EE 450/550
Test # 1 - Oct 20, 2005 in class

1. An extended object of mass $m$ falling downward under gravity (assume constant acceleration due to gravity $g$) is known to experience a resistive force of the air called drag. We assume that the magnitude of this force is proportional to its speed $v$. Write down the dynamics for the object velocity. Solve the dynamical equation to calculate a formula for $v(t)$ assuming that $v(0) = v_0$. Use symbol $k$ for the proportionality constant. Hint: You May Use Laplace. (50)

2. Linearize the following system

$$\dot{x} = x - \cos x$$

about its fixed point(s). Use the following plot if necessary. Hint: First find fixed point $\bar{x}$ and then apply Taylor series expansion up to linear term about that fixed point. (40)

![Diagram of the function $y = \cos(x)$ and $y = x$ with a point marked at (1.2738, 0.7732)](image)

**Figure 1:**

3. Linear systems are characterized by the property of superposition. Explain what is meant by superposition? (10)
\[ \begin{align*}
\text{Solutions - Test 1} \\
1. \\
\begin{align*}
\text{Diagram:} & \quad \text{mass} \ m, \text{velocity} \ v, \text{force} \ F, \\
& \quad \text{mass} \ m, \text{velocity} \ v, \text{force} \ F.
\end{align*}
\end{align*} \\
\begin{align*}
mv^0 + kv &= mg \\
msV(s) - mv(0) + kv &= \frac{mg}{s} \\
V(s)(ms + k) &= mv_0 + \frac{mg}{s} \\
V(s) &= \frac{mv_0}{ms + k} + \frac{mg}{s(ms + k)} \\
V(s) &= \frac{mv_0}{s + \frac{k}{m}} + \frac{g}{s(s + \frac{k}{m})} \\
&= \frac{mv_0}{s + \frac{k}{m}} + mg \left[ \frac{1}{s} - \frac{1}{s + \frac{k}{m}} \right] \\
U(t) &= v_0 \exp(-\frac{k}{mt}) + \frac{mg}{k} - \frac{mg \exp(-\frac{k}{mt})}{k} \\
&= \left( v_0 - \frac{mg}{k} \right) \exp(-\frac{k}{mt}) + \frac{mg}{k}
\end{align*} \]
2. From the graph, we can see that
\[ x = \cos x \] solves for \( \overline{x} = 0.7392 \)

\[ \dot{x} = x - \cos x \]

For fixed point \( x - \cos x = 0 \)

\[ \Rightarrow \overline{x} = 0.7392 \]

Linearize the right hand side, gives

\[ \dot{x} = f(\overline{x}) + \frac{\partial f(x)}{\partial x}(x - \overline{x}) \]

\[ = 0 + \left( 1 + \sin(\overline{x}) \right) \left( \frac{(x - \overline{x})}{\overline{x}} \right) \]

\[ = (1 + \sin(0.7392))(x - 0.7392) \]

\[ = 1.6737(x - 0.7392) \]

\[ = 1.6737x - 1.2372 \]

3. Given \( y = f(x) \), then for any input \( x_1 \) and \( x_2 \), \( y_1 = f(x_1) \), \( y_2 = f(x_2) \), if given an input \( x_3 = x_1 + x_2 \), then \( y_3 = f(x_1 + x_2) = f(x_1) + f(x_2) \), it means superposition applies.