EEL 6616 Spring 2019 Homework 2

1) Show by using the method of Lagrange multiplier that minimizing $||\theta(k+1) - \theta(k)||^2$ under the constraint $y(k) = w^T(k)\,\theta(k+1)$ results in the following *projection algorithm*

$$\theta(k+1) = \theta(k) - w(k)\,\frac{w^T(k)\,\theta(k) - y(k)}{w^T(k)\,w(k)}$$

2) (a) Show that $A \in R^{2\times 2}$ is positive definite (p.d.) iff $a_{11} > 0, a_{22} > 0$, and $a_{11}a_{22} > (a_{12} + a_{21})^2/4$. (b) Find examples of 2×2 symmetric (but not diagonal) matrices such that $A \geq 0, A < 0$ and A is neither positive nor negative semidefinite. In each case, give $\lambda_i(A)$. (c) Find an example of two symmetric positive definite matrices A and B such that the product is neither symmetric nor positive definite. Give $\lambda_i(AB)$ and $\lambda_i((AB) + (AB)^T)$.

3) Let $w(t) = sin(t)$. (a) Find and plot $\lambda(t) = \int_0^t w^2(t)\,dt$. (b) Show that there exist α_1, α_2 and $\delta > 0$ such that $\alpha_2 \geq \int_{t_0}^{t_0+\delta} w^2(t)\,dt \geq \alpha_1$ for all $t_0 \geq 0$. (c) Show that the following limit exists, independently of t_0

$$R_w = \lim_{\delta\to\infty} \frac{1}{\delta} \int_{t_0}^{t_0+\delta} w^2(t)\,dt$$

where $R_w > 0$. d) Repeat part c) with $w(t) = \begin{pmatrix} \sin(t) \\ a\sin(t+\phi) \end{pmatrix}$. What condition must a, ϕ satisfy so that R_w is p.d.