Comparison of BCH and Convolutional Codes in a Direct Sequence Spread Spectrum Multiple Access Packet Radio Network

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Abstract

The most significant indicator of the performance of a multiple access packet radio network is its packet error probability. Packet errors in a multiple access system are a result of noise at the receiver and multiple access interference.

In this paper we compute an upper bound on the packet error probability induced in a direct sequence spread spectrum multiple access packet radio network when convolutional or BCH codes are used for the encoding of the packets. More specifically, an upper bound on the bit error probability is first developed. Then, this upper bound is used to compute upper bounds on the packet error probability for BCH codes. For convolutional codes, we built a simulator package that allows us to compute upper bounds on the packet error probability by utilizing, as in the case of BCH codes, only the upper bound on the bit error probability. The simulator permits the user to enter such parameters as constraint length, bit error probability and decode depth of the utilized Viterbi decoder. At the end of its run the simulator produces the number and the specific positions of the bits that remained uncorrected by the Viterbi decoder from which the packet error probability can be determined.

From our results a fair comparison of the BCH and the convolutional codes in a direct sequence spread spectrum multiple access packet radio network is conducted.

1. INTRODUCTION

One of the most important attributes of spread spectrum signalling is its multiple access capability. The multiple access capability of a Direct Sequence Spread Spectrum (DS-SS) packet radio network is examined in this paper. The multiple access capability of a DS-SS packet radio network is measured by computing the induced packet error probability. The exact evaluation of the packet error probability in direct sequence spread spectrum systems is a difficult task. The difficulty arises from the fact that a multiple integral has to be computed and the complexity of the calculation increases exponentially with the number of interfering packets. Hence most researchers ([1],[2],[3]) have resorted to techniques that upper bound the induced packet error probability.

In this paper the upper bounds on the packet error probability induced in DS-SS systems when BCH or convolutional coding is used for the encoding of the packets is computed. For BCH coding, an upper bound on the bit error probability is first evaluated. This upper bound is then used to calculate an upper bound on the packet error probability. Several problems arise in attempting to bound the packet error probability when convolutional coding is used [1]. The main difficulty arises in evaluating exactly the induced union bound. Most analytical results are based on bounding the union bound in terms of the transfer function of the code. For large signal to noise ratios, upper bounds to the union bound require only the first few terms of the transfer function to be used. For low signal to noise ratios however, such as the case in multiple access networks, these upper bounds cannot be relied upon. As a result, in evaluating the packet error probability of a DS-SS system when convolutional coding is used, we will simulate a convolutional encoder and Viterbi decoder to determine packet error probabilities using the same upper bounds on bit error probabilities as those used for the BCH codes. A comparative performance between BCH and Convolutional codes is then going to be conducted.

2. THE MODEL - PRELIMINARIES

The direct sequence spread spectrum multiple access system of interest is shown in Figure 1. The received signal can be written as follows:

$$
\begin{aligned}
r(t) &= \sum_{i=1}^{K} \sqrt{2P_i}\, s_i(t-\tau_i) + n(t) \\
&= \sum_{i=1}^{K} \sqrt{2P_i}\, b_i(t-\tau_i) a_i(t-\tau_i) \cos(\omega_c t + \phi_i) + n(t)
\end{aligned}
\tag{1}
$$

Where $s_i(t)$ is the spread spectrum signal of the i^{th} transmitter, P_i is the power of the signal of the i^{th} transmitter at the receiver site and $n(t)$ is additive white Gaussian noise with two sided spectral density of $N_0/2$. The spread spectrum signal $s_i(t)$ is the product of the binary data signal, $b_i(t)$, the spreading signal of the i^{th} transmitter, $a_i(t)$, and the term $\cos(\omega_c t + \Theta_i)$ where Θ_i is the phase of the i^{th} transmitter while ω_c is the carrier frequency. The delay between transmitter i and receiver is equal to τ_i and $\phi_i = \Theta_i - \omega_c\tau_i$. The data signal of the i^{th} transmitter can be expressed as

$$
b_i(t) = \sum_{m=-\infty}^{\infty} b_m^{(i)} P_T(t-mT) \tag{2}
$$

Where $b_m^{(i)}$ is the m^{th} bit corresponding to the data signal of the i^{th} transmitter. The spreading signal of the i^{th} transmitter is given by

$$
a_i(t) = \sum_{j=-\infty}^{\infty} a_j^{(i)} P_{T_c}(t-jT_c) \tag{3}
$$

where $\{a_j^{(i)}\}$ is the signature sequence of the transmitter i. In expressions (2) and (3)

$$
P_\Delta(t) = \begin{cases} 1 & \text{for } t \le \Delta \\ 0 & \text{otherwise} \end{cases}
$$

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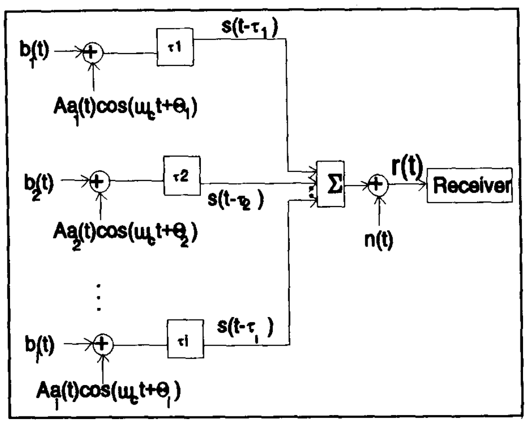

Figure 1. Direct Sequence Spread Spectrum Model.

Hence, in (2) T corresponds to the data bit duration and in (3) T_c corresponds to the chip duration. The signature sequence (i.e. sequence of chips) of every transmitter is assumed to be a sequence of independent, identically distributed, binary random variables, each equally likely to be +1 or -1. The signature sequence assigned to every transmitter in the SS system is assumed to be independent of the sequences assigned to other transmitters.

Let us now assume that we have a slotted channel and transmitters initiate the transmission of a packet of information at the beginning of slots. Let us also assume that K (K>1) packet transmissions occur within a slot and a receiver locks on to packet #1 (i.e., the packet originated from transmitter 1). Arriving packets at the receiver are indexed #1, #2, ..., #K and originate from transmitters 1, 2, ..., K, respectively. A packet is exactly one codeword from an (M, L) BCH code (M = number of codeword bits, L = number of information bits) or a convolutional code with L information bits of rate L/M. The bits of the codeword, in both cases, are indexed from 0 up to M-1. Based on our DS-SS multiple access model we can write that the received signal is

$$r(t) = \sqrt{2P_1}\,b_1(t)a_1(t)\cos(\omega_c t) + \sum_{i=2}^{K}\sqrt{2P_i}\,b_i(t-\tau_i)a_i(t-\tau_i)\cos(\omega_c t + \phi_i) + n(t) \tag{4}$$

where

$$b_i(t) = \sum_{m=0}^{M-1} b_m^{(i)}\, P_T(t-mT) \tag{5}$$

and

$$a_i(t) = \sum_{j=0}^{N(M-1)} a_j^{(i)}\, P_{T_c}(t-jT_c) \tag{6}$$

N represents the number of chips per bit. The receiver is assumed to be a correlation receiver (Figure 2), so in (4), we set $\tau_1=0$, $\phi_1=0$ and we used τ_i and ϕ_i ($2\le i\le K$) to designate the relative delay and phase of packet #i with respect to packet #1, as it is perceived by the receiver.

As we mentioned before, the receiver locks on to packet #1 and tries to decode packet #1 in the presence of additive white Gaussian noise and multiple access interference. Our goal is to determine the packet error probability, that is the probability that packet #1 is decoded incorrectly by the receiver. We denote this probability by $P_e(K)$. Since as we mentioned in the Introduction, exact evaluation of the packet error probability, $P_e(K)$, is computationally intractable we will resort to upper bounds. Our effort to find upper bounds on $P_e(K)$ will be accomplished in two steps. In the first step an expression for the bit error probability is found, and its upper bounds are derived. In the second step the upper bounds on the bit error probability are utilized to compute upper bounds on the packet error probability. The upper bound on the packet error probability is the measure of performance on which comparison among BCH and convolutional codes will be based.

3. BIT ERROR PROBABILITY

The output of the correlation receiver (see Figure 2) corresponding to the m^{th} bit ($0\le m\le M-1$) of packet #1, is the random variable

$$Z_m = n_m + (P_1/2)^{1/2}T\,\{b_m^{(1)} + \sum_{i=2}^{K}(P_i/P_1)^{1/2} I_{i,1}^{m}(b_i^{m}, \tau_i, \phi_i)\}$$
$$0 \le m \le M-1 \tag{7}$$

Each n_m is a Gaussian random variable with zero mean and variance $N_oT/4$. The random variables n_m ($0\le m\le M-1$) are independent. The variable $b_m^{(1)}$ represents the m^{th} bit of packet #1; its value is +1 or -1. The vector b_i^m represents a pair of consecutive bits of packet #i. In particular, $b_i^m = (b_{m-1}^{(i)}, b_m^{(i)})$, and each data bit $b_m^{(i)}$ is either +1 or -1. Each τ_i or ϕ_i is a random variable representing the time delay (modulo T) or the phase angle (modulo 2π), respectively, of packet #i with respect to packet #1. We take the range of each τ_i to be the interval [0, T] and the range of each ϕ_i to be the interval [0, 2π]

The function $I_{i,1}^m$, which appears in (7), represents the normalized multiple-access interference due to packet #i. This function is defined by

$$I_{i,1}^{m}(b_i^{m}, \tau, \phi) = T^{-1}[b_{m-1}^{(i)} R_{i,1}^{m}(\tau) + b_m^{(i)}\hat{R}_{i,1}^{m}(\tau)]\cos\phi \tag{8}$$

where the functions $R_{i,1}^m$ and $\hat{R}_{i,1}^m$ are given by

$$R_{i,1}^{m}(\tau) = \int_{mT}^{mT+\tau} a_i(t-\tau)a_1(t)\,dt \tag{9}$$

$$\hat{R}_{i,1}^{m}(\tau) = \int_{mT+\tau}^{(m+1)T} a_i(t-\tau)a_1(t)\,dt \tag{10}$$

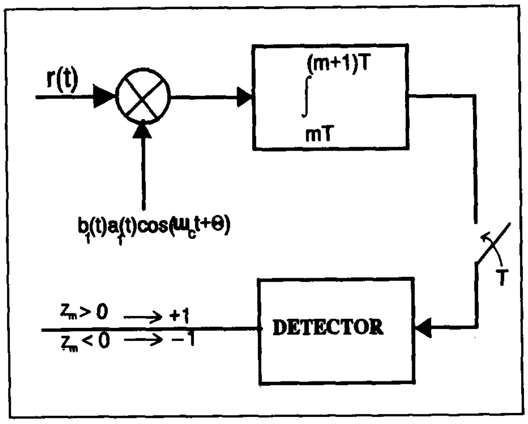

Figure 2. Block diagram of correlation receiver.

The detector decides that the m^{th} bit of packet #1 is +1 or -1 if $Z_m>0$ or $Z_m<0$, respectively (See Figure 2). The m^{th} bit of packet #1 is decoded correctly by the above detector if and only if the random variable

$$X_m = n_m^* + [1 + \sum_{i=2}^{K} b_m^{(1)} (P_i/P_1)^{\frac{1}{2}} I_{i,1}^{m}(b_i^{m}, \tau_i, \phi_i)] \quad (11)$$
$$0 \le m \le M-1$$

is positive. In (11), each n_m^* is a Gaussian random variable with mean 0 and variance $N_o/2E_b$, where $E_b=P_1T$, is the energy per data bit of packet #1. The random variables n_m^* ($0 \le m \le M-1$) are statistically independent.

We will now state two propositions that will be helpful in deriving a more manageable expression for the random variable X_m ($0 \le m \le M-1$).

Proposition 1: For the computation of $P_e(K)$, the τ_i's ($2 \le i \le K$) need be known only to the nearest chip.

Proposition 2: $P_e(K)$ is independent of the values of the data bit sequences $\{b_m^{(i)}\}_{m=0}^{M-1}$ for $1 \le i \le K$.

The validity of propositions 1 and 2 is based on the fact that random signature sequences are utilized. An immediate consequence of Propositions 1 and 2, is that the random variable X_m in (11) assumes the equivalent form

$$\begin{aligned} X_m = n_m^* + 1 + \sum_{i=2}^{K} (P_i/P_1)^{\frac{1}{2}} \{ & [a_{mN-1}^{(i)} a_{mN}^{(1)}] \tau_i / T_c \\ & + [a_{mN}^{(i)} a_{mN}^{(1)} + a_{mN+1}^{(i)} a_{mN+1}^{(1)} + \ldots a_{mN+N-1}^{(i)} a_{mN+N-1}^{(1)}](1 - \tau_i/T_c) \\ & + [a_{mN}^{(i)} a_{mN+1}^{(1)} + a_{mN+1}^{(i)} a_{mN+2}^{(1)} + \ldots + a_{mN+N-2}^{(i)} a_{mN+N-1}^{(1)}] \tau_i/T_c \} \\ & * \cos\phi_i / N \end{aligned}$$
$$0 \le m \le M-1 \quad (12)$$

Given the phase (ϕ_i) and the delay (τ_i) of each transferring transmission ($2 \le i \le K$), the random variables X_m ($0 \le m \le M-1$) can be considered approximately independent [2]. Let us now define the random vectors

$$\tau = (\tau_2, \ldots \tau_K) \qquad \phi = (\phi_2, \ldots \phi_K) \quad (13)a$$

and the vectors

$$\hat{\tau} = (\hat{\tau}_2, \ldots \hat{\tau}_K) \qquad \hat{\phi} = (\hat{\phi}_2, \ldots \hat{\phi}_K) \quad (13)b$$

Let us also denote by $p(\hat{\tau}, \hat{\phi})$ the conditional bit error probability induced in our spread spectrum system given that $\tau=\hat{\tau}$ and $\phi=\hat{\phi}$. We can write

$$\begin{aligned} p(\hat{\tau}, \hat{\phi}) &= Pr(X_o < 0 \mid \tau = \hat{\tau}, \phi = \hat{\phi}) \\ &= Pr(n_o^* + 1 + \sum_{i=2}^{K} (P_i/P_1)^{\frac{1}{2}} I_i^0(\hat{\tau}_i, \hat{\phi}_i) < 0) \end{aligned} \quad (14)$$

with

$$\begin{aligned} I_i^0(\hat{\tau}_i, \hat{\phi}_i) = \{ & a_{-1}^{(i)} a_0^{(1)} \hat{\tau}_i / T_c \\ & + [a_0^{(i)} a_0^{(1)} + \ldots a_{N-1}^{(i)} a_{N-1}^{(1)}](1 - \hat{\tau}_i/T_c) \\ & + [a_0^{(i)} a_1^{(1)} + \ldots + a_{N-2}^{(i)} a_{N-1}^{(1)}] \hat{\tau}_i / T_c \} \cos\hat{\phi}_i / N \end{aligned} \quad (15)$$
$$2 \le i \le K$$

Case 1: Near far ratio = 0dB

The near far ratio equal to 0dB corresponds to the situation where all the spread spectrum signals arrive with equal power at the receiver (i.e. $P_1 = P_2 = \ldots = P_K$). Under this assumption Pursley et al showed in [1] that for every $\hat{\tau}, \hat{\phi}$ the conditional bit error probability, $p(\hat{\tau}, \hat{\phi})$, is upper bounded by q_1, where

$$q_1 = Q[(2E_b/N_o)^{1/2}] + \frac{1}{\pi} \int_0^{\infty} u^{-1} \sin(u) \Phi_2(u) [1 - \Phi_1(u)] du \quad (16)$$

with

$$\Phi_1(u) = \left[\cos\left(\frac{u}{N}\right)\right]^{N(K-1)} \quad (17)$$

$$\Phi_2(u) = e^{[-(N_o/4E_b)u^2]} \quad (18)$$

and

$$Q(x) = (2\pi)^{-1/2} \int_x^{\infty} e^{-u^2/2} du \quad (19)$$

Case 2: Near far ratio ≠ 0dB

The near far ratio ≠ 0dB corresponds to the more realistic situation where the spread spectrum signals arrive with equal or unequal powers at the receiver (i.e., $P_i = P_j$ or $P_i \ne P_j$ for $1 \le i,j \le K$). In this case, Georgiopoulos has shown in [2] that for every $\hat{\tau}, \hat{\phi}$ the conditional bit error probability, $p(\hat{\tau}, \hat{\phi})$, is upper bounded by q_2, where

$$q_2 = \min_{Z \ge 0} \left\{ e^{-Z} E[e^{Z n_o^*}] \prod_{i=2}^{K} E\left[e^{Z (P_i/P_1)^{1/2} J_i}\right] \right\} \quad (20)$$

with

$$J_i = [\sum_{j=0}^{N-1} a_j^{(i)}]/N \quad ; \quad 2 \le i \le K \quad (21)$$

and E in (20) denotes the expectation operator.

The reason that we consider the upper bound q_1 is because, in the unrealistic case where the signals arrive with equal power at the receiver site, q_1 is a tighter upper bound on $p(\hat{\tau}, \hat{\phi})$ than q_2 is.

Upper bounds on $P_e(K)$ -BCH Codes

Let us denote by $f_{\tau,\phi}(\hat{\tau}, \hat{\phi})$ the joint probability density function of the random vectors τ and ϕ; τ, ϕ and $\hat{\tau}, \hat{\phi}$ were defined in (13a) and (13b) respectively. Let us denote by S a random variable which represents the number of bits in packet #1 that are in error. S is also equal to the number of random variables X_m ($0 \le m \le M-1$) that are negative (see discussion prior to the introduction of equation (11)).

If an (M, L) BCH code with error correcting capability e, is used for the encoding of packet #1 we can write

$$P_e(K) = \int_{\hat{\tau}} \int_{\hat{\phi}} Pr(S > e \mid \tau = \hat{\tau}, \phi = \hat{\phi}) f_{\tau,\phi}(\hat{\tau}, \hat{\phi}) d\hat{\phi} d\hat{\tau} \quad (22)$$

Equation (22) can be written as

$$P_e(K) = \int_{\hat{\tau}} \int_{\hat{\phi}} \sum_{i=e+1}^{M} \binom{M}{i} p(\hat{\tau}, \hat{\phi})^i (1 - p(\hat{\tau}, \hat{\phi}))^{M-i} f_{\tau,\phi}(\hat{\tau}, \hat{\phi}) d\hat{\phi} d\hat{\tau} \quad (23)$$

We can now distinguish two cases

Case 1: Near far ratio = 0 dB

We know from (16) that the conditional bit error probability $p(\mathbf{t},\boldsymbol{\phi})$ is upper bounded for every $\mathbf{t}$, $\boldsymbol{\phi}$ by q_1. Furthermore, the quantity

$$\sum_{i=e+1}^{M} \binom{M}{i} p^{i}(1-p)^{M-i}$$

is an increasing function of p. As a result, from (23) we get that

$$P_e(K) \leq \sum_{i=e+1}^{M} \binom{M}{i} q_1^{i}(1-q_1)^{M-i} \triangleq P_e^1(K) \qquad (24)$$

Case 2: Near far ratio ≠ 0dB

We know from (20) that the conditional bit error probability $p(\mathbf{t},\boldsymbol{\phi})$ is upper bounded for every $\mathbf{t}$,$\boldsymbol{\phi}$ by q_2. Hence we can write once more that

$$P_e(K) \leq \sum_{i=e+1}^{M} \binom{M}{i} q_2^{i}(1-q_2)^{M-i} \triangleq P_e^2(K) \qquad (25)$$

Expressions (24) and (25) give upper bounds on the packet error probability $P_e(K)$ for the cases of 0dB and non 0dB near far ratios, respectively. The evaluation of $P_e^1(K)$ and $P_e^2(K)$ requires first the computation of q_1, q_2 via the expressions provided in formulas (16) - (21).

4. THE SIMULATOR

The simulator developed (Figure 3) evaluates upper bounds on the packet error probability induced in the DS-SS system when convolutional codes are used for the encoding of the packets and Viterbi decoding is used for the decoding of the packets. The program was written in the C language and used on the UNIX system.

Data Generator

The Data Generator generates the data bits of length L that are to be transmitted. An all zero bit stream was transmitted. The actual value of the bit stream does not affect the bit or packet error probabilities.

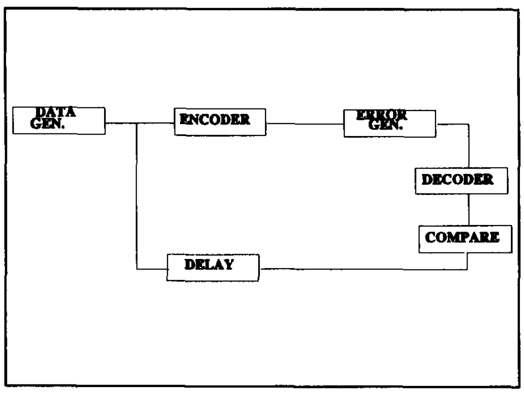

Figure 3. Simulation Model.

Encoder

The encoder takes message symbols from the data generator and shifts them in the register. Code words for the message bit are then calculated using the tap registers specified by the user.

Error Generator

The error module introduces errors into the code word using a random number generator. The probability of an error being introduced (bit error probability) is specified for each simulation by the user. The output of the error module represents the received code while the input at the error module represents the transmitted code.

Decoder

The Decoder generates branches from each state at a given level to the corresponding states at the next level and calculates the path metric for each branch. The decoder then determines the branch with the lowest distance for each state at the next level and labels this branch as the survivor for this state discarding all other branches entering the state. For each path that is terminated there will be another path that splits. If no path has been terminated, then all the paths simply extend themselves. For each path terminated the decoder has to decide which one of the other paths will split. The terminated path is now substituted by the corresponding surviving path that was split. The data symbols associated with each extended path are then added to the path history. After a specified decode depth has been reached the decoder starts producing decoded data. When the decode depth is reached the decoder determines the path with the lowest path metric and selects the oldest symbol of that path from its path history.

Comparator

The comparator compares the data bit generated with the decoded data bit. It also determines the number of data bits that were decoded incorrectly, as well as the position of the bits in error. As a result, the packet error probability can be determined, as well as the number of uncorrected bit errors.

Upper Bounds on $P_e(K)$ - Convolutional Codes

An all zero bit stream was initially produced by the data generator. Every L bits of the data generator corresponds to the information bits of the simulated packet #1. The all zero bit data stream generated an all zero encoded bit stream. The constraint length 7 rate 1/2 binary convolutional code was used to generate the encoded bit stream from the data bit stream. Every M=2L bits of the encoded bit stream represent the coded bits of the simulated packet #1.

The desired transmission is distorted by multiple access interference and additive white Gaussian noise. The cumulative effect of these noise sources results in the worst case scenario into errors produced independently with probability q_1 (see equation 16) for the 0dB near far ratio, and with probability q_2 (see equation 20) for the non-0dB near far ratio. Note that q_1 and q_2 depend on N, K, E_b/N_o and near far ratio (P_i/P_1). The error generator produces independent errors on the input sequence of the encoded bits with probabilities q_1, q_2. The resulting output sequence of the error generator enters the Viterbi decoder where error correction takes place according to the rules specified in the description of the Viterbi decoder. The output of the Viterbi decoder is a data stream that corresponds to the transmitted bit sequence as it is perceived by the receiver. Every L bits of the output data stream corresponds to the information bits of the simulated packet #1. Statistics were gathered with respect to the erroneous data packets in the output bit sequence of the Viterbi decoder. The resulting packet error probability was denoted by $P_e^3(K)$.

The upper bounds on the induced packet error probabilities ($P_e^1(K)$, $P_e^2(K)$ for BCH, $P_e^3(K)$ for convolutional) are based on the same upper bound for the bit error probability. The above scenario represents the right framework to compare the performance of BCH codes ($P_e^1(K)$ in (24) and $P_e^2(K)$ in (25)) and convolutional codes ($P_e^3(K)$) in a DS-SS multiple access environment.

CONCLUSION

In Table I, the performance of the BCH codes ($P_e^1(K)$, $P_e^2(K)$) and the simulated performance of the convolutional codes ($P_e^3(K)$) is shown when N=127, E_b/N_o = 12 or 15dB, near far ratio 0, 3 or 6dB and for large values of the multiple access interference K. From Table I, we observe that for large values of the multiple access interference and small block lengths (i.e., 60) convolutional codes outperform BCH codes. It is worth noting that the numerical results developed in [3] show that BCH codes outperform convolutional codes in DS-SS multiple access environments for small to moderate values of the multiple access interference and small block lengths, as well as for all values of the multiple access interference and larger block lenghts (i.e., 512,1023).

There are specific multiple access environments which seem to favor convolutional codes. Convolutional codes seem to be preferred in indirectly routed packet networks where retransmission delay is an important factor of network performance . The retransmission delay at repeaters is shorter when convolutional codes are used. Block codes encounter a complete block length delay during retransmission. For line of sight packet communications however, the tradeoff presumably favors a long block code, since the transmissions typically occur in bursts. Convolutional codes are best suited for channels requiring transmission of long streams of data. This is because convolutional codes are matched only to blocks of infinite length. For shorter transmission lengths, the block code can be more closely matched to the required block length.

In this work, we compared two encoding schemes, BCH and Convolutional codes in the direct sequence multiple access environment. If the only measure of performance is packet error probability, then it seems that BCH codes are the right choice. If other factors affect the designer's decision as well, such as packet delay or decoding complexity, then convolutional coding might be the more desirable coding technique.

TABLE I

UPPER BOUNDS ON THE PACKET ERROR PROBABILITY $P_e(K)$

(E_b/N_o = 12dB, N = 127)

K =	P_i/P_1	$\min(q_1,q_2)$	BCH (63, 30)	CONV(M=60) rate=1/2
4	6dB	0.018	P_e^2= 1.64E-4	P_e^3= 7.8E-5
8	3dB	0.029	P_e^2= 2.32E-3	P_e^3= 6.4E-4
33	0dB	0.026	P_e^1= 1.43E-3	P_e^3= 3.1E-4

(E_b/N_o = 15dB, N = 127)

K =	P_i/P_1	$\min(q_1,q_2)$	BCH (63, 30)	CONV(M=60) rate=1/2
5	6dB	0.028	P_e^2= 2.20E-3	P_e^3= 4.7E-4
9	3dB	0.029	P_e^2= 2.32E-3	P_e^3= 6.4E-4
34	0dB	0.028	P_e^1= 2.05E-3	P_e^3= 4.7E-4

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