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ANALYSIS OF A MULTI-HOP CDMA PACKET RADIO NETWORK

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ABSTRACT

A multi-hop packet radio network employing Code Division Multiple Access (CDMA) is considered. The network is modeled as a system of interacting queues, and sufficient conditions for the stability of the queueing system model are obtained. Also, an upper bound on the network mean packet delay is found. The obtained results are used to characterize the throughput performance of a radio network employing frequency-hop spread-spectrum signaling.

I. INTRODUCTION

Multi-hop CDMA packet radio networks have been treated in [2-4] and elsewhere. When buffered users are considered, such networks are modeled as systems of highly interacting and interdependent queues. The stability analysis and throughput-delay performance of such multiqueue systems is an open problem, even for the simplest network models.

When confronted with a difficult problem, approximations are often useful. This is reflected in the work of Silvester [4], who assumes that queues are independent and parameterizes their interaction with a steady-state estimate of their state. In their recent study of non-CDMA networks [1], Tsybakov and Bakirov take a different approach, and construct an easy-to-analyze auxiliary network of queues, whose stability implies that of the original network; this approach provides sufficient (but not necessary) conditions for network stability.

In this paper some of the results in [1] are extended to CDMA networks with receiver directed codes, and an upper bound on the average packet delay is found. A particular frequency-hopping (FH) transmission scheme with memoryless FH patterns is then introduced and the probability of successful packet reception in the presense of multi-access interference is found. Finally, some numerical results on the throughput performance are presented for symmetric networks.

The paper is organized as follows. Section II defines the basic network model. Network stability is dealt with in Section III. Section IV treats frequency-hopping.

II. NETWORK MODEL

The network model is specified in terms of topology, input traffic, channel access, and routing.

Topology

The network has M users ($M<\infty$) indexed $1,2,\ldots,M$. Users i and j are called neighbors if they are in the transmission range of each other. Let V_i denote the set of neighbors of user i, and v_i denote their number. The collection of sets, $T=(V_1,\ldots,V_M)$, will be referred to as the network's topology. We assume that T is arbitrary but fixed. Given T, the network can be represented by a non-directional graph with M nodes (node i represents user i, $i=1,\ldots,M$), in which nodes i and j are connected by a link if, and only if, they are neighbors.

Input Traffic

The time axis is divided in unit length intervals, called slots. Let the interval $[k,k+1]$ denote the k-th slot, $k=0,1,2,\ldots$ We assume that, during each slot, user i generates a packet with probability g_i and generates no packets with probability $1-g_i$, $i=1,\ldots,m$. All packets have fixed length, and the packet transmission time is equal to z, where $z<1$.

Channel Access

Users initiate transmission only at slot boundaries. If at the beginning of a slot the queue of user i is non-empty, then user i transmits a packet with probability p and remains silent with probability 1-p. The priority with which packets are transmitted (i.e., the queue service discipline) may be arbitrary but fixed.

In receiver oriented CDMA each of the neighbors of user i, $i=1,\ldots,M$ is assigned a unique code, and, when i transmits a packet intended for his neighbor m, he encodes this packet in m's code. Let c_j denote the code of user j. We assume that $c_j\neq c_m$, $j\neq m$, $j,m\in V_i$, $i=1,\ldots,M$, and that each user has his receiver tuned to his own code.

A packet transmitted by user i, $i=1,\ldots,M$ to user $m\in V_i$ during slot k, is successfully received by m if, and only if, all three of the following events occur during slot k.

E_1 = {user m is not transmitting}
E_2 = {user i is the only neighbor of transmitting in m's code}

E_3 = {m's receiver decodes correctly i's packet}.

We will assume that, given E_1, E_2, and given the event I_j = {the total number of packets transmitted by the neighbor of m during slot k is j+1}, the probability of E_3 depends only on the number j of interfering packets, i.e.,

$$\Pr(E_3|E_1,E_2,I_j) = c(j), \quad j=0,1,2... \tag{1}$$

The sequence {c(j), j=0,1,2,...} depends on the particular code division scheme and forward-error correction codes used. We will elaborate on this in Section IV, where we discuss frequency-hop spread spectrum signaling. At this point, we will only make the following assumption:

(A.1) c(j) is non-increasing in j.

Upon successful reception of a packet from i, m sends an acknowledgement to i encoded in i's code. (Like packets, acknowledgments are also encoded in the receiver's code.) Note that an acknowledgement from m to i might interfer with an acknowledgement transmitted from user ℓ to user ℓ', where $\ell \epsilon V_i$-{m} and $\ell' \epsilon V_\ell$-{m,i}; however, since only the acknowledgement from m to i uses i's code, this interference is limited. For simplicity in the analysis, we will assume that acknowledgements are always decoded correctly. We will also assume that the acknowledgement transmission time, propagation delays, and processing delays are such that their sum does not exceed 1-z units of time, where z is the packet transmission time. Thus, the acknowledgement for a packet that was successfully transmitted in slot k is decoded by the transmitter of the packet by the end of the slot, and the packet leaves the transmitter's queue by time k+1.

Routing

A simple random routing model is considered: a packet transmitted by user i is destined for his neighbor m (i.e., the packet is encoded in m's code) with probability $1/v_i$, $m \epsilon v_i$, i=1,...,M. A packet successfully received by user i leaves the network (i.e., user i is its final destination) with probability f, and remains in the network (i.e., enters the queue of user i to be forwarded) with probability 1-f.

III. NETWORK STABILITY

Let $N_i(k)$ denote the number of packets in the queue of user i, i=1,...,M, at time k, k=0,1,.... Under the model of Section II, the vector process $N(k)=[N_1(k),...,N_M(k)]$ is a Markov chain. The network will be called stable if N(k) is ergodic.

Determining explicit conditions for the ergodicity of vector Markov chains, such as N(k), is a hard open problem [5]. Here we follow the approach taken by Tsybakov and Bakirov [1]. This approach uses an easy-to-analyze auxiliary Markov chain whose ergodicity implies that of N(k). The analysis in [1] refers to the model of Section II with c(0)=1, c(j)=0 for j≠0 (i.e., a narrowbound Aloha-type network); however, the key results in [1] can be extended to the case where c(j) is non-increasing in j, but otherwise arbitrary.

Let $\bar{N}(k)=[\bar{N}_1(k),..., \bar{N}_M(k)]$, k=0,1,..., where $\bar{N}_i(k)$ is the number of packets in the queue of user i, at time k, in the network specified by the following model.

Dominant network model. This model coincides with the model of Section II in all respects except for the following: a user with an empty queue transmits a fictitious packet with probability p and remains silent with probability 1-p. Until its first successful transmission, a fictitious packet is not included in the number of packets in the queue of the user that generated it; otherwise, fictitious packets are in no way different from real ones.

The role of the fictitious packets in the dominant network model is to make the outcome of a packet transmission from a user independent of the state of the queue (empty vs. non-empty) of any other user in the network. As a result, $\bar{N}_i(k)$ is a Markov chain for each i, i=1...,M. What is important, however, is that $\bar{N}(k)$ dominates N(k) in the sense of the following inequality:

$$\Pr\left\{\sum_{i=1}^{M} \bar{N}_i(k)>m \,\middle|\, \bar{N}(0)=\rho\right\} \geq \Pr\left\{\sum_{i=1}^{M} N_i(k)>m \,\middle|\, N(0)=\rho\right\} \tag{2}$$

for every k, m, and initial state $\rho=[\rho_1,...,\rho_M]$.

Inequality (2) is the basis in the proof of the following key theorem.

Theorem A [Thm. 1, [1]]: If $\bar{N}_i(k)$ is ergodic for all i, i=1,...,m, then N(k) is also ergodic.

The ergodicity analysis of $\bar{N}_i(k)$ is standard, and the following result is as expected.

Proposition 1. $\bar{N}_i(k)$ is ergodic if, and only if,

$$g_i+r_i < s_i, \tag{3}$$

where s_i is the probability of successful packet transmission, and r_i is the probability of successful packet reception of user i in the dominant network.

The probabilities s_i and r_i are as follows:

$$s_i = (1-p) \sum_{m \epsilon V_i} \sum_{j=0}^{v_m-1} c(j) q_{im}(j), \tag{4}$$

$$r_i = (1-p)(1-f) \sum_{m \epsilon V_i} \sum_{j=0}^{v_i-1} c(j) q_{mi}(j) \tag{5}$$

where $q_{im}(j)$ is the probability that (j+1) of m's neighbors transmit and only the neighbor i uses m's code; $q_{im}(j)$ is as given below:

$$q_{im}(j) = p^{j+1}(1-p)^{V_m-j-1} \; v_i^{-1} \sum_K \left(1-v_{k_1}^{-1}\right) \cdots \left(1-v_{k_j}^{-1}\right),$$

where $K=(k_1,...,k_j)$, $k_1<k_2<...<k_j$, $k_n \epsilon V_m$-{i},

n=1,...,j.

In view of Proposition 1, Theorem A yields the following corollary.

Corollary 1: The network is stable if $g_i+r_i<s_i$, for all i, i=1,...,M.

While Corollary 1 gives a sufficient condition for network stability, the following theorem gives a sufficient condition for network instability.

Theorem B [Thm. 3,[1]]: If $g_i+r_i>s_i$, for all i, i=1,...,M, then $N_i(k)\to\infty$, with probability 1 for all i.

For the special case of a symmetric network in which $v_i=v$ and $g_i=g$, for all i, we have that $s_i=s$ and $r_i=r=(1-f)s$, for all i, where

$$s = \sum_{j=0}^{v-1}\binom{v-1}{j}\left(1-v^{-1}\right)^j p^{j+1}(1-p)^{v-j-1}c(j) \quad (6)$$

By Corollary 1 and Theorem B, we have that for given v, f, and c(j), j=1,2,..., such a symmetric network is stable if, and only if,

$$g < g^* = fs^* \quad (7)$$

where s^* is the maximum s, attained for $p=p^*$ in (6). We refer to g^* as the user maximum stable throughput of the symmetric network.

We close this section by giving an upper bound on the mean packet delay in the network. Let (3) hold for all i, and let

$$L = \lim_{k\to\infty} E\left[\sum_{i=1}^{M} N_i(k)\right], \bar{L}_i = \lim_{k\to\infty} E\left[\tilde{N}_i(k)\right].$$

Using standard analysis techniques for the Markov chain $N_i(k)$, yields $\bar{L}_i = (r_i+g_i(1-g_i)/(s_i-r_i-g_i)$. Therefore, by (2) and Little's result, we have that the mean steady state packet delay, $D(=L/(g_1+...+g_M))$, is bounded from above as follows:

$$D \le \left[\sum_{i=1}^{M}(r_i+g_i(1-g_i))/(s_i-r_i-g_i)\right]/\sum_{i=1}^{M} g_i \quad (8)$$

For a symmetric network (8) becomes

$$D \le [(1-f)s+g(1-g)]/(fs-g)g.$$

where s is as given in (6).

IV. NETWORK PERFORMANCE WITH FREQUENCY-HOPPING

In this section, we first evaluate the probability of correct packet decoding in the presense of j interfering packets, c(j), defined in (1), for a particular FH transmission scheme; the results of the previous section are then used to evaluate the throughput performance of symmetric FH-CDMA networks.

We assume that the available bandwidth is divided into q orthogonal frequency sub-bands and that packets are divided into n bytes each. For the transmission of a packet to user m, the frequency hopping pattern $F_m=\{f_k^{(m)}, 1\le k\le n\}$ is used; i.e., the k-th byte is transmitted in sub-band $f_k^{(m)}$, where $f_k^{(m)}$ is one of the q sub-bands. The frequency hopping patterns satisfy the following properties: i) For each m, m=1,...,M, F_m is a collection of i.i.d. random variables, which are uniformly distributed over the set of the q frequency sub-bands. ii) The patterns $F_1,\dots,F_m$ are statistically independent.

Packet transmissions are slot-synchronous, but synchronization at the byte level is not assumed. We say that the k-th byte of a packet transmitted to user m is hit if, at any time during the reception of the k-th byte, user m can hear another packet at the same frequency sub-band. We assume that receivers have side information which enables them to determine which bytes have been hit [2]; bytes with hits are erased, and erasure correction decoding is used to recover the packet. We will assume that extended Reed-Solomon (n,ℓ) codes are used, with block length n a power of 2. The alphabet size is equal to n and the erasure-correction capbility, e, is equal to $n-\ell$.

Consider a silent receiver, say m, which during a slot hears j+1 packet transmissions, indexed 1,2,...,j+1, and assume that only packet 1 utilizes m's hopping pattern. Define the random variables H_k, k=1,...,n, such that $H_k=1$ if the k-th byte of packet 1 is hit at receiver m, and $H_k=0$ otherwise. Clearly,

$$c(j) = \sum_{i=0}^{e} \Pr\left(\sum_{k=1}^{n} H_k = i\right) \quad (9)$$

The right hand side of (9) will be evaluated under the following assumption:

(A.2) The frequency hopping patterns corresponding to packets 2,...,j+1 are independent.

This assumption does not hold if two or more of the involved packets use the frequency hopping pattern of the same receiver, say ℓ, where $\ell\ne m$. However, in certain cases, (A.2) is pessimistic, i.e., c(j) is underestimated. Such is the case in networks where the maximum propagation delay between any two neighbor users does not exceed the time to transmit a byte (i.e., the length of the hop interval). In any case, relaxing (A.2) requires a model for the propagation delays between neighbor users.

Under (A.2), the sequence $(H_1,\dots,H_n)$ is Markov: $\Pr(H_k|H_{k-1},\dots,H_1)=\Pr(H_k|H_{k-1})$, $2\le k\le n$. This is in contrast to the hop-synchronous case, where the H_k's are independent. The transition probabilities $t_{\ell m}=\Pr(H_k=m|H_{k-1}=\ell)$, $2\le k\le n$, are as given below (details can be found in [6]):

$$t_{00} = q^{-1}(1-q^{-1})^j+(1-q^{-1})(1-2q^{-1})^j \quad (10.a)$$

$$t_{10} = (1-t_{00})P_0/P_1,\ t_{11}=1-t_{10}, t_{01}=1-t_{00}, \quad (10.b)$$

where

$$P_0 = \Pr(H_k=0)=(1-q^{-1})^{2j}, 1\le k\le n, P_1=1-P_0.$$

Remark: Note from (10) that byte hits are independent only when j=2, or as $q\to\infty$. Nevertheless, in practice, q is large, and assuming independent byte hits is a good approximation.

Using (10), $\Pr(H_1+\dots+H_n=i)$ was computed recursively [6], and used in (9) to obtain c(j). Figure 1 gives c(j) for q=25, n=32, and ℓ/n=1/4, 1/2, and 3/4.

The user maximum stable throughput g^*, given in (7), for a symmetric network with q=50, f=1, n=32, and n/ℓ=1/2 and 3/4 is given in Table 1, for various values of v (number of neighbors). In the same table we have included the user maximum normalized stable throughput, $\bar{g}^* = g(\ell/n)(1/q)$, to account for the bandwidth expansion, and the throughput $\hat{g}^*$ of the corresponding non-CDMA network, in which $c(j)=0$ for $j\geq 1$ and $c(0)=1$. The values of p that maximize the throughput in each case are also given; they are denoted by p^*.

From Table 1 we observe the following:
i) The normalized throughput $\bar{g}^*$ of the FH-CDMA network is always smaller than the throughput $\hat{g}^*$ of the corresponding non-CDMA network.

ii) Given n/ℓ, the ratio $\bar{g}^*/\hat{g}^*$ increases with the number of neighbors.
iii) The value of g^* is less sensitive than $\hat{g}^*$ to changes in the number of neighbors; the same is true when the transmission probability deviates from its optimal value, although this cannot be observed from the given table.

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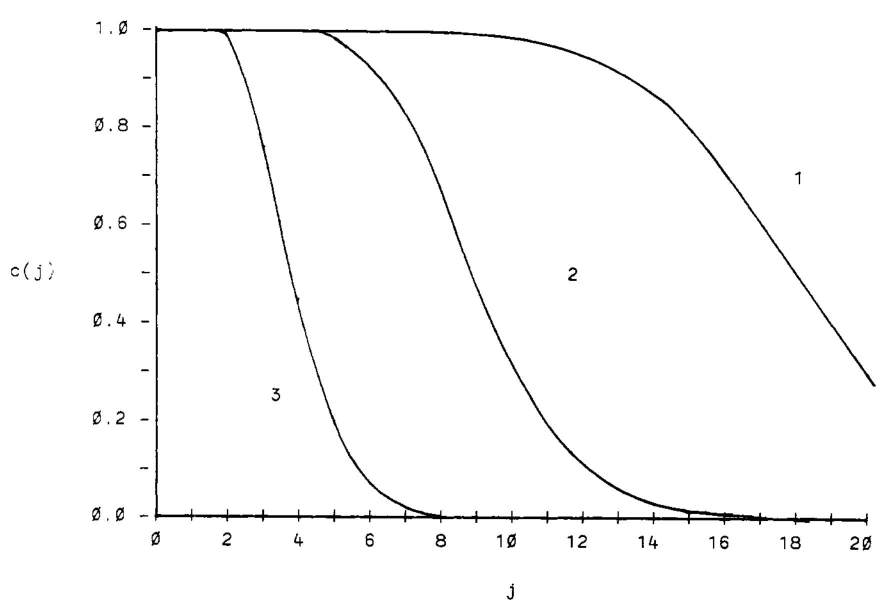

Figure 1. The probability c(j) for q=25, n=32.
1. 1/n=1/4
2. 1/n=1/2
3. 1/n=3/4

		FH-CDMA			Non-CDMA	
v	1/n	p*	g*	$\bar{g}$*	p*	$\hat{g}$*
2	1/2	0.42	0.1924	0.1924 E-2	0.33	0.1481
	3/4	0.42	0.1924	0.2886 E-2		
4	1/2	0.40	0.1749	0.1749 E-2	0.20	0.8192 E-1
	3/4	0.40	0.1748	0.2623 E-2		
6	1/2	0.39	0.1700	0.1700 E-2	0.14	0.5663 E-1
	3/4	0.39	0.1693	0.2539 E-2		
25	1/2	0.36	0.1600	0.1600 E-2	0.04	0.1441 E-1
	3/4	0.21	0.1125	0.1687 E-2		
50	1/2	0.24	0.1296	0.1296 E-2	0.02	0.7283 E-2
	3/4	0.12	0.6918 E-1	0.1037 E-2		
100	1/2	0.13	0.8357 E-1	0.8357 E-3	0.01	0.3660 E-2
	3/4	0.06	0.3847 E-1	0.5771 E-3		

Table 1. The user maximum stable throughputs $\bar{g}$*, g*, and $\hat{g}$*, for q=50 and n=32.