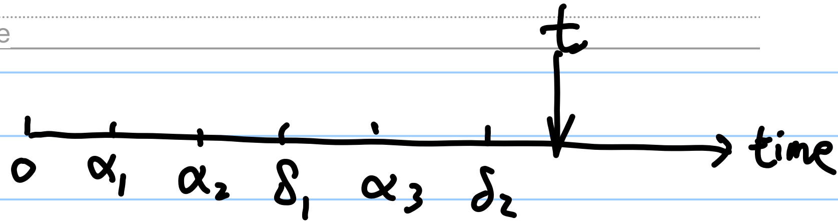
CDA6530, lecture 15

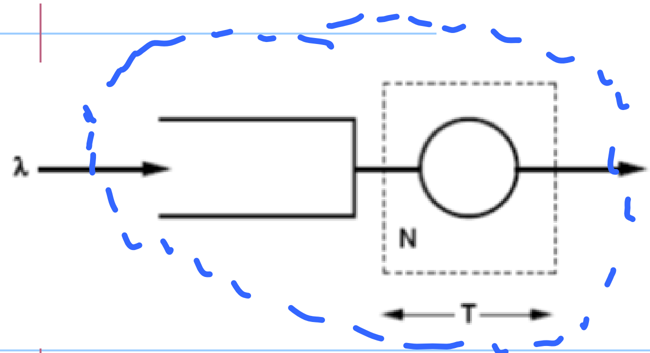
Note Title 10/11/2012

$$\gamma(t) = \sum_{n=1}^{\alpha(t)} \min\{d_n, t\} - a_n = \int_0^t N(s)ds$$



$$\gamma(t) = (\delta_1 - \alpha_1) + (\delta_2 - \alpha_2) + (t - \alpha_3)$$

- $T_t = \gamma(t)/\alpha(t)$
- $N_t = \gamma(t)/t$ $\Rightarrow$ $N_t = \frac{\alpha(t)\cdot T_t}{t} = \lambda_t \cdot T_t$

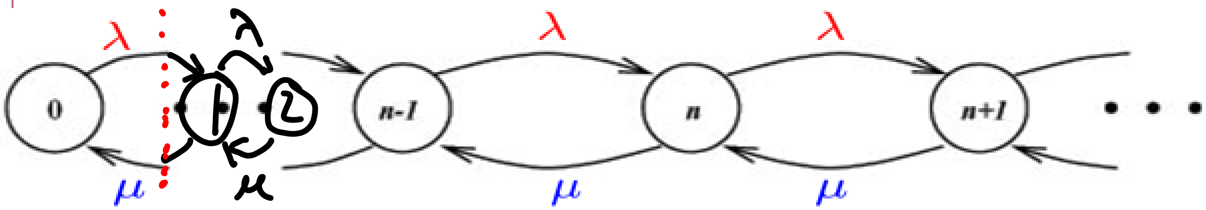


$N = \lambda T$

$\Rightarrow E[N] = \lambda \cdot E[T]$

N: # of customer in (queue + server)

T: sojourn time of customer



$$\pi_0 \cdot \lambda = \pi_1 \cdot \mu \Rightarrow \pi_1 = \frac{\lambda}{\mu} \cdot \pi_0 = \rho \pi_0$$

$$\sum_{i=0}^{\infty} \pi_i = 1$$

$$\pi_1 \lambda = \pi_2 \cdot \mu \Rightarrow \pi_2 = \frac{\lambda}{\mu} \pi_1 = \rho^2 \pi_0$$

$$\pi_n = \rho^n \cdot \pi_0$$

$$\pi_0 + \pi_1 + \cdots = \pi_0 + \rho\pi_0 + \rho^2\pi_0 + \cdots = 1$$

$$\Rightarrow \pi_0 \underbrace{(1 + \rho + \rho^2 + \rho^3 + \cdots)}_{\frac{1}{1-\rho}} = 1 \Rightarrow \pi_0 = 1 - \rho$$

$\Rightarrow \rho = 1 - \pi_0$: Prob. server is busy

$$E[N] = \sum_{k=1}^{\infty} k\pi_k = \pi_0 \underbrace{\sum_{k=1}^{\infty} k\rho^k}_{S} = \frac{\rho}{1-\rho}$$

$$S = \rho + 2\rho^2 + 3\rho^3 + \cdots$$

$$-)\ \rho S = \rho^2 + 2\rho^3 + 3\rho^4 \cdots$$

$$(1-\rho)S = \rho + \rho^2 + \rho^3 + \cdots = \frac{\rho}{1-\rho}$$

$$\pi_0 S = \frac{\rho}{(1-\rho)^2} \cdot (1-\rho) = \frac{\rho}{1-\rho}$$

$$E[W] = E[(N-1)X] + E[R] = E[N] \cdot E[X]$$

$E[N] \hookrightarrow \frac{\rho}{1-\rho}$ $\quad$ $E[X] \hookrightarrow \frac{1}{\mu}$ $\quad$ $\rho = \lambda / \mu$

$$E[W] = \frac{\lambda}{\mu - \lambda} \cdot \frac{1}{\mu}$$

$$E[T] = \frac{1}{\mu} + E[W] = \frac{1}{\mu - \lambda}$$

$$\hookrightarrow \frac{1}{\mu} + \frac{1}{\mu} \cdot \frac{\lambda}{\mu - \lambda} = \frac{1}{\mu} \cdot \left(\frac{\mu - \lambda + \lambda}{\mu - \lambda} \right) = \frac{1}{\mu - \lambda}$$

little law : $E[N] = \lambda \cdot E[T]$

$$\Rightarrow E[T] = \frac{E[N]}{\lambda} = \frac{\lambda / \mu}{1 - \lambda / \mu} \cdot \frac{1}{\lambda} = \frac{\lambda}{\mu - \lambda} \cdot \frac{1}{\lambda} = \frac{1}{\mu - \lambda}$$

$\lambda = 80 \quad \mu = 100$

$\Rightarrow \rho = 0.8$

① $E[N] = \frac{\rho}{1-\rho} = \frac{0.8}{0.2} = 4$

② $E[W] = \frac{\lambda}{\mu - \lambda} \cdot \frac{1}{\mu} = \frac{80}{20} \times \frac{1}{100} = 0.04\,sec$

③ $E[T] = \frac{1}{\mu - \lambda} = \frac{1}{20} = 0.05\,sec = 50\,ms$.

④ $P(N=0) = \pi_0 = 1 - \rho = 0.2$ $\quad$ ⑤ $P(N > 5) = 1 - P(N \leq 5)$

$$= 1 - [\pi_0 + \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5]$$

Q6: μ? $\Longrightarrow E[T] = 0.02$ sec

$E[T] = \frac{1}{\mu - \lambda}$ $\quad \frac{1}{\mu - 80} = 0.02 \Longrightarrow \mu = 50 + 80$

$= 130$

$\Longrightarrow$ bandwidth $= 130$ kbps