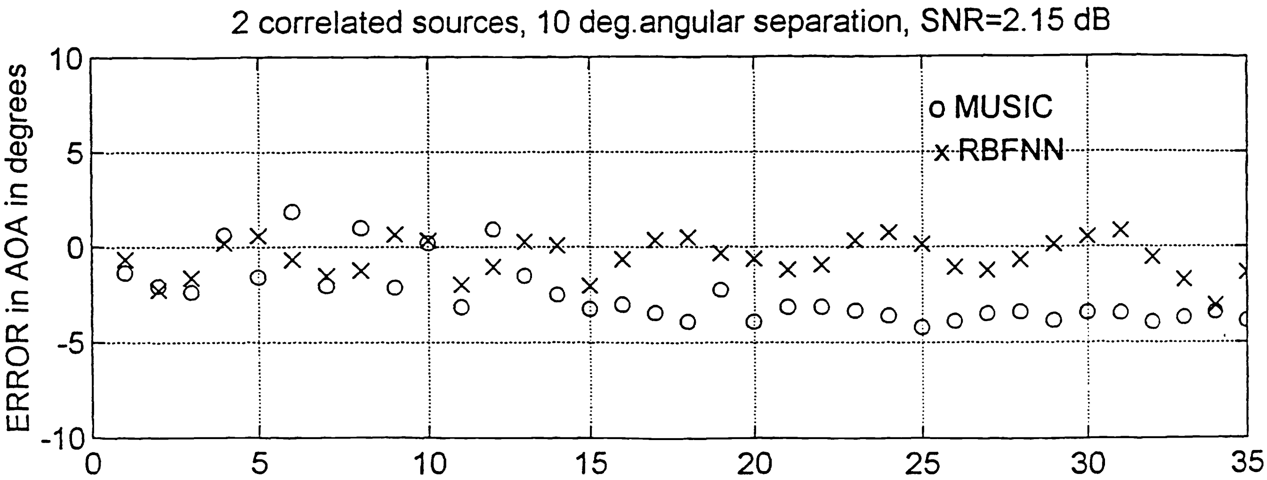
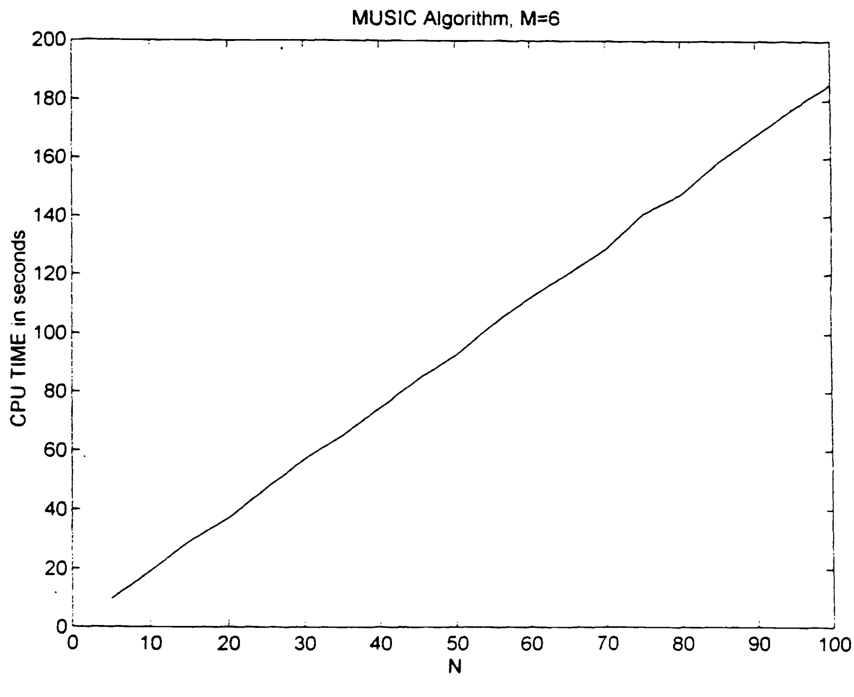


Figure (5) Correlated signals $\Delta\theta = 10^0$.



Figure(6) Estimation Error in MUSIC and RBFNN algorithms.



Figure(7)