**EEL 4612 Spring 2013 Test 1** **2/21/13 Duration: 1 hour**

1) Write the input-output transfer function for the state-space system represented by

$$A = \begin{bmatrix} -14 & -56 & -160 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T \quad C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \quad D = 0 \qquad (30)$$

$$\frac{Y(s)}{U(s)} = C(sI-A)^{-1}B + D = \frac{s}{s^3+14s^2+56s+160}$$

2) Given a system defined by: $y^{(3)} + 6y^{(2)} + 11y^{(1)} + 6y^{(0)} = 2u^{(1)} + u^{(0)}$, write it in the following canonical forms (a) Controllable, (b) Observable, (c) Jordan/Diagonal. (30)

$$a_1 = 6,\ a_2 = 11,\ a_3 = 6, \quad b_0 = b_1 = 0,\ b_2 = 2,\ b_3 = 1$$

(a) $$\begin{pmatrix} \dot{x}_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u; \quad y = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

(b) $$\begin{pmatrix} \dot{x}_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} u; \quad y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

(c) $$\frac{Y(s)}{U(s)} = \frac{2s+1}{(s+1)(s+2)(s+3)} = \frac{-1}{2(s+1)} + \frac{3}{(s+2)} - \frac{5}{2(s+3)}$$

$$\begin{pmatrix} \dot{x}_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} u; \quad y = \begin{bmatrix} -\frac{1}{2} & 3 & -\frac{5}{2} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

3) Show that eigenvalues of a matrix are invariant under a linear transformation. (20)

$$|\lambda I - P^{-1}AP| = |\lambda P^{-1}P - P^{-1}AP| = |P^{-1}(\lambda I - A)P|$$
$$= |P^{-1}||(\lambda I - A)||P|$$
$$= |\lambda I - A|$$

4) Given a state-space system represented by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \quad C = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \quad D = 0$$

where $c_i$ are not all zero, find a set of values for $c_1, c_2, c_3$ to make the system unobservable. (20)

$$[C^T \; A^T C^T \; (A^T)^2 C^T] = \begin{bmatrix} c_1 & -6c_3 & -6(c_2 - 6c_3) \\ c_2 & c_1 - 11c_3 & -11c_2 + 60c_3 \\ c_3 & c_2 - 6c_3 & c_1 - 6c_2 + 25c_3 \end{bmatrix}$$

Infinitely many solutions are possible

eg. $\underline{C} = [1 \quad 1 \quad 0] \Rightarrow \begin{bmatrix} 1 & 0 & -6 \\ 1 & 1 & -11 \\ 0 & 1 & -5 \end{bmatrix}$ Rank = 2

$\underline{C} = (1 \quad 1 \quad 2/9)$ $\quad \because Col_3 = -6Col_1 - 5Col_2$

$\underline{C} = (6 \quad 5 \quad 1)$

$\underline{C} = [1 \quad 1 \quad 0.25)$