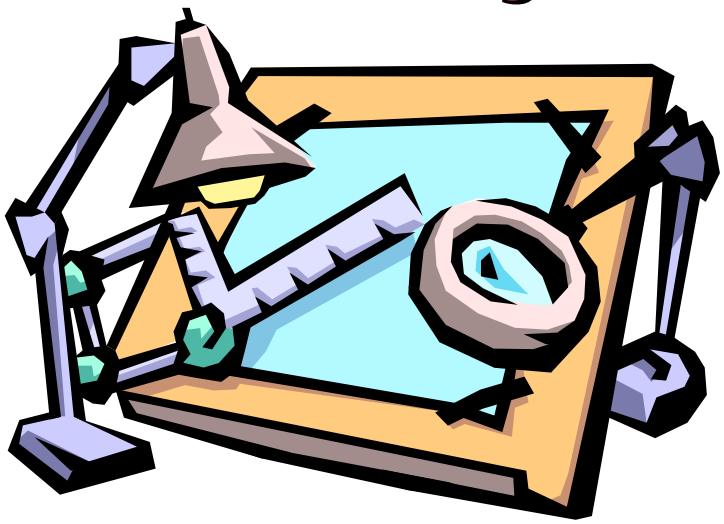
Synchronous Sequential Circuit Design



Sequential circuit design

- In sequential circuit design, we turn some description into a working circuit
  - We first make a state table or diagram to express the computation
  - Then we can turn that table or diagram into a sequential circuit

Sequential circuit design procedure

Step 1:

Make a state table based on the problem statement. The table should show the present states, inputs, next states and outputs. (It may be easier to find a state diagram first, and then convert that to a table)

Step 2:

Assign binary codes to the states in the state table, if you haven't already. If you have n states, your binary codes will have at least $\lceil \log_2 n \rceil$ digits, and your circuit will have at least $\lceil \log_2 n \rceil$ flip-flops

Step 3:

For each flip-flop and each row of your state table, find the flip-flop input values that are needed to generate the next state from the present state. You can use flip-flop excitation tables here.

Step 4:

Find simplified equations for the flip-flop inputs and the outputs.

Step 5:

Build the circuit!

Sequence recognizer (Mealy)

- A sequence recognizer is a special kind of sequential circuit that looks for a special bit pattern in some input
- The recognizer circuit has only one input, X
  - One bit of input is supplied on every clock cycle
  - This is an easy way to permit arbitrarily long input sequences
- There is one output, Z, which is 1 when the desired pattern is found
- Our example will detect the bit pattern “1001”:

  Inputs: 1 1 1 0 0 1 1 0 1 0 0 1 0 0 1 1 0 …
  Outputs: 0 0 0 0 0 1 0 0 0 0 0 1 0 0 1 0 0 …

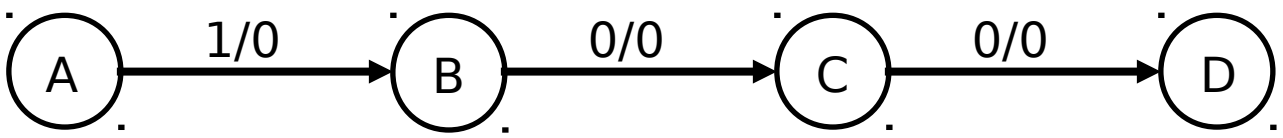
- A sequential circuit is required because the circuit has to “remember” the inputs from previous clock cycles, in order to determine whether or not a match was found

Step 1: Making a state table

- The first thing you have to figure out is precisely how the use of state will help you solve the given problem
  - Make a state table based on the problem statement. The table should show the present states, inputs, next states and outputs
  - Sometimes it is easier to first find a state diagram and then convert that to a table
- This is usually the most difficult step. Once you have the state table, the rest of the design procedure is the same for all sequential circuits

A basic Mealy state diagram

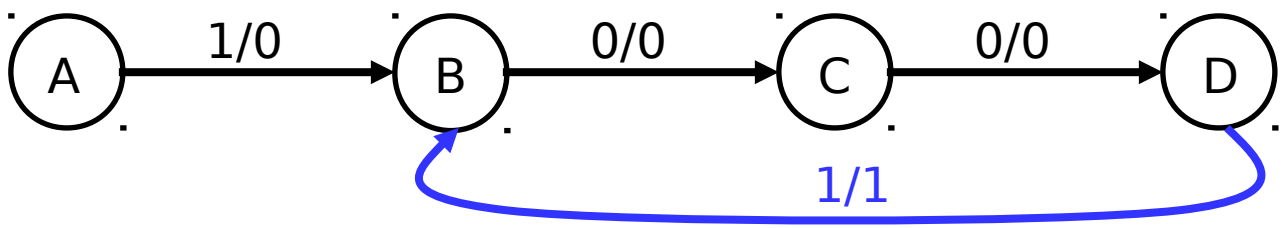
- What state do we need for the sequence recognizer?
  - We have to "remember" inputs from previous clock cycles
  - For example, if the previous three inputs were 100 and the current input is 1, then the output should be 1
  - In general, we will have to remember occurrences of parts of the desired pattern—in this case, 1, 10, and 100
- We'll start with a basic state diagram:



State	Meaning
A	None of the desired pattern (1001) has been input yet.
B	We've already seen the first bit (1) of the desired pattern.
C	We've already seen the first two bits (10) of the desired pattern.
D	We've already seen the first three bits (100) of the desired pattern.

Overlapping occurrences of the pattern

- What happens if we're in state D (the last three inputs were 100), and the current input is 1?
  - The output should be a 1, because we've found the desired pattern
  - But this last 1 could also be the start of another occurrence of the pattern! For example, 1001001 contains *two* occurrences of 1001
  - To detect overlapping occurrences of the pattern, the next state should be B.



State	Meaning
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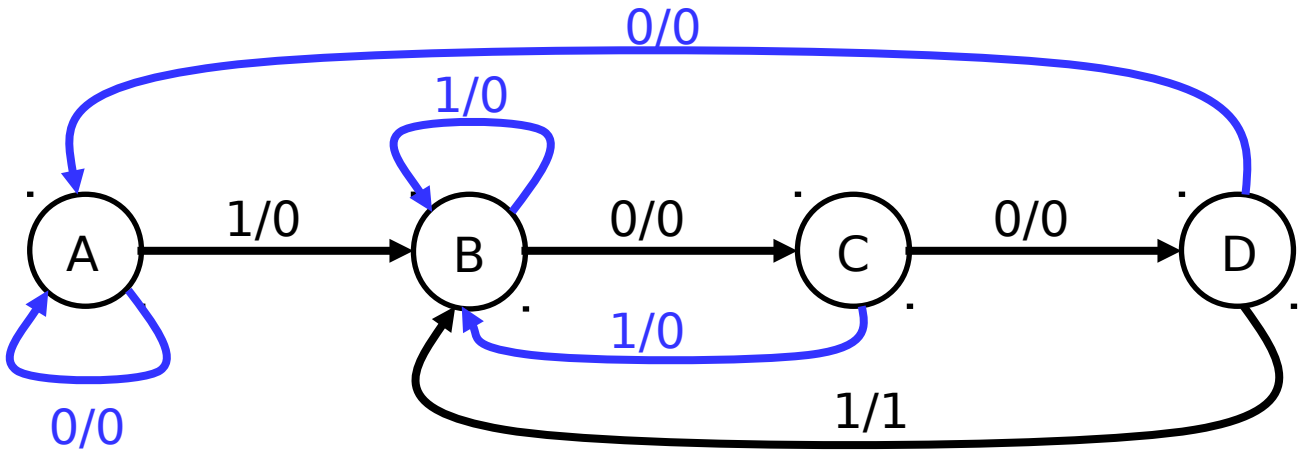
B	We've already seen the first bit (1) of the desired pattern.
C	We've already seen the first two bits (10) of the desired pattern.
D	We've already seen the first three bits (100) of the desired pattern.

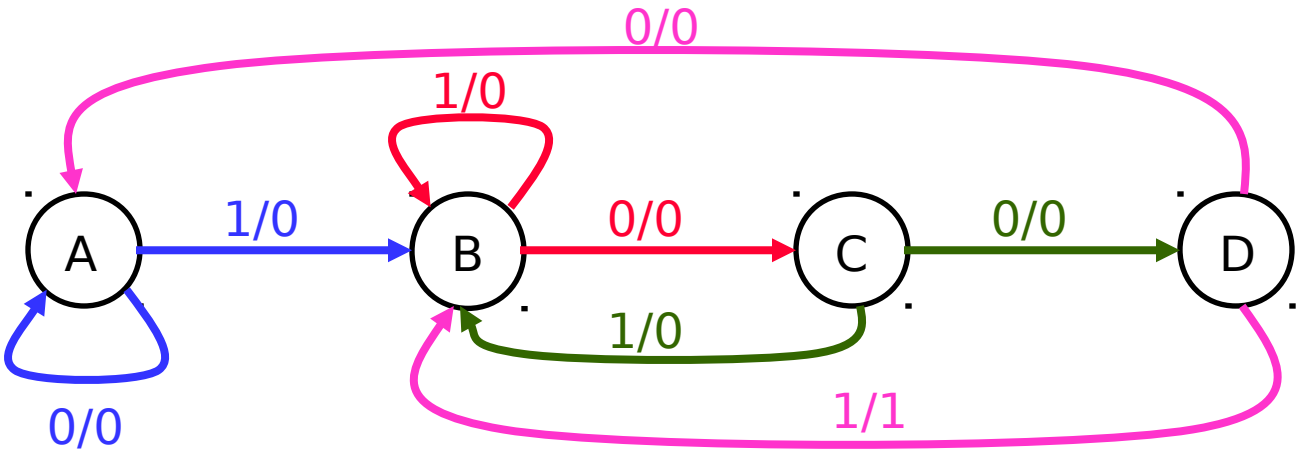
Filling in the other arrows

- *Two* outgoing arrows for each node, to account for the possibilities of X=0 and X=1
- The remaining arrows we need are shown in blue. They also allow for the correct detection of overlapping occurrences of 1001.



State	Meaning
A	None of the desired pattern (1001) has been input yet.
B	We've already seen the first bit (1) of the desired pattern.
C	We've already seen the first two bits (10) of the desired pattern.
D	We've already seen the first three bits (100) of the desired pattern.

Mealy state diagram & table

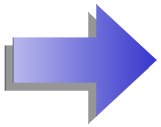


Present State	Input	Next State	Output
A	0	A	0
A	1	B	0
B	0	C	0
B	1	B	0
C	0	D	0
C	1	B	0
D	0	A	0
D	1	B	1

Step 2: Assigning binary codes to states

- We have four states ABCD, so we need at least two flip-flops Q_1Q_0
- The easiest thing to do is represent state A with Q_1Q_0 = 00, B with 01, C with 10, and D with 11
- The state assignment can have a big impact on circuit complexity, but we won't worry about that too much in this class

Present State	Input	Next State	Output
A	0	A	0
A	1	B	0
B	0	C	0
B	1	B	0
C	0	D	0
C	1	B	0
D	0	A	0
D	1	B	1



Present State Q_1	Present State Q_0	Input X	Next State Q_1	Next State Q_0	Output Z
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	0	1	0
1	1	0	0	0	0
1	1	1	0	1	1

Step 3: Finding flip-flop input values

- Next we have to figure out how to actually make the flip-flops change from their present state into the desired next state
- This depends on what kind of flip-flops you use!
- We'll use two JKs. For each flip-flip Q_i, look at its present and next states, and determine what the inputs J_i and K_i should be in order to make that state change.

Present State		Input	Next State		Flip flop inputs				Output
Q_1	Q_0	X	Q_1	Q_0	J_1	K_1	J_0	K_0	Z
0	0	0	0	0					0
0	0	1	0	1					0
0	1	0	1	0					0
0	1	1	0	1					0
1	0	0	1	1					0
1	0	1	0	1					0
1	1	0	0	0					0
1	1	1	0	1					1

Finding JK flip-flop input values

- For JK flip-flops, this is a little tricky. Recall the characteristic table:

J	K	Q(t+1)	Operation
0	0	Q(t)	No change
0	1	0	Reset
1	0	1	Set
1	1	Q'(t)	Complement

- If the present state of a JK flip-flop is 0 and we want the next state to be 1, then we have *two* choices for the JK inputs:
  - We can use JK= 10, to explicitly set the flip-flop's next state to 1
  - We can also use JK=11, to complement the current state 0
- So to change from 0 to 1, we must set J=1, but K could be *either* 0 or 1
- Similarly, the other possible state transitions can all be done in two different ways as well

JK excitation table

- An excitation table shows what flip-flop inputs are required in order to make a desired state change

Q(t)	Q(t+1)	J	K	Operation
0	0	0	x	No change/reset
0	1	1	x	Set/complement
1	0	x	1	Reset/complement
1	1	x	0	No change/set

- This is the same information that's given in the characteristic table, but presented "backwa

J	K	Q(t+1)	Operation
0	0	Q(t)	No change
0	1	0	Reset
1	0	1	Set
1	1	Q'(t)	Complement

Back to the example

- Use the JK excitation table on the right to find the correct values for *each* flip-flop's inputs, based on its present and next states

Q(t)	Q(t+1)	J	K
0	0	0	x
0	1	1	x
1	0	x	1
1	1	x	0

Present State		Input	Next State		Flip flop inputs				Output
Q_1	Q_0	X	Q_1	Q_0	J_1	K_1	J_0	K_0	Z
0	0	0	0	0	0	x	0	x	0
0	0	1	0	1	0	x	1	x	0
0	1	0	1	0	1	x	x	1	0
0	1	1	0	1	0	x	x	0	0
1	0	0	1	1	x	0	1	x	0
1	0	1	0	1	x	1	1	x	0
1	1	0	0	0	x	1	x	1	0
1	1	1	0	1	x	1	x	0	1

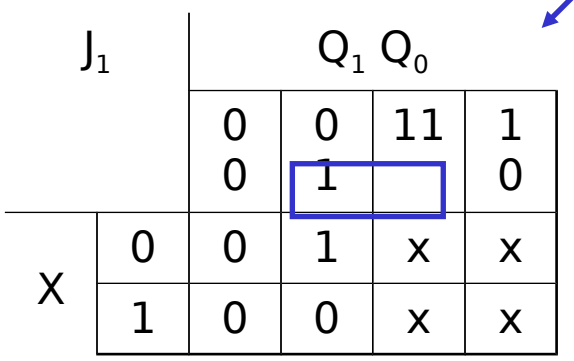
Step 4: Find equations for the FF inputs and output

- Now you can make K-maps and find equations for each of the four flip-flop inputs, as well as for the output Z
- These equations are in terms of the present state and the inputs
- The advantage of using JK flip-flops is that there are many don't care conditions, which can result in simpler MSP equations

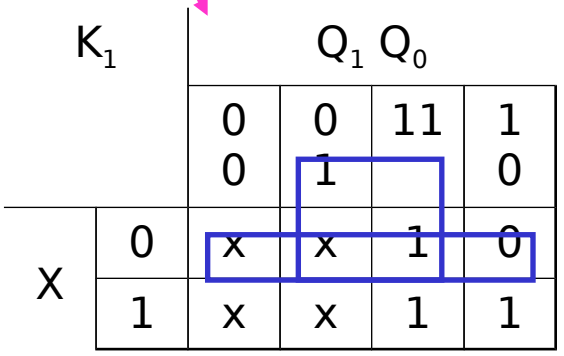
Present State		Input	Next State		Flip flop inputs				Output
Q_1	Q_0	X	Q_1	Q_0	J_1	K_1	J_0	K_0	Z
0	0	0	0	0	0	x	0	x	0
0	0	1	0	1	0	x	1	x	0
0	1	0	1	0	1	x	x	1	0
0	1	1	0	1	0	x	x	0	0
1	0	0	1	1	x	0	1	x	0
1	0	1	0	1	x	1	1	x	0
1	1	0	0	0	x	1	x	1	0
1	1	1	0	1	x	1	x	0	1

FF input equations

Present State Q_1	Present State Q_0	Input X	Next State Q_1	Next State Q_0	Flip flop inputs J_1	Flip flop inputs K_1	Flip flop inputs J_0	Flip flop inputs K_0	Output Z
0	0	0	0	0	0	x	0	x	0
0	0	1	0	1	0	x	1	x	0
0	1	0	1	0	1	x	x	1	0
0	1	1	0	1	0	x	x	0	0
1	0	0	1	1	x	0	1	x	0
1	0	1	0	1	x	1	1	x	0
1	1	0	0	0	x	1	x	1	0
1	1	1	0	1	x	1	x	0	1



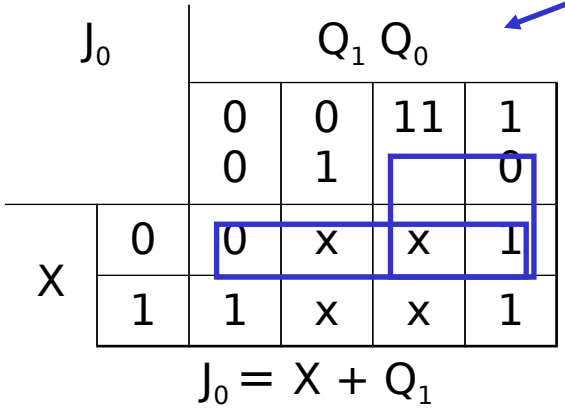
$J_1 = X' Q_0$



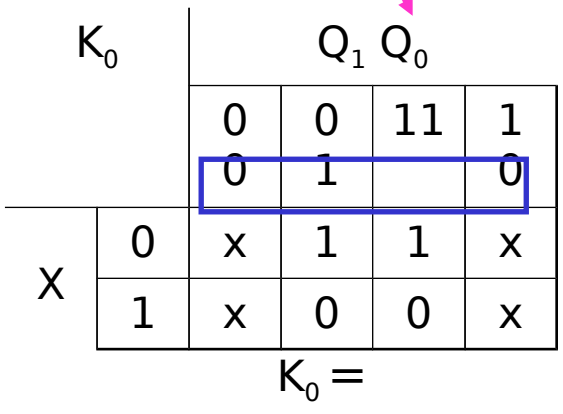
$K_1 = X + Q_0$

FF input equations

Present State Q_1	Present State Q_0	Input X	Next State Q_1	Next State Q_0	Flip flop inputs J_1	Flip flop inputs K_1	Flip flop inputs J_0	Flip flop inputs K_0	Output Z
0	0	0	0	0	0	x	0	x	0
0	0	1	0	1	0	x	1	x	0
0	1	0	1	0	1	x	x	1	0
0	1	1	0	1	0	x	x	0	0
1	0	0	1	1	x	0	1	x	0
1	0	1	0	1	x	1	1	x	0
1	1	0	0	0	x	1	x	1	0
1	1	1	0	1	x	1	x	0	1



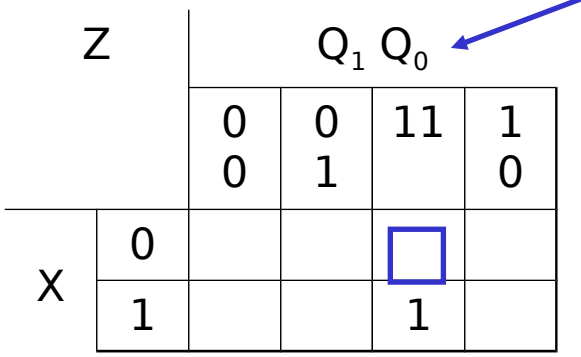
$J_0 = X + Q_1$



$K_0 = X'$

Output equation

Present State Q_1	Present State Q_0	Input X	Next State Q_1	Next State Q_0	Flip flop inputs J_1	Flip flop inputs K_1	Flip flop inputs J_0	Flip flop inputs K_0	Output Z
0	0	0	0	0	0	x	0	x	0
0	0	1	0	1	0	x	1	x	0
0	1	0	1	0	1	x	x	1	0
0	1	1	0	1	0	x	x	0	0
1	0	0	1	1	x	0	1	x	0
1	0	1	0	1	x	1	1	x	0
1	1	0	0	0	x	1	x	1	0
1	1	1	0	1	x	1	x	0	1



$Z = X Q_1 Q_0$

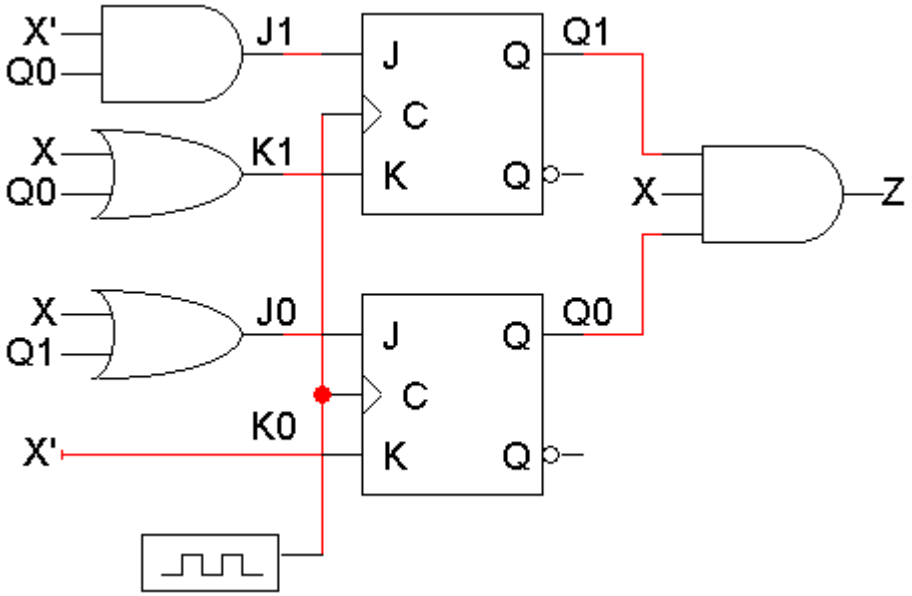
Step 5: Build the circuit

- Lastly, we use these simplified equations to build the completed circuit

$J_1 = X' Q_0$
$K_1 = X + Q_0$

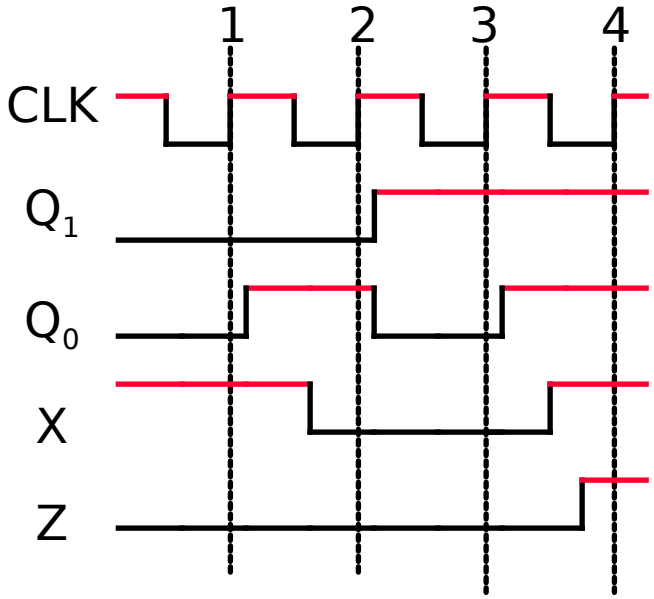
$J_0 = X + Q_1$
$K_0 = X'$

$Z = Q_1 Q_0 X$



Timing diagram

- Here is one example timing diagram for our sequence detector
  - The flip-flops Q_1Q_0 start in the initial state, 00
  - On the first three positive clock edges, X is 1, 0, and 0. These inputs cause Q_1Q_0 to change, so after the third edge $Q_1Q_0 = 11$
  - Then when X=1, Z becomes 1 also, meaning that 1001 was found
- The output Z does not have to change at positive clock edges. Instead, it may change whenever X changes, since $Z = Q_1Q_0X$



Building the same circuit with D flip-flops

- What if you want to build the circuit using D flip-flops instead?
- We already have the state table and state assignments, so we can just start from Step 3, finding the flip-flop input values
- D flip-flops have only one input, so our table only needs two columns for D_1 and D_0

Present State		Input	Next State		Flip-flop inputs		Output
Q_1	Q_0	X	Q_1	Q_0	D_1	D_0	Z
0	0	0	0	0			0
0	0	1	0	1			0
0	1	0	1	0			0
0	1	1	0	1			0
1	0	0	1	1			0
1	0	1	0	1			0
1	1	0	0	0			0
1	1	1	0	1			1

D flip-flop input values (Step 3)

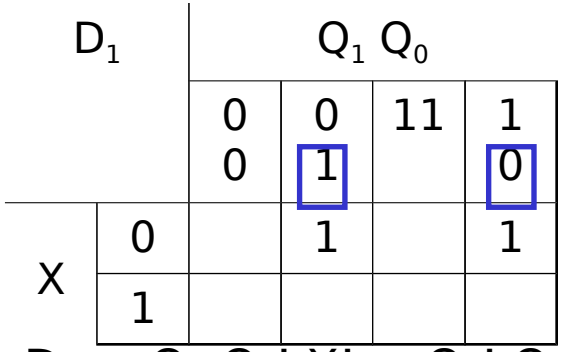
- The D excitation table is pretty boring; set the D input to whatever the next state should be
- You don't even need to show separate columns for D_1 and D_0; you can just use the Next State columns

Q(t)	Q(t+1)	D	Operation
0	0	0	Reset
0	1	1	Set
1	0	0	Reset
1	1	1	Set

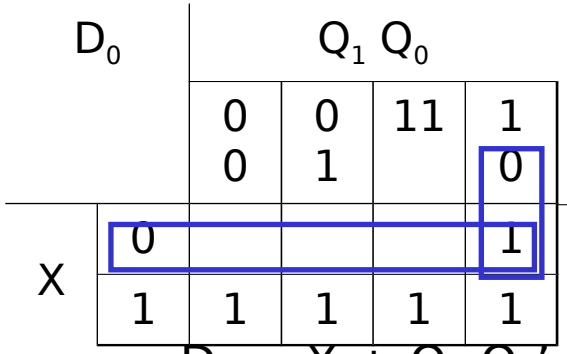
Present State		Input	Next State		Flip flop inputs		Output
Q_1	Q_0	X	Q_1	Q_0	D_1	D_0	Z
0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0
0	1	0	1	0	1	0	0
0	1	1	0	1	0	1	0
1	0	0	1	1	1	1	0
1	0	1	0	1	0	1	0
1	1	0	0	0	0	0	0
1	1	1	0	1	0	1	1

Finding equations (Step 4)

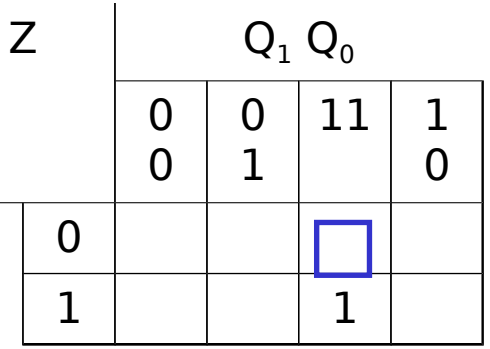
Present State Q_1	Present State Q_0	Input X	Next State Q_1	Next State Q_0	Flip flop inputs D_1	Flip flop inputs D_0	Output Z
0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0
0	1	0	1	0	1	0	0
0	1	1	0	1	0	1	0
1	0	0	1	1	1	1	0
1	0	1	0	1	0	1	0
1	1	0	0	0	0	0	0
1	1	1	0	1	0	1	1



$D_1 = Q_1 Q_0' X' + Q_1' Q_0 X'$

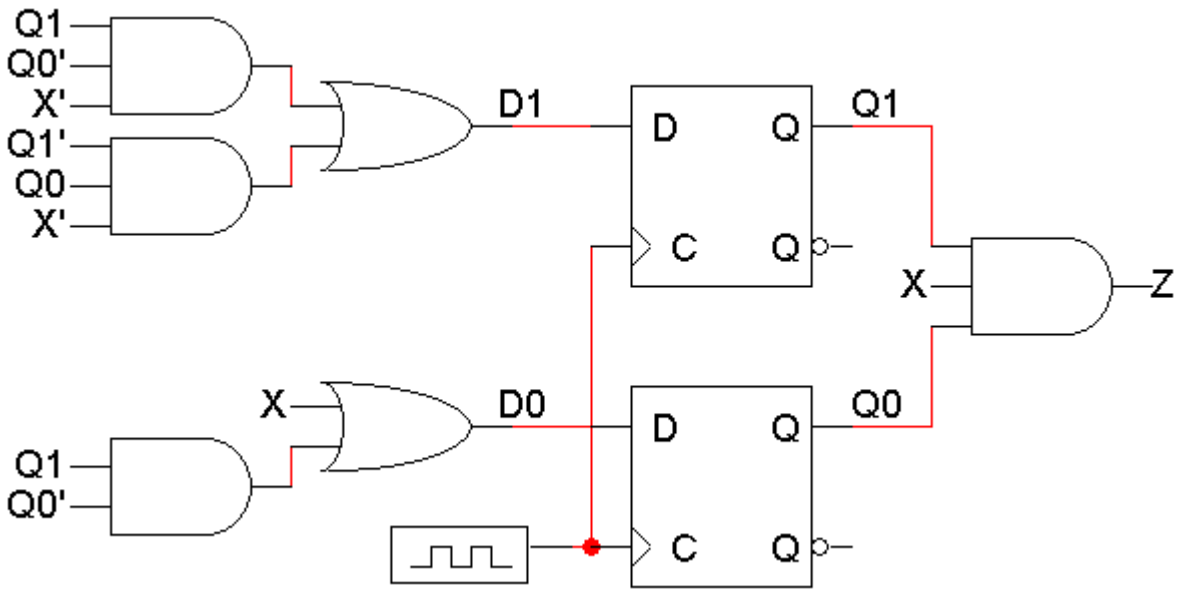


$D_0 = X + Q_1 Q_0'$



$Z = X Q_1 Q_0$

Building the circuit (Step 5)



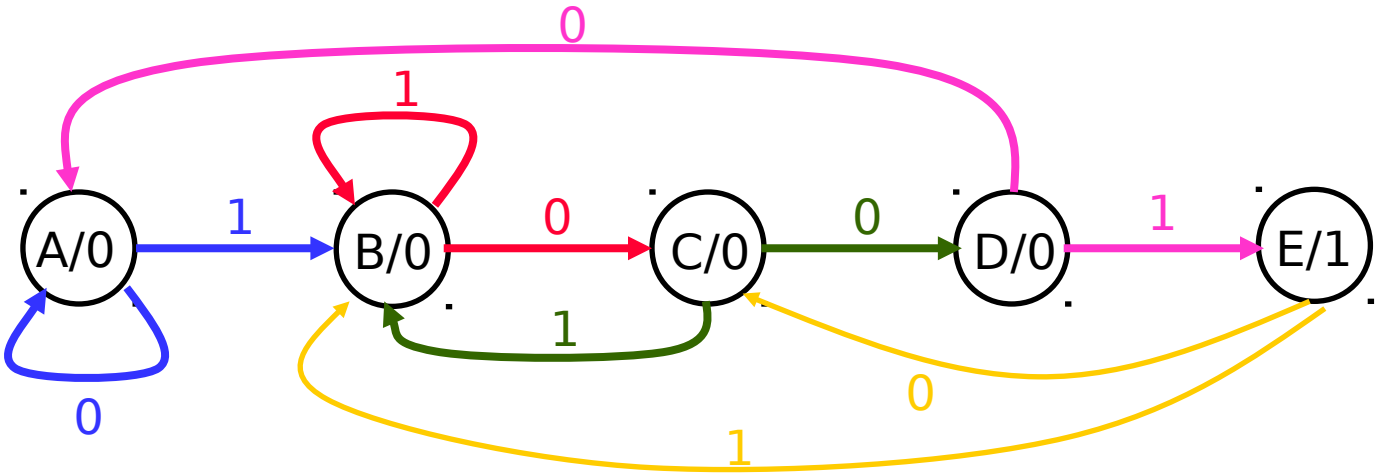
Sequence recognizer (Moore)

- A sequence recognizer is a special kind of sequential circuit that looks for a special bit pattern in some input
- The recognizer circuit has only one input, X
  - One bit of input is supplied on every clock cycle
  - This is an easy way to permit arbitrarily long input sequences
- There is one output, Z, which is 1 when the desired pattern is found
- Our example will detect the bit pattern “1001”:

  Inputs: 1 1 1 0 0 1 1 0 1 0 0 1 0 0 1 1 0 …
  Outputs: 0 0 0 0 0 1 0 0 0 0 0 1 0 0 1 0 0 …

- A sequential circuit is required because the circuit has to “remember” the inputs from previous clock cycles, in order to determine whether or not a match was found

Moore state diagram & table



Present State	Input	Next State	Output
A	0	A	0
A	1	B	0
B	0	C	0
B	1	B	0
C	0	D	0
C	1	B	0
D	0	A	0
D	1	E	0
E	0	C	1
E	1	B	1

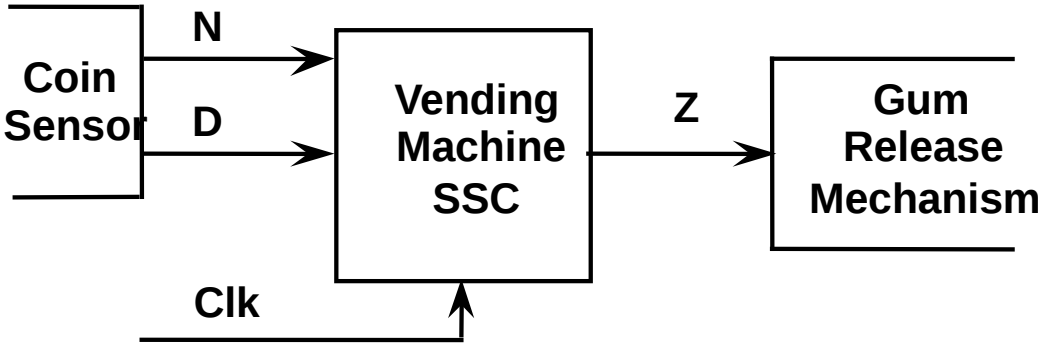
A: 000 D: 100
B: 001 E: 101
C: 010

Z	$Q_2 Q_1$			
	00	01	11	10
Q_0 0				
Q_0 1				1

$Z = Q_2 Q_1' Q_0$

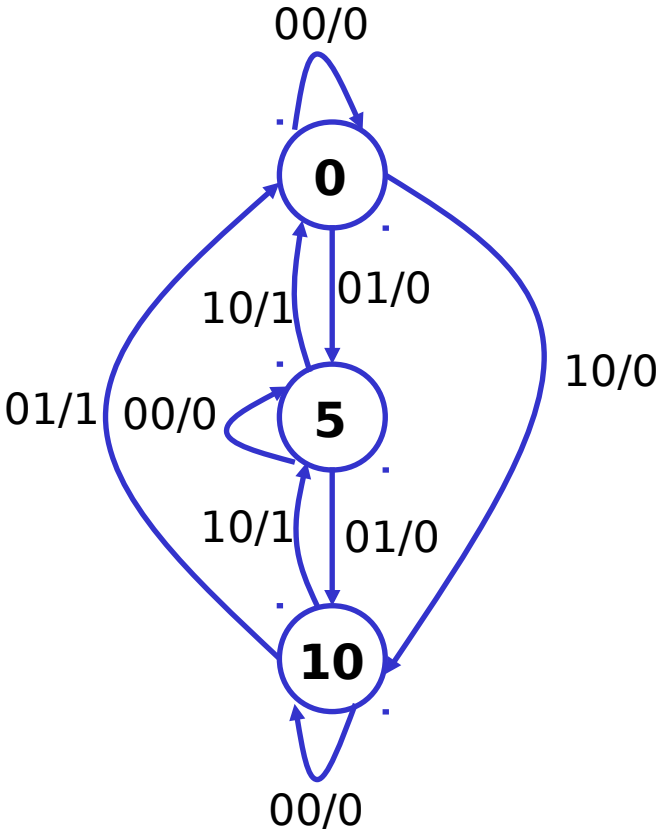
Vending Machine

- Design a vending machine which meets the following specs:
  - Deliver a package of gum after 15 cents deposited
  - Single coin slot for dimes, nickels
    - N =1 => A nickel is deposited
    - D =1 => A dime is deposited
  - No change

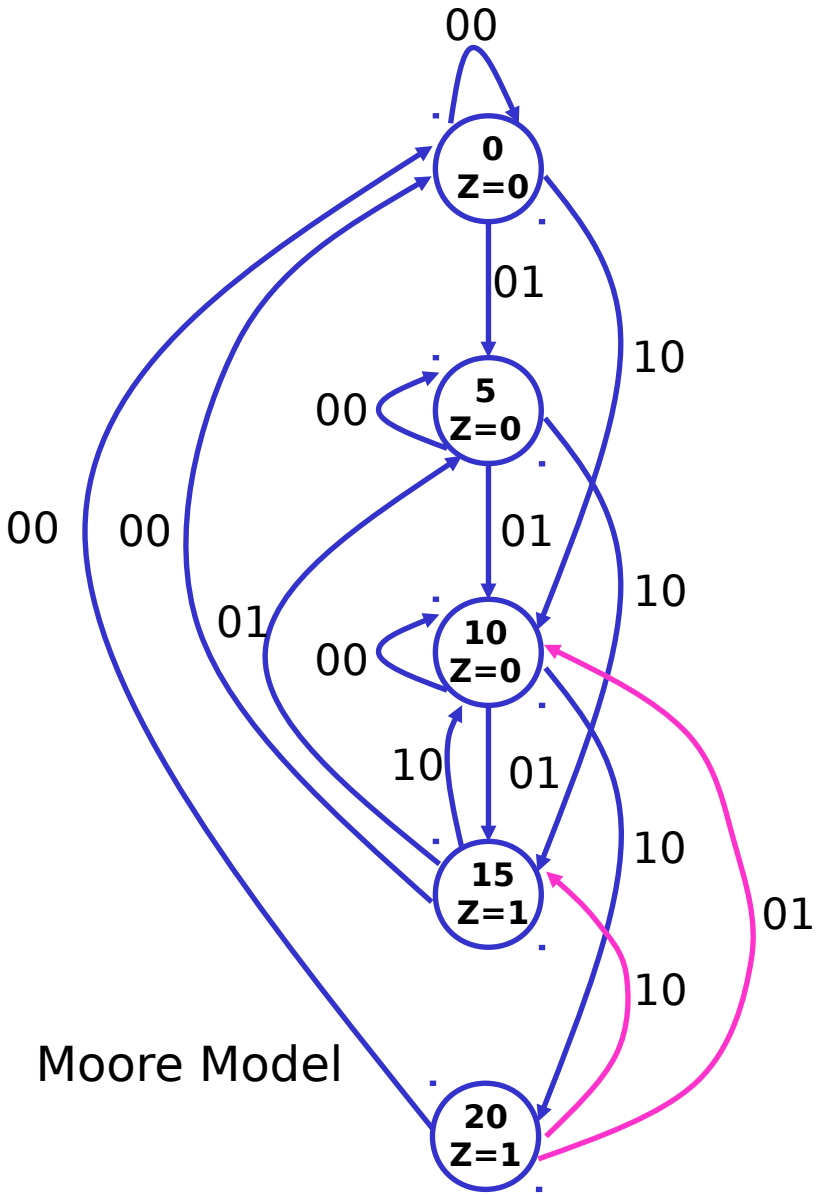


Block Diagram of the System

Vending Machine



Mealy Model



Moore Model

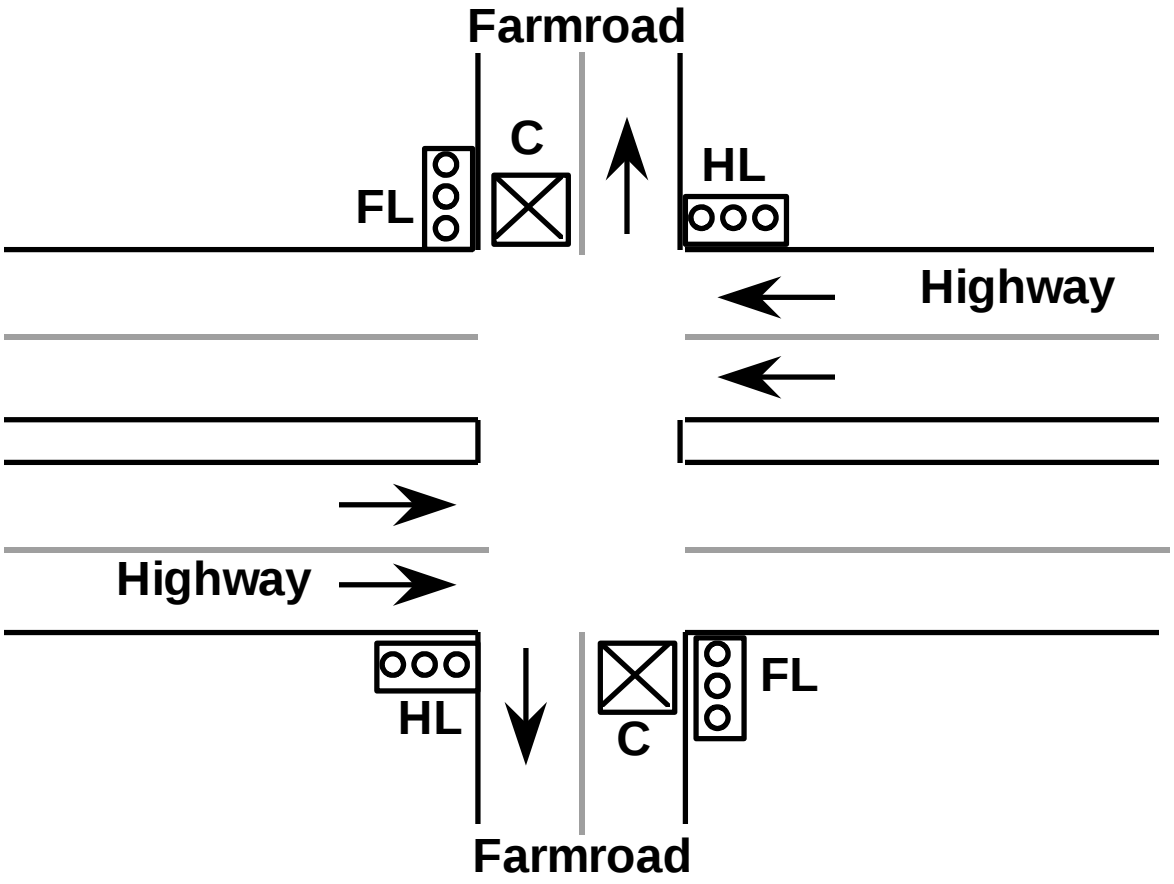
Traffic Light Controller

A busy highway is intersected by a little used farmroad. Detectors C sense the presence of cars waiting on the farmroad. With no car on farmroad, light remain green in highway direction. If vehicle on farmroad, highway lights go from Green to Yellow to Red, allowing the farmroad lights to become green. These stay green only as long as a farmroad car is detected but never longer than a set interval. When these are met, farm lights transition from Green to Yellow to Red, allowing highway to return to green. Even if farmroad vehicles are waiting, highway gets at least a set interval as green.

Assume you have an interval timer that generates a short time pulse (TS) and a long time pulse (TL) in response to a set (ST) signal. TS is to be used for timing yellow lights and TL for green lights.

Note: The interval timer is just another sequential circuit!

Traffic Light Controller

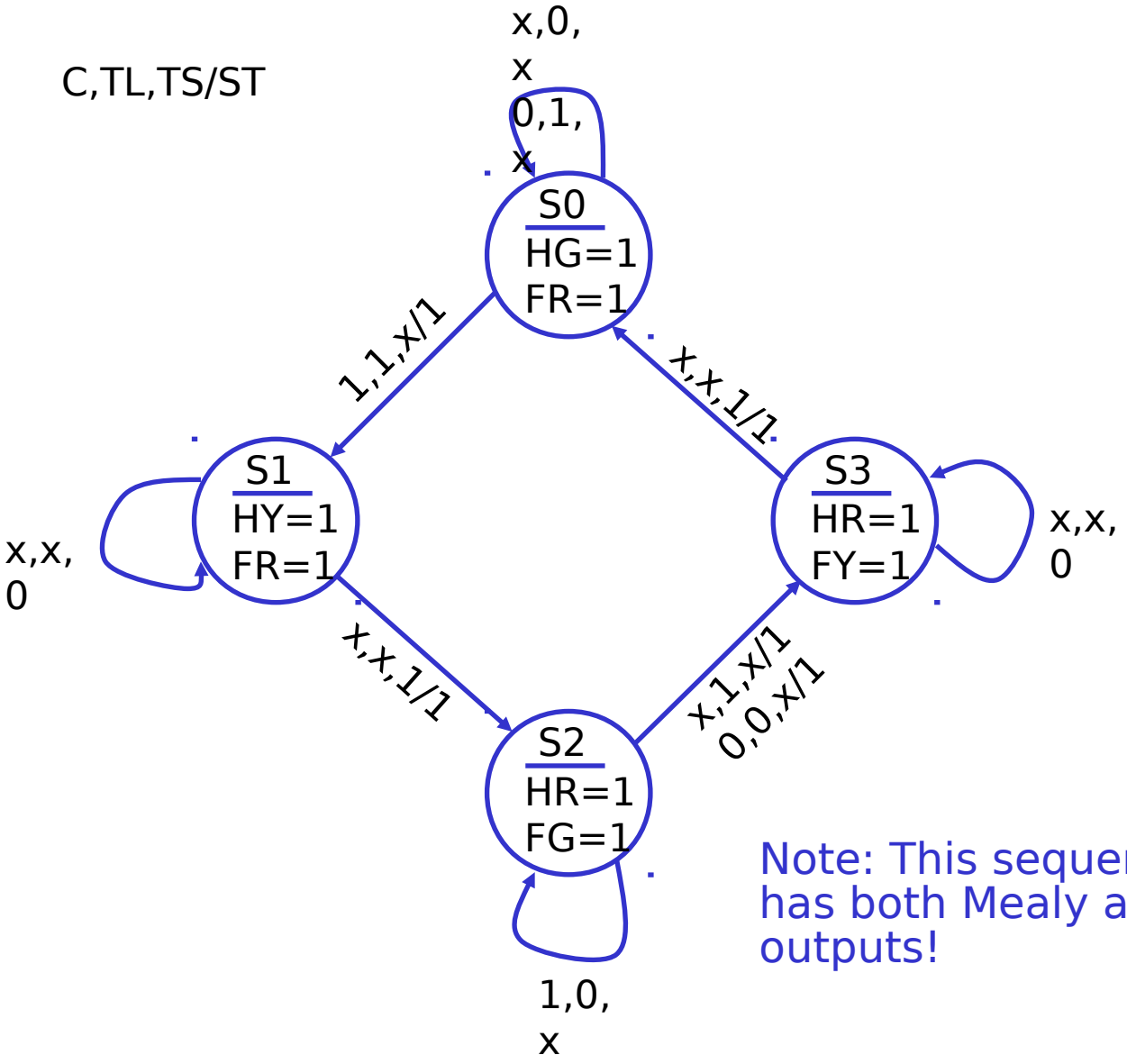


Traffic Light Controller

Input Signals	
C	detect vehicle on farmroad
TS	short time interval expired
TL	long time interval expired
Output Signals	
HG, HY, HR	assert green/yellow/red highway lights
FG, FY, FR	assert green/yellow/red farmroad lights
ST	start timing a short or long interval

State	
S0	Highway green (farmroad red)
S1	Highway yellow (farmroad red)
S2	Farmroad green (highway red)
S3	Farmroad yellow (highway red)

Traffic Light Controller



Note: This sequential circuit has both Mealy and Moore outputs!

Digital Combination Lock

"3 bit serial lock controls entry to locked room. Inputs are RESET,
ENTER, 2 position switch for bit of key data. Locks generates an
UNLOCK signal when key matches internal combination. ERROR
light illuminated if key does not match combination.

Sequence is:

(1) Press RESET, (2) enter key bit, (3) Press ENTER,
(4) repeat (2) &(3) two more times.

Problem specification is incomplete:

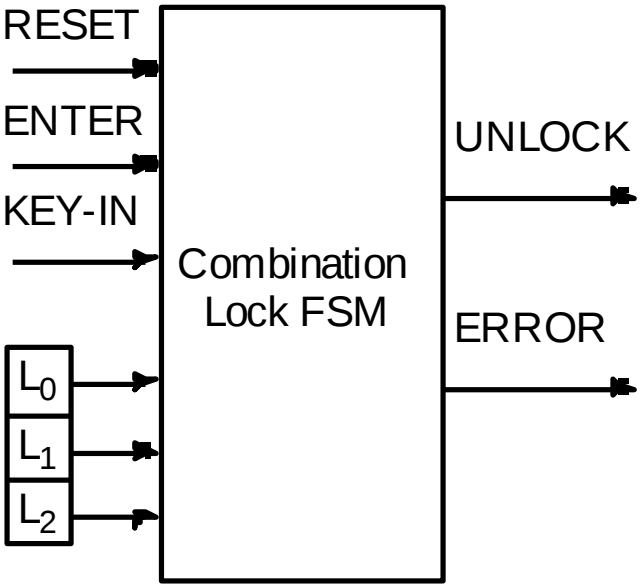
- how do you set the internal combination?
- exactly when is the ERROR light asserted?

Make reasonable assumptions:

- hardwired into next state logic vs. stored in internal register
- assert as soon as error is detected vs. wait until full combination has been entered

Our design: registered combination plus error after full combination

Digital Combination Lock



Operator Data

Internal Combination

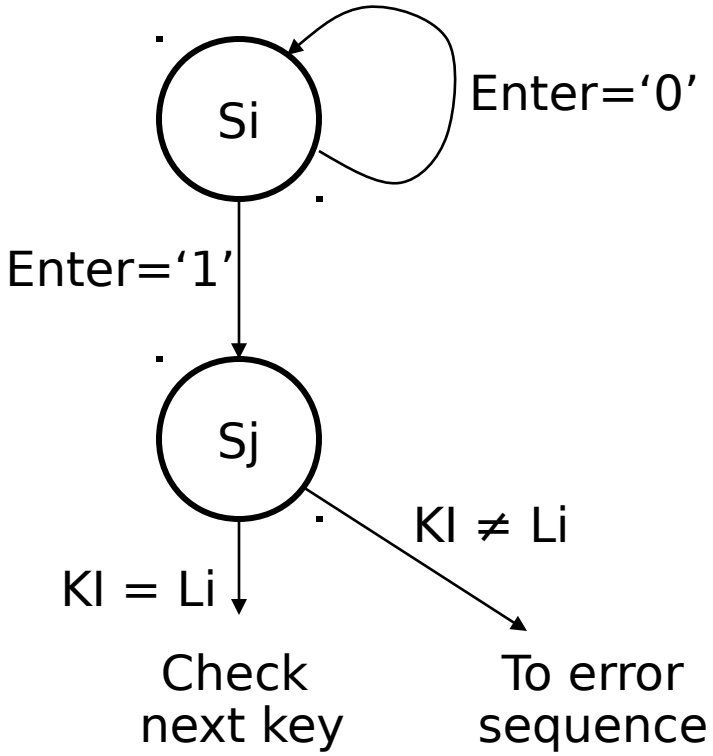
Inputs:
Reset
Enter
Key-In
L0, L1, L2

Outputs:
Unlock
Error

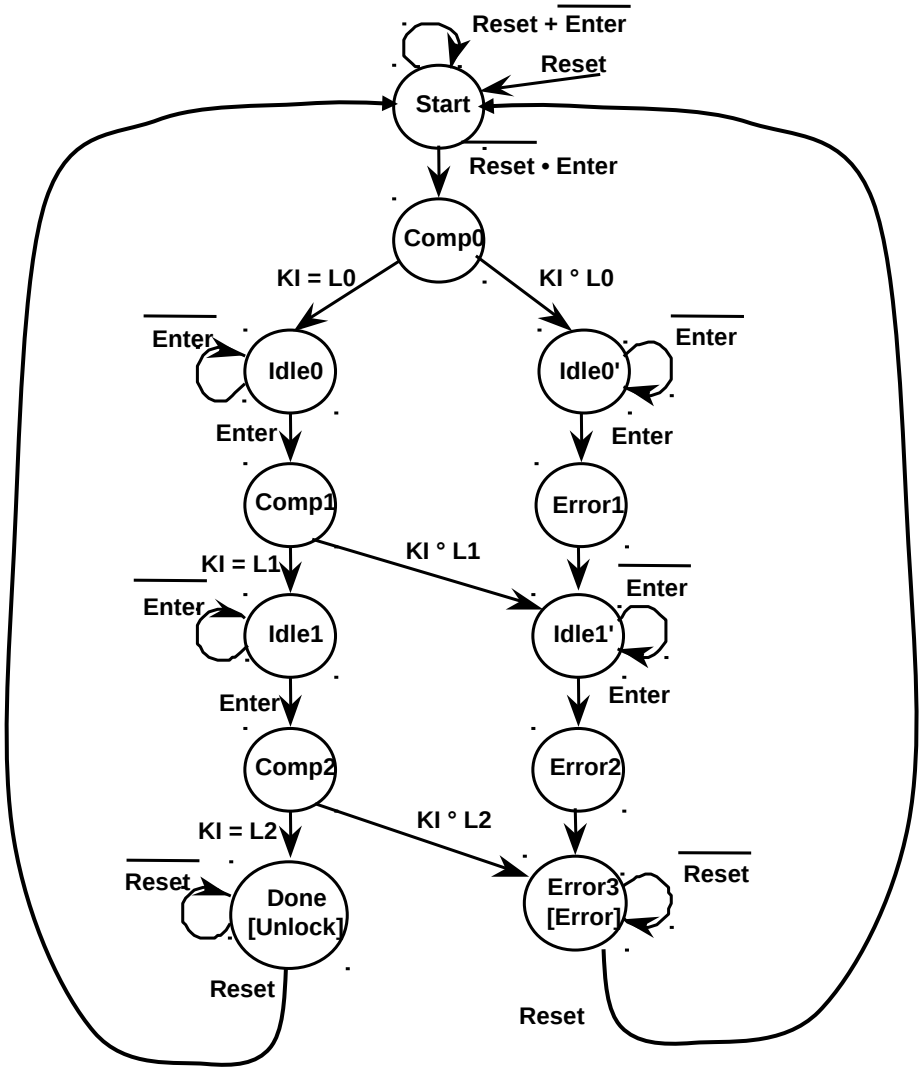
Digital Combination Lock

Note that each key entry is really a two-step process

1. Wait for the enter key
2. Check if correct key was selected



Digital Combination Lock



Summary

- The basic sequential circuit design procedure:
  - Make a state table and, if desired, a state diagram. This step is usually the hardest
  - Assign binary codes to the states if you didn’t already
  - Use the present states, next states, and flip-flop excitation tables to find the flip-flop input values
  - Write simplified equations for the flip-flop inputs and outputs and build the circuit