M_X: liability for claims w/o deductible for one week

Note Title 9/25/2012

M_Y: " " " with " " "

X: Poisson process for claims w/o deductible
\# of arrivals

A: claim amount $\sim$ expo. distr. $E[A] = 700$

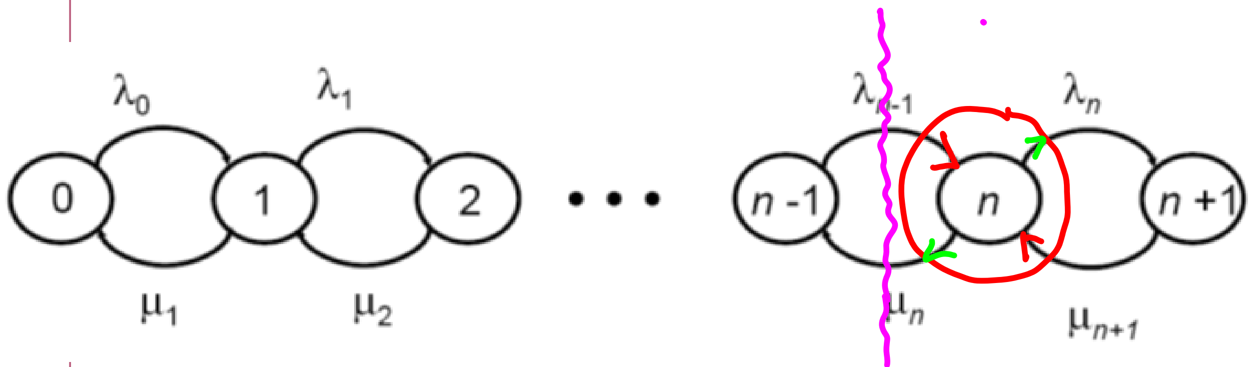
$E[M_X] = E[A] \cdot E[X]$

$= \frac{1}{3} \times 100 \times 700$

$E[M_Y] = (E[A] - 250) \cdot E[Y]$

$= 450 \times \frac{2}{3} \times 100$

avg. liability for 13 weeks $= 13 \times \left(E[M_X] + E[M_Y] \right)$



$\mu_{n+1} \cdot P_{n+1} + \lambda_{n-1} \cdot P_{n-1}$

$= P_n \lambda_n + \mu_n \cdot P_n$

$$\lambda_{n-1} P_{n-1} + \mu_{n+1} P_{n+1} = (\lambda_n + \mu_n) P_n, \qquad n \geq 1$$

$P_{n-1} \cdot \lambda_{n-1} = P_n \cdot \mu_n$

$$
\begin{aligned}
P_{ij}^{n+m} &= P(X_{n+m} = j \mid X_0 = i), \\
&= \sum_{k=0}^{\infty} P(\underbrace{X_{n+m} = j}_{A}, \underbrace{X_n = k}_{B} \mid X_0 = i), \\
&= \sum_{k=0}^{\infty} P(X_{n+m} = j \mid X_n = k, X_0 = i) P(X_n = k \mid X_0 = i),
\end{aligned}
$$

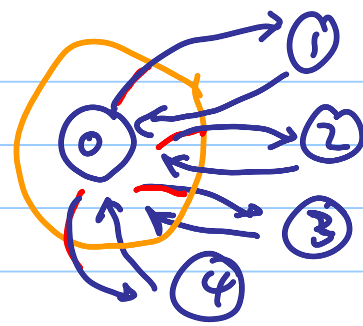
$P(A \cap B)$

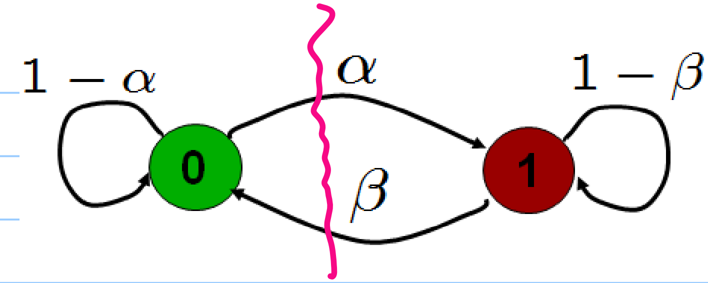
$P(A \mid B) \cdot P(B)$

$$
\begin{aligned}
\pi &= \pi P, \\
\pi \mathbf{1} &= 1
\end{aligned}
$$

Why?

$\pi = (\pi_0, \pi_1, \pi_2, \dots \pi_n)$





$$\pi_0(P_{01} + P_{02} + \dots) = \pi_0(1 - P_{00})$$

$$= \pi_1 \cdot P_{10} + \pi_2 P_{20} + \dots$$

$$\begin{cases} \pi_0 \alpha = \pi_1 \cdot \beta \\ \pi_0 + \pi_1 = 1 \end{cases}$$

$$\Rightarrow \pi_0 = \pi_0 P_{00} + \pi_1 P_{10} + \pi_2 P_{20} + \dots$$

$$\pi_0 = [\pi_0 \ \pi_1 \ \pi_2 \ \dots] \cdot \begin{bmatrix} \boxed{\begin{matrix} P_{00} \\ P_{10} \\ P_{20} \\ \vdots \end{matrix}} & P \end{bmatrix}$$

- $\pi(P - I) = 0$
- $\pi P = \pi$, $\pi[1\ 1\ 1 \ldots 1]^T = 1$

$\pi = \pi P$

$$(\pi_0 \quad \pi_1 \quad \pi_2) \cdot \begin{pmatrix} P_{00}-1 & P_{01} & P_{02} \\ P_{10} & P_{11}-1 & P_{12} \\ P_{20} & P_{21} & P_{22}-1 \end{pmatrix} = (0 \quad 0 \quad 0)$$

$$(\pi_0 \quad \pi_1 \quad \pi_2) \underbrace{\begin{bmatrix} 1 & P_{01} & P_{02} \\ 1 & P_{11}-1 & P_{12} \\ 1 & P_{21} & P_{22}-1 \end{bmatrix}}_{A} = [1 \quad 0 \quad 0]$$

$$\pi = [1 \quad 0 \quad 0] \cdot A^{-1}$$