Warm Ups

1) If x is 25% larger than z, and y is 50% larger than z, then y is what percent larger than x?

2) Given that $x^2 + y^2 = 12x + 4y - 40$, what is x + 2y?

3) Three standard six-sided dice are rolled. What is the probability that all three values showing are different?

4) Two cars start 100 miles apart, driving towards each other. The first car drives at a steady rate of 30 mph while the second car drives at a steady rate of 20 mph. A bird, flying at 55 mph starts at the first car and flies until it reaches the second car, then reverses direction and goes back to the first car, and so forth, until the cars meet. Assuming that the bird can instantaneously change direction without changing speed, how far did the bird fly in the time that it took the two cars to meet?

5) Jenny has some Pokemon cards she wants to distribute to her friends. If she tries to give an equal number of cards to each of five friends, she's left with two cards. If she tries to give an equal number of cards to each of seven friends, she's left with three cards. If she tries to give an equal number of cards to each of nine friends, she's left with four cards. If Jenny has less than 1000 cards, list all possible number of cards Jenny could have.

Exercises

1) Let a and b be positive integers with a < b such that their greatest common divisor is 12 and their least common multiple is 4320. What are all of the possible values of the ordered pair (a, b)?

2) Circle A is centered at (2, 3) with a radius of 6 and circle B is centered at (8, 3) with a radius of 4. What is the area of the intersection of circles A and B?

3) Two professors arrive at the break room, randomly, in between 10 am and 11 am. Each stays in the break room for exactly *m* minutes. If the probability that they run into each other in the break room is .6. What is the value of *m*?

4) David is taking a matching test where he is matching n words with n definitions. (Each word maps to exactly one of the definitions given.) Unfortunately, David forgot to study and will generate a random matching of words to definitions. What is the expected number of correct pairings he'll choose?

5) What is the sum of the real roots of the following equation?

$$3^{49x+3} + 3^{147x} = 3^{98x+3} + 1$$

<u>Investigations</u>

1) Suppose that α, β, and γ are angles such that

$$\tan(\alpha + \beta) = 2, \tan(\beta + \gamma) = 3, \text{ and } \tan(\alpha + \gamma) = 4.$$

What are possible value(s) of $\tan(\alpha + \beta + \gamma)$?

2) Suppose that α, β, γ, and δ are complex numbers satisfying

$$\alpha + \beta + \gamma + \delta = 2,$$

$$\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = 3,$$

$$\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = 5,$$

$$\alpha\beta\gamma\delta = 7.$$

Find the values of:

(a) $(\alpha + \beta + \gamma)(\alpha + \beta + \delta)(\alpha + \gamma + \delta)(\beta + \gamma + \delta)$
(b) $(\alpha^2 + \beta^2 + \gamma^2)(\alpha^2 + \beta^2 + \delta^2)(\alpha^2 + \gamma^2 + \delta^2)(\beta^2 + \gamma^2 + \delta^2)$

3) How many solutions are there to the inequality

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 200$$

where each of x_1, x_2, x_3, x_4, x_5, and x_6 is a positive odd integer.

4) There are n teams in a round robin tournament. Each team plays every other team exactly once. In each match the chance of each team winning is exactly 50%. What is the probability that there is neither an undefeated team nor a winless team at the end of the tournament?

5) Let a, b, c be positive real numbers. Prove that

$$\frac{a}{b + c} + \frac{b}{c + a} + \frac{c}{a + b} \ge \frac{3}{2}$$

Please either email your solutions in a .pdf attachment to <u>ucfomc@gmail.com</u> with the subject "UCF-OMC January 2015 Submission – Name", where you place your first and last name in place of Name. Alternatively, mail a hard-copy of your solutions (if you do this, please retain the originals just in case something gets lost in the mail) to

UCF-OMC
Attn: Arup Guha
4328 Scorpius Street
Orlando, FL 32816

The deadline for receipt for either method is <u>Friday, February 6, 2015.</u> Good luck!

UCF-OMC Staff