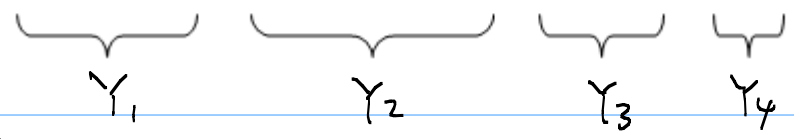


②



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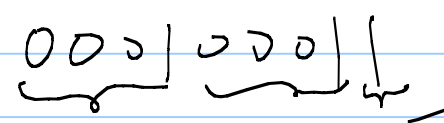


9/8/2011

$Y_i$ : # of trials in cycle  $i$

③

||



$$P(N=n) = P(Y_1 \geq 3 \ \& \ Y_2 \geq 3 \ \dots$$

$$\ \& \ Y_n \leq 2)$$

$Y_i \sim$  Geometric with  $p$

$$P(N=n) = P(Y_1 \geq 3) \cdot P(Y_2 \geq 3) \ \dots \ P(Y_n \leq 2)$$

$$P(Y \leq 2) = P(Y=1) + P(Y=2) = p + (1-p) \cdot p = 2p - p^2$$

$$P(Y \geq 3) = 1 - P(Y \leq 2) = 1 - 2p + p^2 = (1-p)^2$$

$$P(N=n) = (1-p)^{2(n-1)} \cdot (2p - p^2)$$

$$E[N] = \sum_{n=2}^{\infty} P(N=n) \cdot n = p(2-p) \cdot \sum_{n=2}^{\infty} n(1-p)^{2n-2}$$

$$\alpha = (r-p)^2 \quad \hookrightarrow \quad \sum_{n=2}^{\infty} n \alpha^{n-1} = S$$

$$S = 2\alpha + 3\alpha^2 + 4\alpha^3 + 5\alpha^4 + \dots$$

$$\alpha S = 2\alpha^2 + 3\alpha^3 + 4\alpha^4 + 5\alpha^5 + \dots$$

$$(1-\alpha)S = \alpha + \alpha^2 + \alpha^3 + \dots = \alpha + \frac{\alpha}{1-\alpha}$$

$$P_n(t + \Delta t) = P_{n-1}(t)\lambda\Delta t + P_n(t)(1 - \lambda\Delta t) + o(\Delta t)$$

$$P_n(t + \Delta t) - P_n(t) = P_{n-1}(t)\lambda\Delta t - P_n(t)\lambda\Delta t + o(\Delta t)$$

$$\frac{P_n(t + \Delta t) - P_n(t)}{\Delta t} = P_{n-1}(t)\lambda - P_n(t)\lambda + \frac{o(\Delta t)}{\Delta t}$$

①.  $(n-1)$  arrived by  $t$ , 1 arrives  $(t, t+\Delta t)$

②.  $n$  arrived by  $t$ , 0 arrives in  $(t, t+\Delta t)$

$$\square \quad dP_0(t)/dt = -\lambda P_0(t) \quad \Rightarrow \quad P_0(t) = c e^{-\lambda t} \quad P_0(0) = 1$$

$$\Rightarrow \quad c = 1$$