University of Central Florida
Online Mathematics Contest
Problems: January 2014 (Year 1, Round 1)

Warm Ups

1) Johnny walks to from home to school at a pace of 3 miles per hour. On the same day, excited to get home, he runs home from school at a pace of 8 miles per hour. What was his average speed in miles per hour for this round trip?

2) Samantha is traveling from city A to city B to city C and back to city A. If she averages 40 mph on the first leg of the trip, 50 mph on the second leg of the trip, 44 mph on the last leg of the trip, the trip from city A to city B was 100 miles, the trip from city B to city C was 200 miles, and the whole trip average was 45 mph, how long was her trip from city C back to city A.

3) What is the coefficient of a^4b^4 is the expansion of $(2a – 3b)^8$?

4) If $sin\theta = \frac{2}{3}$ and $0 < \theta < 90°$, what are $\sin(2\theta), \cos(2\theta), and \tan(2\theta)$.

5) What is $(1 + i)^5(2 – 2i)^5$?

Exercises

1) You are given that $x + y = 17$ and $xy = 60$. What is $x^3 + y^3$?

2) How many permutations of "CENTRALFLORIDA" do not contain any double letters? Note: A double letter in a string is the same letter next to itself.

3) What are the sums of the roots of the equation $4(81^x) – 43(9^x) + 108 = 0$?

4) In a group project, there is a "to-do" list with 13 different items. Seven of the items involve writing reports while the other six involve building a robot. In a group of four students, if everyone must write at least one report, how many ways can you distribute the work? We count two distributions of work differently if at least one student in the two distributions has a different *number* of a particular task. *Don't* distinguish between distributions where each student has the same number of reports to write and robot tasks in both distributions. (As an example, one possible distribution would be student A writes 2 reports and does 2 robot tasks, student B writes 2 report and does 2 robot tasks, student C writes 2 reports and does 2 robot tasks, and student D writes 1 report and does no robot tasks. This particular distribution should only be counted once, no matter *which* reports student A writes. In short, the students are distinguishable, but the reports are indistinguishable and the robot tasks are indistinguishable from one another.)

5) Let θ be any acute angle such that $tan\theta = \frac{n}{12}$, for some positive integer n. For how many different values of θ will sin θ be rational.

Investigations

1) You are given 3^n coins, where n is a positive integer. All of these coins except 1, which is heavier than the rest, weigh the exact same. Using mathematical induction, prove that n weighings on a balance are sufficient to determine which coin is the heavy one. Once again, using mathematical induction, show that $n - 1$ weighings are insufficient to always determine the heavy coin.

2) Elise and Ellie are identical twins who really like powers of two. Not so surprisingly, they share a car with a six-digit odometer and like odometer readings that only contain digits that are perfect powers of 2. (Thus, the first odometer reading they like is 111111 and the 8^{th} one is 111128, for example.) Determine the 2013^{th} odometer reading they like.

3) Let circle C be the inscribed circle in a right triangle with side lengths 3, 4 and 5. Now, create circles X, Y and Z where each circle is tangent to two sides of the triangle and C. What is the sum of the radii of X, Y and Z?

4) Let $f(x) = x^3 - ax^2 + bx - c = 0$ with non-zero real roots, of which exactly 2 are equal to r_1, with $a^2 - 4b = 0$. What are a, b and c in terms of r_1?

5) Let P_n be a regular polygon with unit area of n sides for any integer $n \geq 3$. Define Q_n to be the regular polygon created by connecting the midpoints of each side of P_n. In terms of n, what is the area of Q_n?

Directions for submission: If you have not done so yet, please register:

http://eecs.ucf.edu/ucf-omc/register.php

Please either email your solutions in a .pdf attachment to ucfomc@gmail.com with the subject "UCF-OMC January 2014 Submission – Name", where you place your first and last name in place of Name. Alternatively, mail a hard-copy of your solutions (if you do this, please retain the originals just in case something gets lost in the mail) to

UCF-OMC
Attn: Arup Guha
4328 Scorpius Street
Orlando, FL 32816

The deadline for receipt for either method is Friday, January 24, 2014. Good luck!

UCF-OMC Staff