

# **Numerical Program Optimization for High-Level Synthesis**

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# Introduction

## Floating-point operations...

- **are easy to use**

High dynamic range

- **are ubiquitous**

Altera introduced new FPGAs (Arria 10 and Stratix 10) with hardened floating-point DSP elements

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## Floating-point operations...

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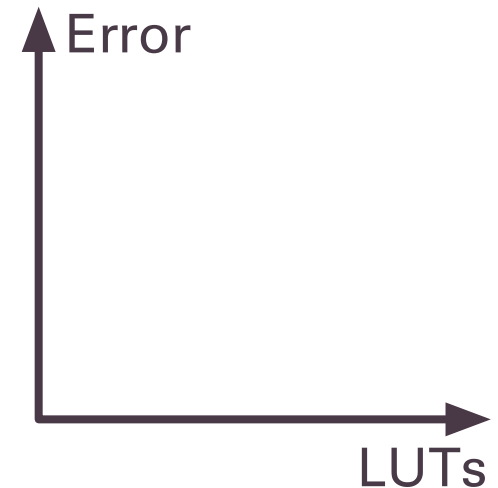
## However, floating-point operations

- use a lot of resources
- have round-off errors

# Introduction

Our tool exploits the rules of equivalence to **automatically** optimize the structure of numerical programs, for example:

- $(a + b) + c \equiv a + (b + c)$
- $(a + b) * c \equiv a * b + a * c$
- and many more



# Introduction

GCC / LLVM /  
Vivado HLS  
**-ffast-math**

- Simple transformations
- What about accuracy?

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SOAP  
Arithmetic  
Expressions

- Simple transformations
- What about accuracy?
- Deep transformations
- Resource usage & Accuracy!

# Introduction

GCC / LLVM /  
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SOAP  
Arithmetic  
Expressions

SOAP2  
Full  
Programs

- Simple transformations
- What about accuracy?
- Deep transformations
- Resource usage & Accuracy!
- Full program transformations

# Example

## Program

```
if (x < 20) {  
    x = x + (y + 500);  
} else {  
    x = (x + y) + 500;  
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```



# Example

## Specification

```
float x, y;  
assume(0 <= x <= 500);  
assume(err(x) == 0);  
assume(0 <= y <= 30);  
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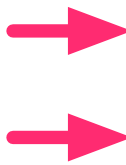
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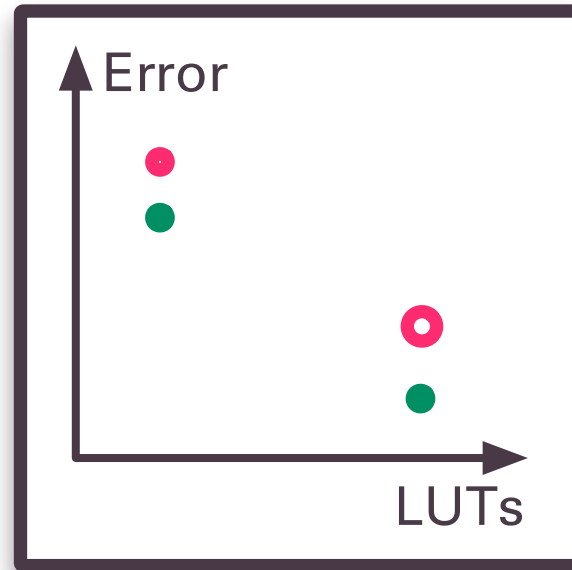
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- Pareto optimal
- Pareto suboptimal

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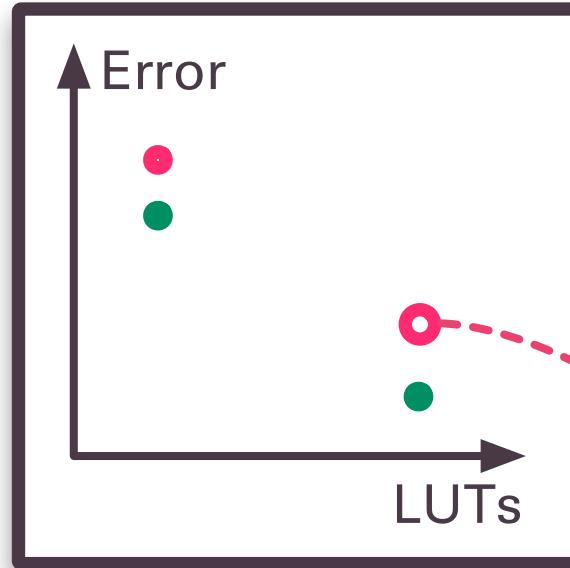
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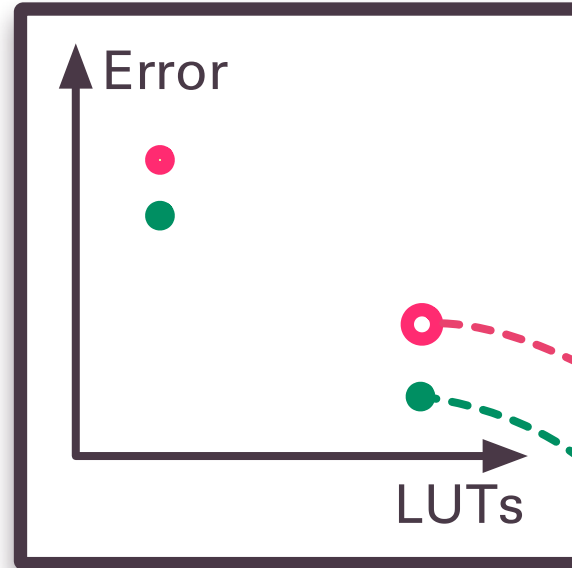
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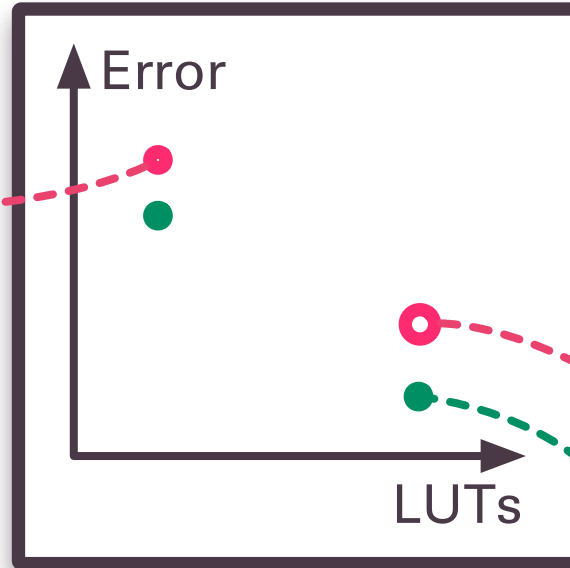
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# Example

Fewest resources but less accurate

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x = x + (y + 500);
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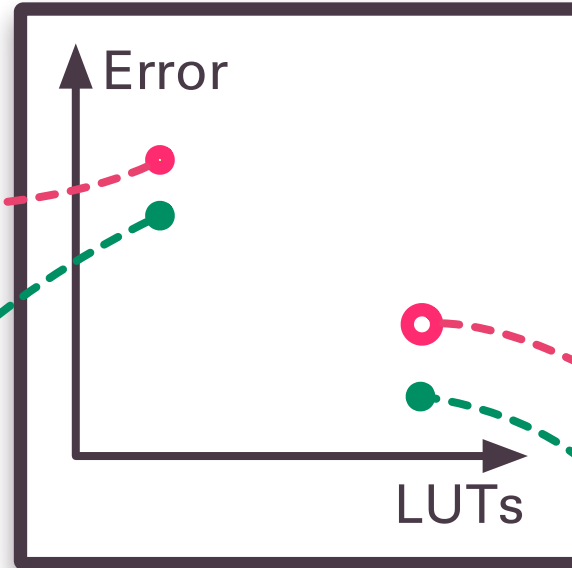
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# And there is more...

There are a lot of things we did not cover:

- **how we do that**  
a more complex example program
- **work flow**  
how it fits in the traditional HLS work flow
- **results**  
~60% better accuracy

All of these above are in the poster!

**Thank you!**

Join us in the poster session