

1. (10 points) [UseModels] Write an iterative function

```
MyLength: <fun {$ <List T>}: <Int> >
```

that takes a list *Lst* of values of some type *T* and returns the number of elements in *Lst*, that is, it returns the size of *Lst*.

Your solution must have iterative behavior, and must be written using tail recursion. Don't use the built-in Oz function `Length`, don't use any higher-order functions, and don't use the Oz `for` loop syntax in your solution. (You are supposed to know what these directions mean.)

The following are examples, that use the `Test` procedure from the homework.

```
\insert 'MyLength.oz'  
{Test {MyLength nil} '==' 0}  
{Test {MyLength [c]} '==' 1}  
{Test {MyLength [b a]} '==' 2}  
{Test {MyLength [b b a]} '==' 3}  
{Test {MyLength [a b b a]} '==' 4}  
{Test {MyLength [b d a c a b b a]} '==' 8}  
{Test {MyLength [um no no no no yes no yes no]} '==' 9}  
{Test {MyLength [4 0 7 5 5 5 1 2 1 2]} '==' 10}
```

2. (10 points) [UseModels] Write a function

SquareAll: `<fun {$ <List <Number> >}: <List <Number> >`

that takes a list of numbers, Nums, and produces a list containing the squares of each number in Nums, in the original order. The following are examples.

```
\insert 'SquareAll.oz'  
{Test {SquareAll nil} '==' nil}  
{Test {SquareAll [9 8 7 6 5 4 3 2 1 0 10]}  
  '==' [81 64 49 36 25 16 9 4 1 0 100]}  
{Test {SquareAll [10 3 10]} '==' [100 9 100]}  
{Test {SquareAll [11 22 2009]} '==' [121 484 4036081]}
```

3. (15 points) [UseModels] Using Filter and Map, write the function

```
SelectAndMap: <fun {$ <List S> <fun {$ S}: Bool> <fun {$ S}: T>}: <List T> >
```

that for some types S and T takes three arguments: Ls, which is a list of values of type S, Pred, which is a boolean-valued function of type <fun {\$ S}: Bool>, and F, which is a function of type <fun {\$ S}: T>. The function you are to write, SelectAndMap, selects elements E of Ls for which {Pred E} returns true, and returns a list of the result of applying F to those selected elements (preserving the original order). The following are examples.

```
\insert 'SelectAndMap.oz'
local
  fun {AlwaysTrue _} true end
  fun {Odd N} N mod 2 == 1 end
  fun {Even N} N mod 2 == 0 end
  fun {Double X} X*2 end
in
  {Test {SelectAndMap nil AlwaysTrue Double} '==' nil}
  {Test {SelectAndMap 4|5|6|7|nil AlwaysTrue Double} '==' 8|10|12|14|nil}
  {Test {SelectAndMap 4|5|6|7|nil Odd Double} '==' 10|14|nil}
  {Test {SelectAndMap 4|5|6|7|nil Even Double} '==' 8|12|nil}
  {Test {SelectAndMap 1|2|3|1|4|5|6|7|nil fun {$ X} X>3 end Double}
    '=='8|10|12|14|nil}
  {Test {SelectAndMap 1|2|3|1|4|5|6|7|nil fun {$ X} X>3 end fun {$ X} X*10 end}
    '==' 40|50|60|70|nil}
  {Test {SelectAndMap 1|2|3|1|4|5|6|7|nil fun {$ X} X <= 3 end fun {$ X} X end}
    '==' 1|2|3|1|nil}
  {Test {SelectAndMap 1|2|3|1|4|5|6|7|nil fun {$ _} false end fun {$ X} X end}
    '==' nil}
  {Test {SelectAndMap [o o p s l a] AlwaysTrue fun {$ X} [X] end}
    '==' [[o] [o] [p] [s] [l] [a]]}
end
```

Your solution must use both Filter and Map (but you can also write additional helping functions if you wish).

4. (15 points) [UseModels] Write a function

Project: `<fun {$ <List <Pair S T> > S} : <List T> >`

that for some types *S* and *T*, takes an association list, *Pairs* (that is, a list of #-tuples of *S* and *T* elements), and an element of type *S*, *E*, and returns a list of all the elements of type *T* to which *E* is associated in *Pairs* (by being in the same pair). That is, whenever there is a pair *SVal#TVal* in *Pairs* and *E == SVal*, then *TVal* is in the result. The relative order of the values of type *T* that are in the result should be the same as their relative order in *Pairs*. The following are examples.

```
\insert 'Project.oz'
\insert 'TestingNoStop.oz'
{StartTesting 'Project'}
{Test {Project [1#2 2#3 3#4 2#5 1#6] 2} '==' [3 5]}
{Test {Project [a#1 a#0 b#2 b#~1 c#3 c#4 d#4 d#5] a} '==' [1 0]}
{Test {Project [a#1 a#0 b#2 b#~1 c#3 c#4 d#4 d#5] b} '==' [2 ~1]}
{Test {Project [a#1 a#0 b#2 b#~1 c#3 c#4 d#4 d#5] b} '==' [2 ~1]}
{Test {Project [a#1 a#0 b#2 b#~1 c#3 c#4 d#4 d#5] c} '==' [3 4]}
{Test {Project [a#1 a#0 b#2 b#~1 c#3 c#4 d#4 d#5] z} '==' nil}
{Test {Project nil q} '==' nil}
{Test {Project [q#ok] q} '==' [ok]}
```

5. (10 points) [Concepts] [UseModels] Write a curried version of the function `Project`, from question 4 on the previous page. The function you are to write should be called `CurriedProject`. That is, write

```
CurriedProject: <fun {$ <List <Pair S T> >}: <fun {$ S}: <List T> > >
```

that for some types `S` and `T`, takes an association list, named `Pairs`, of type `<List <Pair S T> >`, and returns a function that takes an element, `E`, of type `S`, and returns a list of all the elements of type `T` to which `E` is associated in `Pairs` (by being in the same pair). To save time, just call `Project` in your answer, instead of writing out the code for `Project` again.

The following are examples.

```
\insert 'CurriedProject.oz'
\insert 'TestingNoStop.oz'
{StartTesting 'CurriedProject'}
{Test {{CurriedProject [1#2 2#3 3#4 2#5 1#6]} 2} '==' [3 5]}
{Test {{CurriedProject [a#1 a#0 b#2 b#~1 c#3 c#4 d#4 d#5]} a} '==' [1 0]}
{Test {{CurriedProject [a#1 a#0 b#2 b#~1 c#3 c#4 d#4 d#5]} b} '==' [2 ~1]}
{Test {{CurriedProject [a#1 a#0 b#2 b#~1 c#3 c#4 d#4 d#5]} b} '==' [2 ~1]}
{Test {{CurriedProject [a#1 a#0 b#2 b#~1 c#3 c#4 d#4 d#5]} c} '==' [3 4]}
{Test {{CurriedProject [a#1 a#0 b#2 b#~1 c#3 c#4 d#4 d#5]} z} '==' nil}
{Test {{CurriedProject nil} q} '==' nil}
{Test {{CurriedProject [q#ok]} q} '==' [ok]}
```

Please write your answer below.

```
\insert 'Project.oz' % so you can use Project from the previous problem
```

6. (20 points) [UseModels] This problem is about the following grammar for boolean expressions:

```

⟨Bexp⟩ ::=
  andExp(⟨Bexp⟩ ⟨Bexp⟩)
  | orExp(⟨Bexp⟩ ⟨Bexp⟩)
  | notExp(⟨Bexp⟩)
  | comp(⟨Comp⟩)
⟨Comp⟩ ::= equals(⟨Atom⟩ ⟨Atom⟩)
  | notequals(⟨Atom⟩ ⟨Atom⟩)

```

Write a function `Subst`: `<fun {$ <BExp> <Atom> <Atom>}: <BExp> >` that takes a `⟨BExp⟩` record, `E`, and two atoms `Old` and `New`, and returns a `⟨BExp⟩` that is just like `E`, except that in each `⟨Comp⟩` record, each `⟨Atom⟩` that is equal to `Old` is replaced by `New`. The following are examples using the `Test` function from the homework.

```

\insert 'Subst.oz'
{StartTesting 'Subst'}
{Test {Subst andExp(comp(notequals(o n))
  comp(equals(a o)))
  o sym} '==' andExp(comp(notequals(sym n))
  comp(equals(a sym)))}}
{Test {Subst orExp(comp(equals(q r))
  andExp(comp(notequals(o n)) comp(equals(r q))))
  r newvar}
  '==' orExp(comp(equals(q newvar))
  andExp(comp(notequals(o n)) comp(equals(newvar q))))}}
{Test {Subst notExp(comp(equals(x y))) x a} '==' notExp(comp(equals(a y)))}}
{Test {Subst notExp(andExp(comp(equals(q oldr))
  andExp(comp(notequals(o n)) comp(equals(oldr q))))))
  oldr newvar}
  '==' notExp(andExp(comp(equals(q newvar))
  andExp(comp(notequals(o n)) comp(equals(newvar q))))))}}
{Test {Subst comp(equals(x y)) y z} '==' comp(equals(x z))}}
{Test {Subst comp(equals(x y)) x a} '==' comp(equals(a y))}}
{Test {Subst comp(notequals(o n)) o sym} '==' comp(notequals(sym n))}}

```

Be sure to follow the grammar!

7. (20 points) [UseModels] This problem is about the following grammar for “unit tests” for functions of type `<fun {$ S}: T>`, where S and T are arbitrary types.

```

<UnitTest S T> ::=
  testcase(arg: <S> expect: <T>)
  | testsuite(<list <UnitTest S T> >)

```

Write a function `AllPass: <fun {$ <UnitTest S T> <fun {$ S}: T}: Bool>` that takes a `<UnitTest S T>` record, `TestRec` and a function `F` (which takes arguments of type `S` and returns results of type `T`), and returns `true` when `F` passes each test in `TestRec`. `AllPass` returns `false` otherwise. A unit test record of the form `testcase(arg: X expect: E)` passes for a function `F` when `{F X} == E`. A unit test record of the form `testsuite([UT1 ... UTn])` passes for a function `F` when each unit test `UT1 ... UTn` passes for function `F`.

To simplify this problem, we will make the somewhat unrealistic assumption that the function given to `AllPass` never throws an exception and always terminates normally when called. The following are examples using the `Test` function from the homework.

```

\insert 'AllPass.oz'
{Test {AllPass testcase(arg: 1 expect: 2) fun {$ X} X+1 end} '==' true}
{Test {AllPass testsuite(nil) fun {$ X} X+1 end} '==' true}
{Test {AllPass testsuite(testcase(arg: 1 expect: 3)|nil) fun {$ X} X+2 end}
'==' true}
{Test {AllPass testsuite([testcase(arg: 1 expect: 3) testsuite([testcase(arg: 4 expect: 6)])
                        testsuite(nil) testcase(arg: 10 expect:12)
                        testsuite([testcase(arg: 9 expect: 11) testsuite([testcase(arg: 0 expect: 2)])])])
      fun {$ X} X+2 end}
'==' true}
% Some tests where AllPass returns false (below)
{Test {AllPass testcase(arg: 1 expect: 3) fun {$ X} X+10 end} '==' false}
{Test {AllPass testsuite([testcase(arg: 1 expect: 3)
                        testsuite([testcase(arg: 4 expect: 6)])])
      fun {$ _} 3 end}
'==' false}
{Test {AllPass testsuite([testcase(arg: 1 expect: 3) testsuite([testcase(arg: 4 expect: 6)])
                        testsuite(nil) testcase(arg: 10 expect:12)
                        testsuite([testcase(arg: 9 expect: 11) testsuite([testcase(arg: 0 expect: 2)])])])
      fun {$ _} 3 end}
'==' false}

```

Be sure to follow the grammar!