Name: $\qquad$

# COP 4020 - Programming Languages I <br> Test on Haskell and Functional Programming 

## Special Directions for this Test

This test has 8 questions and pages numbered 1 through 8 .
This test is open book and notes, but no electronics.
If you need more space, use the back of a page. Note when you do that on the front.
Before you begin, please take a moment to look over the entire test so that you can budget your time.
Clarity is important; if your programs are sloppy and hard to read, you may lose some points. Correct syntax also makes a difference for programming questions. Take special care with indentation and capitalization in Haskell.
When you write Haskell code on this test, you may use anything we have mentioned in class that is built-in to Haskell. But unless specifically directed, you should not use imperative features (such as the IO type). You are encouraged to define functions not specifically asked for if they are useful to your programming; however, if they are not in the standard Haskell Prelude, then you must write them into your test. (That is, your code may not import modules other than the Prelude.)

## Hints

If you use functions like filter and map whenever possible, then you will have to write less code on the test, which will mean fewer chances for making mistakes and will leave you more time to be careful. The problem will note explicitly if you are prohibited from using such functions, but by default you can. In the "follow the grammar" problems the examples may be very extensive and take a long time to read if you read every detail. But the basic idea of the recursion is usually clear from the description, simple examples, and the "follow the grammar" idea of recursing everywhere possible. So look to the examples to confirm your understanding and don't spend too much time reading examples after you understand the problem.

## For Grading

| Question: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Points: | 5 | 10 | 10 | 15 | 15 | 15 | 15 | 15 | 100 |
| Score: |  |  |  |  |  |  |  |  |  |

1. (5 points) [Concepts] In Haskell, which of the following is equivalent to the list [5, $0,2,1]$ ? Circle the letter of the correct answer.
A. $(((([]: 5): 0): 2): 1)$
B. $(5:(0:(2:(1:[]))))$
C. $(5: 0)++(2:(1:[]))$
D. $(((5++0)++2)++1)$
E. $(5++(0++(2++1)))$
2. (10 points) [UseModels] In Haskell, write the function:
```
scale :: Integer -> [Integer] -> [Integer]
```

that takes an Integer, factor, and a list of Integers, 1st, and returns a list of Integers that is like 1st, except that each element in the result is factor times the corresponding element of list. The following are examples, written using the Testing module from the homework.

```
tests :: [TestCase [Integer]]
tests = [(eqTest (scale 32 []) "==" [])
        ,(eqTest (scale 17 [10]) "==" [170])
        ,(eqTest (scale 6 [1,2,1]) "==" [6,12,6])
        ,(eqTest (scale 50 [1 .. 7]) "==" [50, 100, 150, 200, 250, 300, 350])
        ,(eqTest (scale 3 [1,2 .. 100]) "==" [3,6 .. 300])
        ,(eqTest (scale 31 [10,11 .. 1000]) "==" [310,341 .. 31000]) ]
```

3. (10 points) [Concepts] [UseModels] Consider the data type Outcome defined below.
```
data Outcome = StrictlyIncreasing | AllEqual | StrictlyDecreasing | Unordered
    deriving (Eq,Show)
```

In Haskell, write the polymorphic function

```
order :: (Integer, Integer, Integer) -> Outcome
```

which takes a triple of integers, ( $i, j, k$ ), and returns an Outcome that says how $i, j$, and $k$ are ordered from left to right. That is: it returns StrictlyIncreasing if $i<j$ and $j<k$, it returns AllEqual if $i==j$ and $j==k$, it returns StrictlyDecreasing if $i>j$ and $j>k$, and otherwise it returns Unordered. The following are examples, written using the Testing module from the homework.

```
tests :: [TestCase Outcome]
tests = [(eqTest (order (3,4,10)) "==" StrictlyIncreasing)
    ,(eqTest (order (7,7,7)) "==" AllEqual)
    ,(eqTest (order (7,2,0)) "==" StrictlyDecreasing)
    ,(eqTest (order (9999,100,10000000)) "==" Unordered)
    ,(eqTest (order (99999,99999,99999)) "==" AllEqual)
    ,(eqTest (order (3,7,4)) "==" Unordered) ]
```

4. [UseModels] In this problem you will write a function in two different ways. The function you are to write in Haskell is
```
averages :: [(Double, Double, Double)] -> [Double]
```

which takes a list of triples of Doubles, 1st, and returns a list of Doubles such that each element of the result is the average (i.e., the arithmetic mean) of the corresponding triple. The following are examples, written using the FloatTesting module from the homework.

```
tests :: [TestCase [Double]]
tests =
    [(vecWithin (averages []) "~=~" [])
    ,(vecWithin (averages [(3.0,4.0,5.0)]) "~=~" [4.0])
    ,(vecWithin (averages [(4.0,5.0,3.0)]) "~=~" [4.0])
    ,(vecWithin (averages [(4.0,5.0,3.0),(5.0,15.0,10.0)]) "~=~" [4.0,10.0])
    ,(vecWithin (averages [(4.0,5.0,3.0),(5.0,15.0,10.0)]) "~=~" [4.0,10.0])
    ,(vecWithin (averages [(4.0,5.0,3.0),(5.0,15.0,10.0),(81.3,86.0,99.0)])
                            "~=~" [4.0,10.0,(81.3+86.0+99.0)/3.0])
    ,(vecWithin (averages [(81.3,86.0,99.0), (107.0,100.0,105.3),(0.0,0.0,0.0),
                                    (22.5,34.4,9.0), (8.0,16.0,32.0), (1.0,-0.5,-0.25)])
    "~=~" [(81.3+86.0+99.0)/3.0,(107.0+100.0+105.3)/3.0,0.0,
                                    (22.5+34.4+9.0)/3.0,(8.0+16.0+32.0)/3.0,0.25/3.0]) ]
```

(a) (5 points) Write averages using a list comprehension.
(b) (10 points) Write averages without using a list comprehension, by writing out the recursion explicitly (without using map or other higher-order functions).
5. (15 points) [Concepts] [UseModels] In Haskell write a function

```
duplicate :: [a] -> [a]
```

which for all types a, takes a list of elements of type a, 1st, and returns a list that is twice as long as lst and in which each element of lst is duplicated, and so appears twice in the result. The following are tests. In the tests, recall that Strings in Haskell are lists of Chars, so it is the Chars in the list that are to be duplicated.

```
tests :: [TestCase String] -- recall String = [Char]
tests =
    [(eqTest (duplicate []) "==" [])
    ,(eqTest (duplicate ['a','b','a']) "==" ['a','a','b','b','a','a'])
    ,(eqTest (duplicate "acid") "==" "aacciidd")
    ,(eqTest (duplicate "mississippi") "==" "mmiissssiissssiippppii")
    ,(eqTest (duplicate "even spaces!") "==" "eevveenn ssppaacceess!!")
    ]
```

6. (15 points) [UseModels] In Haskell, write the function
```
count :: Char -> [Char] -> Integer
```

which takes a Char, what, and a list of Chars (i.e., a String), s, and returns the number of times that what occurs in s . The following are examples.

```
tests :: [TestCase Integer]
tests = [(eqTest (count 'c' []) "==" 0)
    ,(eqTest (count 'c' ['d','a','d','a']) "==" 0)
    ,(eqTest (count 'c' "cade") "==" 1)
    ,(eqTest (count 'c' "brocade") "==" 1)
    ,(eqTest (count 'c' "succinct") "==" 3)
    ,(eqTest (count 'x' "xylem") "==" 1)
    ,(eqTest (count 'i' "mississippi") "==" 4)
    ,(eqTest (count 's' "mississippi") "==" 4)
    ,(eqTest (count ' ' "now is the time for all") "==" 5) ]
```

7. (15 points) [UseModels] This problem uses the type BinaryRelation
```
type BinaryRelation a b = [(a,b)]
```

In this problem you will write the function

```
update :: (Eq a) => a -> b -> (BinaryRelation a b) -> (BinaryRelation a b)
```

This function take a key (of some equality type a), and a val (of some type b), and binary relation (i.e., a list of pairs), rel, and returns a binary relation that is just like rel except that all pairs ( $k, v$ ) in rel such that key $==\mathrm{k}$ are replaced in the result by (key, val). The following are examples.

```
tests :: [TestCase (BinaryRelation String Integer)]
tests =
    [(eqTest (update "happy" 6000 []) "==" [])
    ,(eqTest (update "happy" 6000 [("happy",3),("sad",2)])
        "==" [("happy",6000),("sad",2)])
    ,(eqTest (update "Scott" 2014 [("Scott",2010),("Crist",2006),("Bush",2002)])
        "==" [("Scott", 2014), ("Crist", 2006), ("Bush", 2002)])
    ,(eqTest (update "Voyager" 13000000000
                            [("Cassini",12000000000), ("Voyager",7), ("Curiosity", 60000000)])
        "==" [("Cassini",12000000000),("Voyager",13000000000),("Curiosity", 60000000)])
    ]
```

8. (15 points) [UseModels] In Haskell, write the function
```
sumSublists :: [[Integer]] -> Integer
```

that takes a list of lists of Integers, nss, and returns the sum of all the Integers in nss. The following are examples.

```
tests :: [TestCase Integer]
tests =
    [(eqTest (sumSublists []) "==" 0)
    ,(eqTest (sumSublists [[], []]) "==" 0)
    ,(eqTest (sumSublists [[4,0],[2,0]]) "==" 6)
    ,(eqTest (sumSublists [[2,0]]) "==" 2)
    ,(eqTest (sumSublists [[1..10],[10..20]]) "==" 220)
    ,(eqTest (sumSublists [[5,-10,15],[35,30,25,20],[55,50,40,45,10]]) "==" 320)
    ,(eqTest (sumSublists [[1],[2],[3],[],[4],[5,6],[7,8,9,10]]) "==" 55) ]
```

