CDA 6530

Note Title

11/6/2012

$$G_X(z) \equiv E[z^X] = \sum_{k=0}^{\infty} p_k z^k$$

$$\frac{\pi_1 z + \pi_2 z^2 + \cdots}{z} = \pi_1 + \frac{1}{z}(\pi_2 z^2 + \pi_3 z^3 \cdots)$$

$$\rho = \frac{\lambda}{\mu}$$

$$(\lambda+\mu)\pi_i = \lambda\pi_{i-1} + \mu\pi_{i+1}, \quad i = 1, \ldots$$

$$\lambda\pi_0 = \mu\pi_1 \Rightarrow \rho\pi_0 = \pi_1$$

$$(1+\rho)\pi_i = \rho\pi_{i-1} + \pi_{i+1} \quad (\times z^i)$$

$$(1+\rho)\pi_i z^i = \rho\pi_{i-1} \cdot z^{i-1} \cdot z + \frac{\pi_{i+1} \cdot z^{i+1}}{z}$$

$i = 1, 2, 3, \ldots$

$$(1+\rho)\pi_0 = \pi_1 + \pi_0$$

$$(1+\rho)\sum_{k=0}^{\infty} \pi_k z^k = \rho z \sum_{k=0}^{\infty} \pi_k z^k + \frac{1}{z}\left(\sum_{k=0}^{\infty} \pi_k z^k - \pi_0\right) + \pi_0$$

$$\Rightarrow (1+\rho)\, G_N(z) = \rho z \cdot G_N(z) + \frac{1}{z}\left(G_N(z) - \pi_0\right) + \pi_0$$

$$G_N(z=1) = \sum_{k=0}^{\infty} \pi_k \cdot 1^k = \sum_{k=0}^{\infty} \pi_k = 1$$

$$G_N(z) = \frac{\pi_0}{1-\rho z}, \qquad = \frac{1-\rho}{1-\rho z}$$

Let $z = 1 \Rightarrow 1 = \frac{\pi_0}{1-\rho} \Rightarrow \pi_0 = 1-\rho$

$$F_X^*(s) \equiv E[e^{-sX}] = \int_0^\infty f_X(x)e^{-sx}dx$$

$$E[X] = -\frac{d}{ds}F_X^*(s)\Big|_{s=0}$$

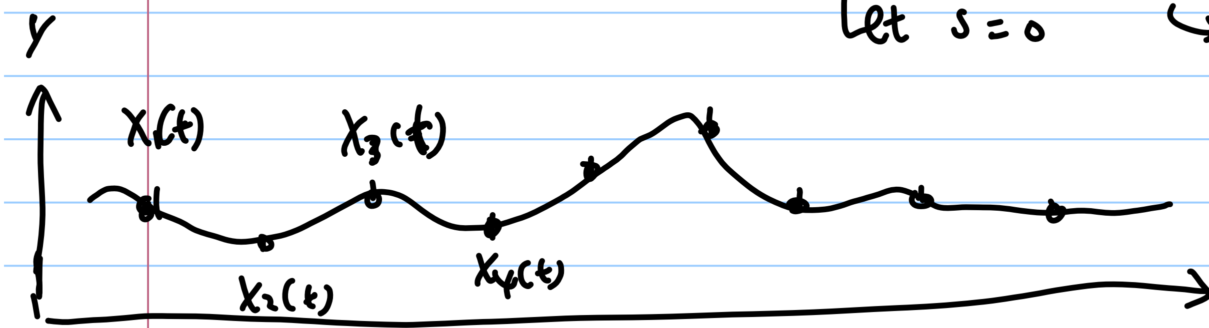
$$E[X^i] = (-1)^i\frac{d}{ds}F_X^*(s)\Big|_{s=0}$$

$$E[X] = \int_0^\infty f_X(x) \cdot x \, dx$$

$$\frac{dF_X^*(s)}{ds} = \int_0^\infty f_X(x)\frac{de^{-sx}}{ds}dx$$

$$= \int_0^\infty f(x) \cdot (-x) e^{-sx} dx$$

Let $s = 0$ $\hookrightarrow \int_0^\infty f(x)(-x) \cdot 1 \, dx = -E[X]$



- $X_i(t) = (U-0.5) + (X_{i-1}(t-1) + X_{i+1}(t-1)) / 2$

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Simul_N = 1000; n=100; X = ones(n,1);
for k=1:Simul_N,
    U = rand(n,1);
    X(1) = (U(1) - 0.5) + X(2)/2;
    for i=2:n-1,
        X(i) = (U(i) - 0.5) + (X(i-1) + X(i+1)) / 2;
    end
    X(n) = (U(n) - 0.5) + X(n-1) / 2;
    % display or save X value for time k
end
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$$X_2(t) = U-0.5 + \frac{X_1(t-1) + X_3(t-1)}{2}$$

$i=2$

$$X_2(t) = U-0.5 + \frac{X_1(t) + X_3(t-1)}{2}$$

$i=3$

$$X_3(t) = U-0.5 + \frac{X_2(t) + X_4(t-1)}{2}$$