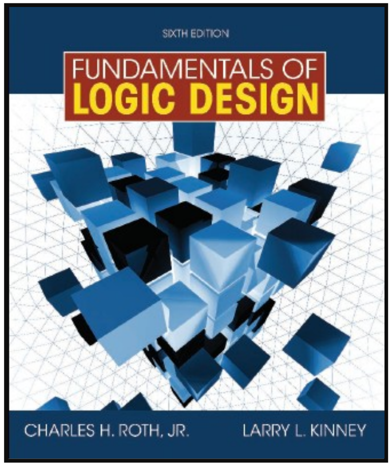
CHAPTER 5

KARNAUGH MAPS



This chapter in the book includes:

Karnaugh Maps

Switching functions can generally be simplified by using the algebraic techniques described in Unit 3. However, two problems arise when algebraic procedures are used:

1. The procedures are difficult to apply in a systematic way.

2. It is difficult to tell when you have arrived at a minimum solution.

The Karnaugh map method is generally faster and easier to apply than other simplification methods.

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Minimum Forms of Switching Functions

Given a minterm expansion, the minimum sum-of-products format can often be obtained by the following procedure:

1. Combine terms by using $XY' + XY = X$. Do this repeatedly to eliminate as many literals as possible. A given term may be used more than once because $X + X = X$.

2. Eliminate redundant terms by using the consensus theorem or other theorems.

Unfortunately, the result of this procedure may depend on the order in which the terms are combined or eliminated so that the final expression obtained is not necessarily minimum.

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Example

Find a minimum sum-of-products expression for

$$F(a, b, c) = \Sigma\, m\,(0, 1, 2, 5, 6, 7)$$

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

$$= a'b' + b'c + bc' + ab \qquad (5\text{-}1)$$

None of the terms in the above expression can be eliminated by consensus. However, combining terms in a different way leads directly to a minimum sum of products:

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

$$= a'b' + bc' + ac \qquad (5\text{-}2)$$

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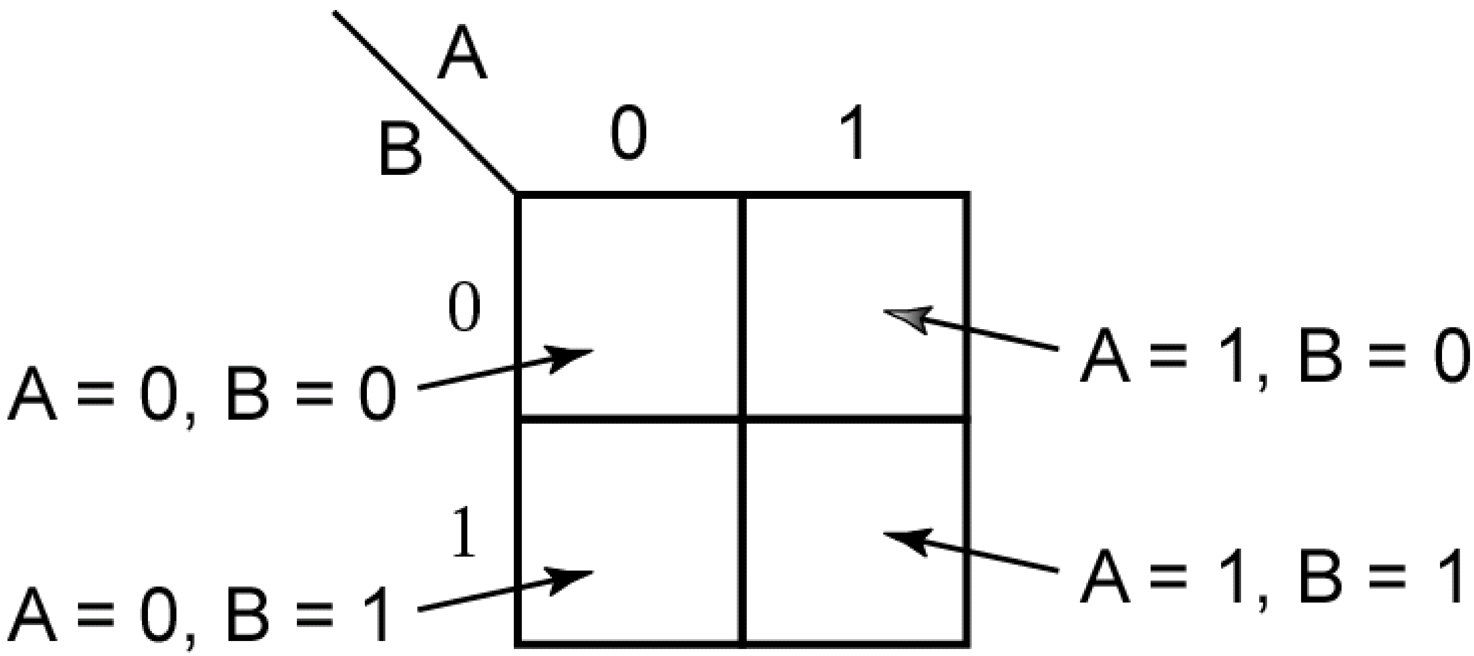
Two- and Three- Variable Karnaugh Maps

Just like a truth table, the Karnaugh map of a function specifies the value of the function for every combination of values of the independent variables.



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(a)

A B	F
0 0	1
0 1	1
1 0	0
1 1	0

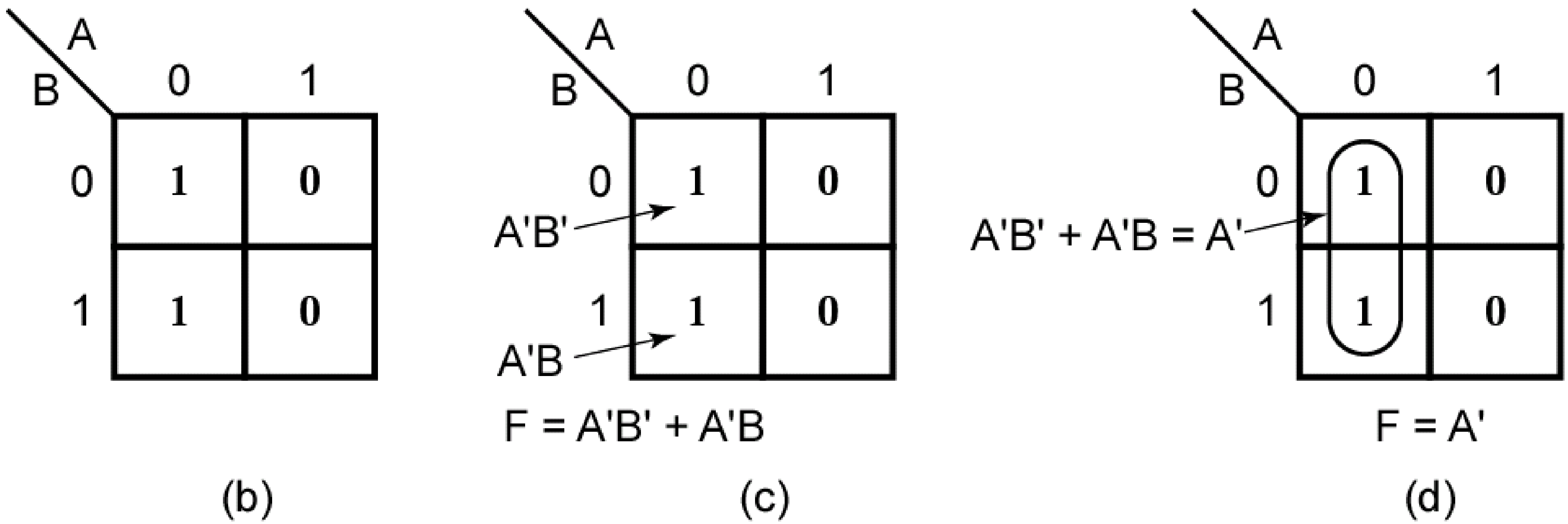
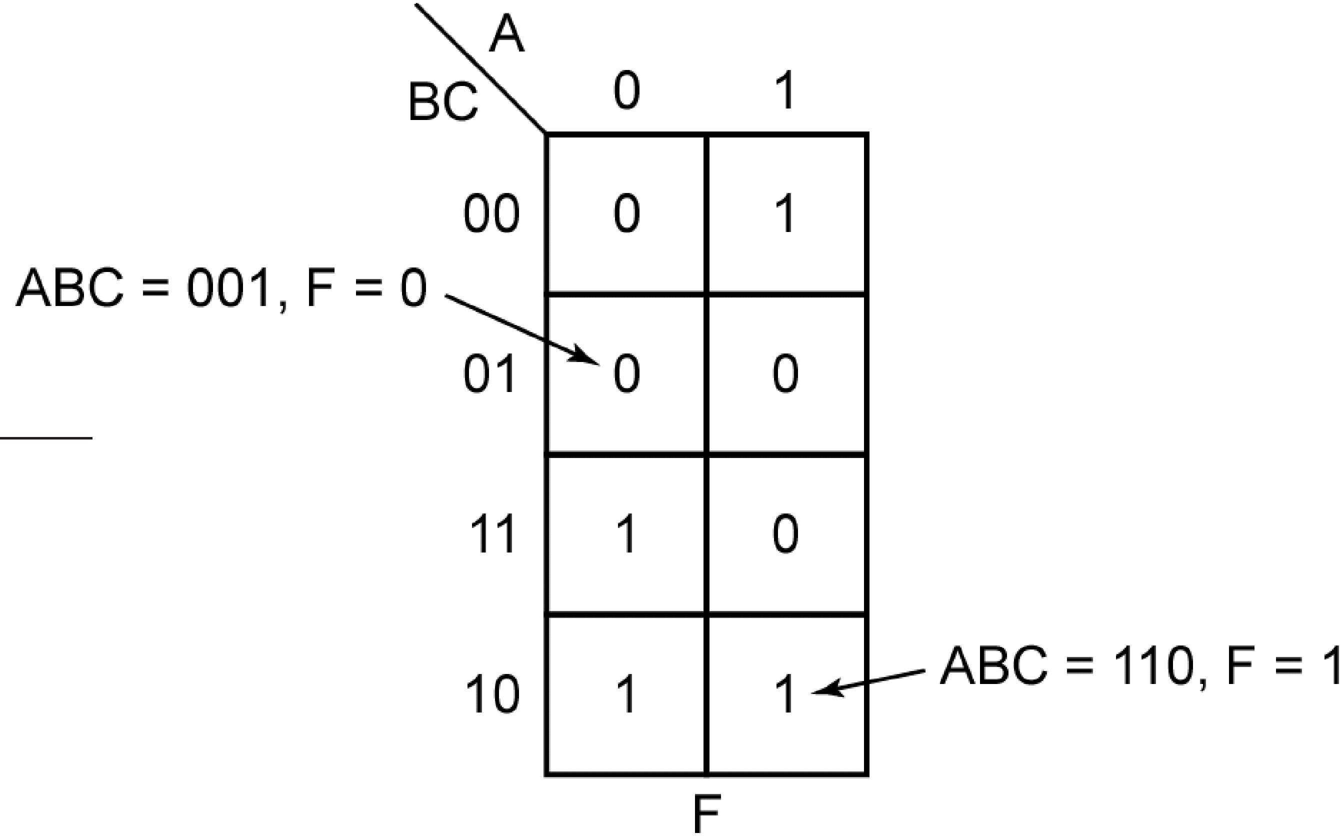


Figure 5-1

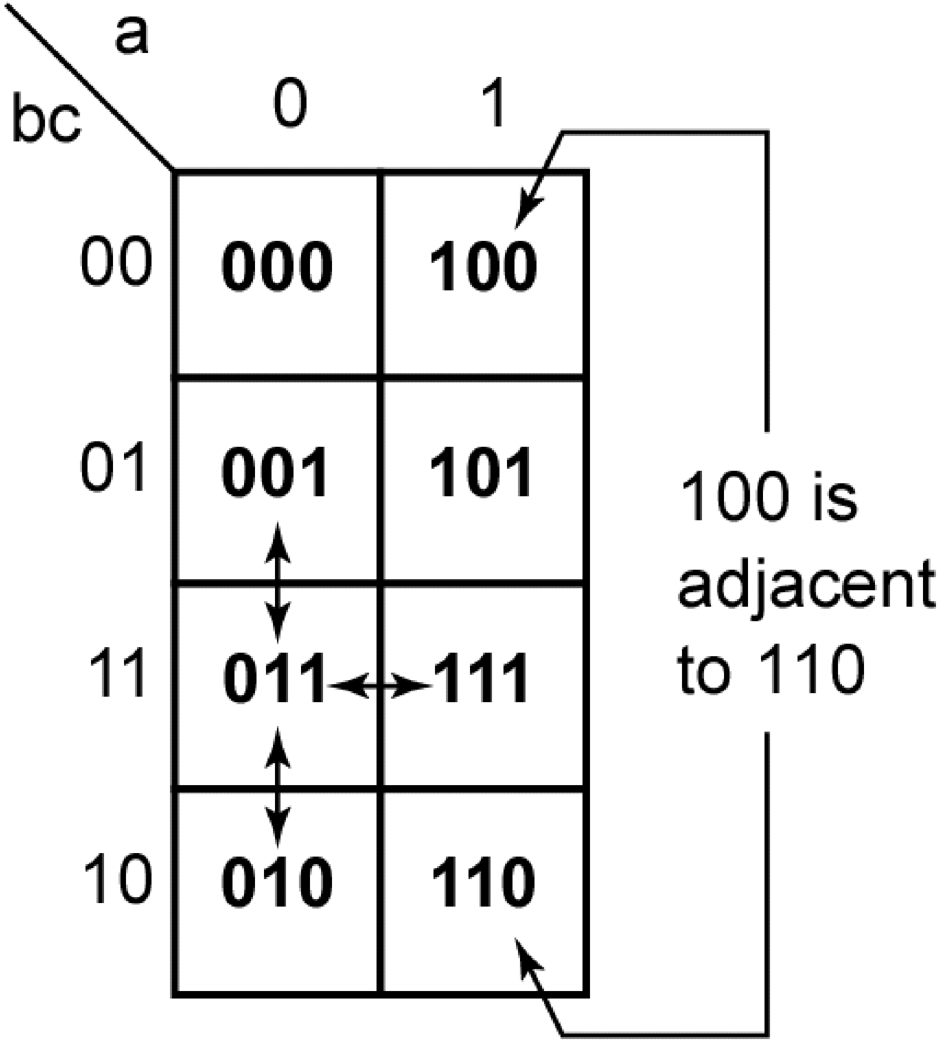
A B C	F
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	0

(a)

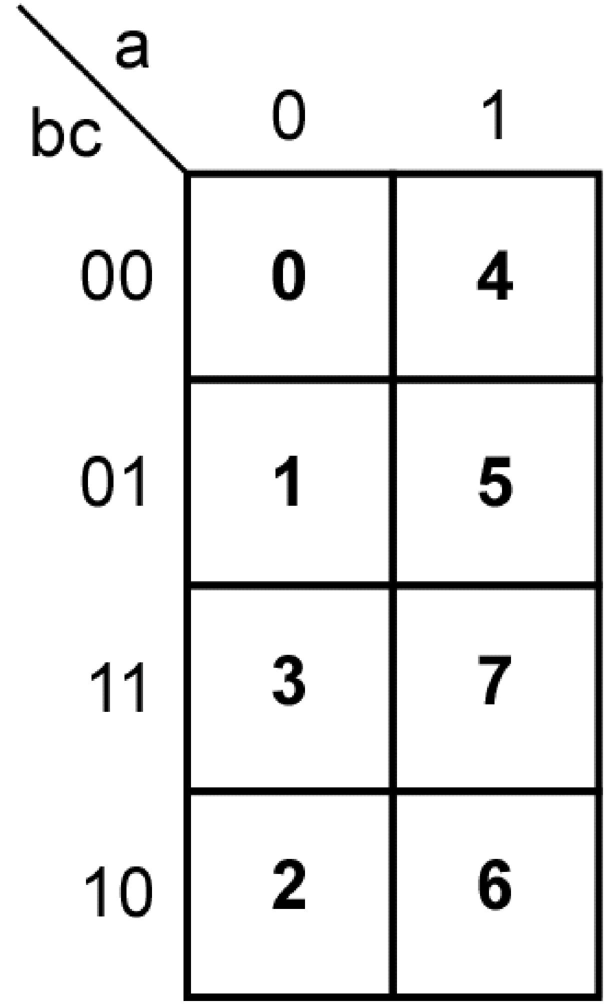


(b)

Figure 5-2: Karnaugh Map for Three-Variable Function



(a) Binary notation



(b) Decimal notation

Figure 5-3: **Location of Minterms on a Three-Variable Karnaugh Map**

bc \ a	0	1
00	**0** (0)	**0** (4)
01	**1** (1)	**1** (5)
11	**1** (3)	**0** (7)
10	**0** (2)	**0** (6)

Figure 5-4: **Karnaugh Map of $F(a, b, c) = \Sigma m(1, 3, 5) = \prod M(0, 2, 4, 6, 7)$**

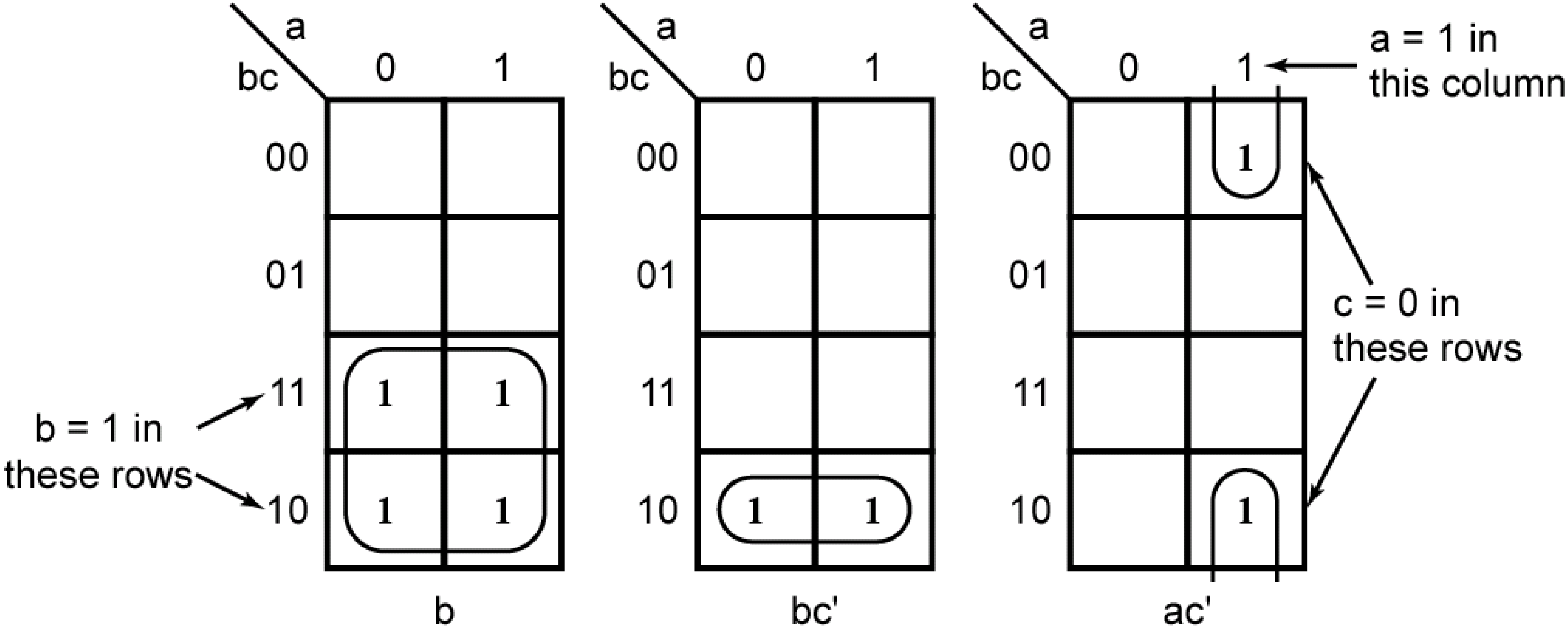
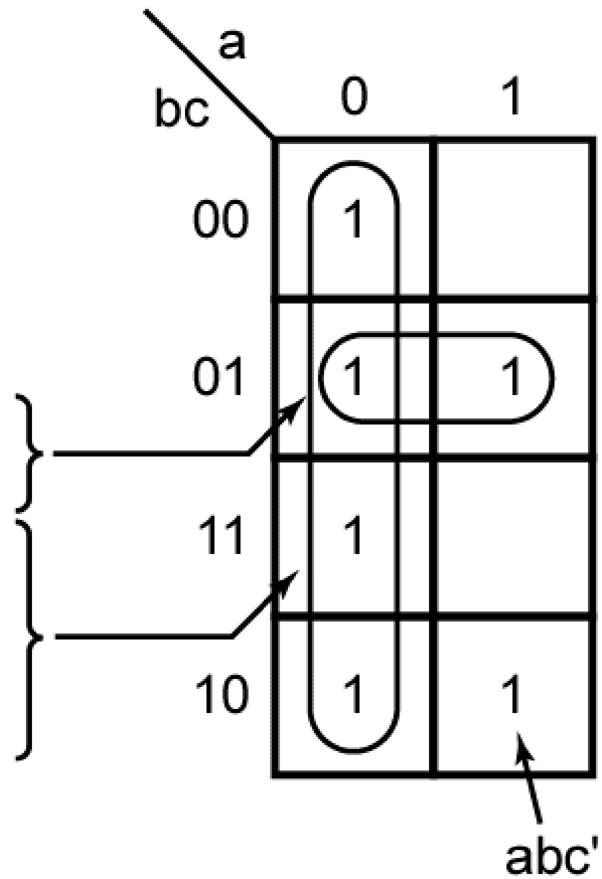


Figure 5-5: **Karnaugh Maps for Product Terms**

$f(a,b,c) = abc' + b'c + a'$

1. The term abc' is 1 when a = 1 and bc = 10, so we place a 1 in the square which corresponds to the a = 1 column and the bc = 10 row of the map.
2. The term b'c is 1 when bc = 01, so we place 1's in both squares of the bc = 01 row of the map.
3. The term a' is 1 when a = 0, so we place 1's in all the squares of the a = 0 column of the map. (Note: Since there already is a 1 in the abc = 001 square, we do not have to place a second 1 there because x + x = x.)



Section 5.2 (p. 131)

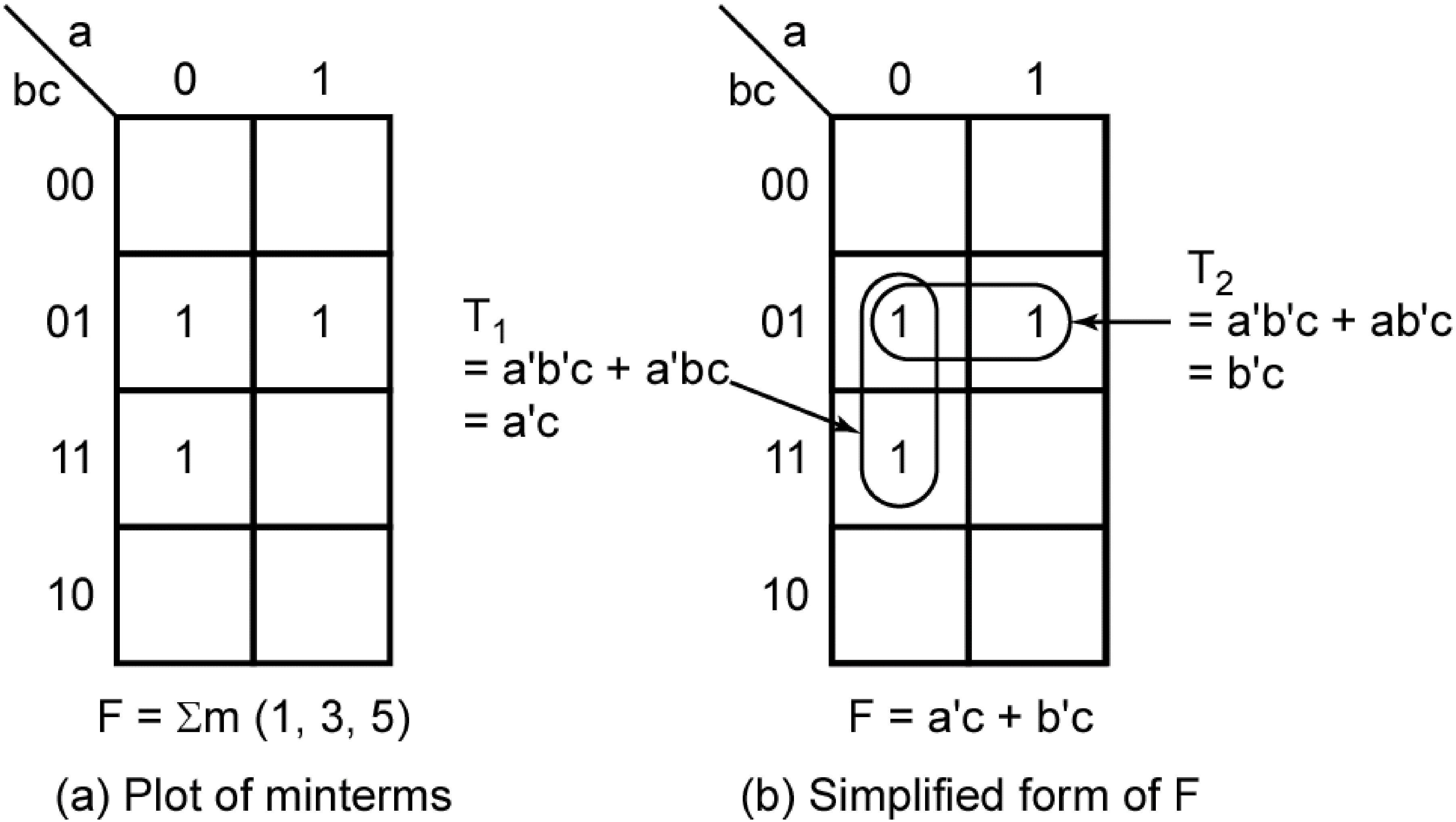
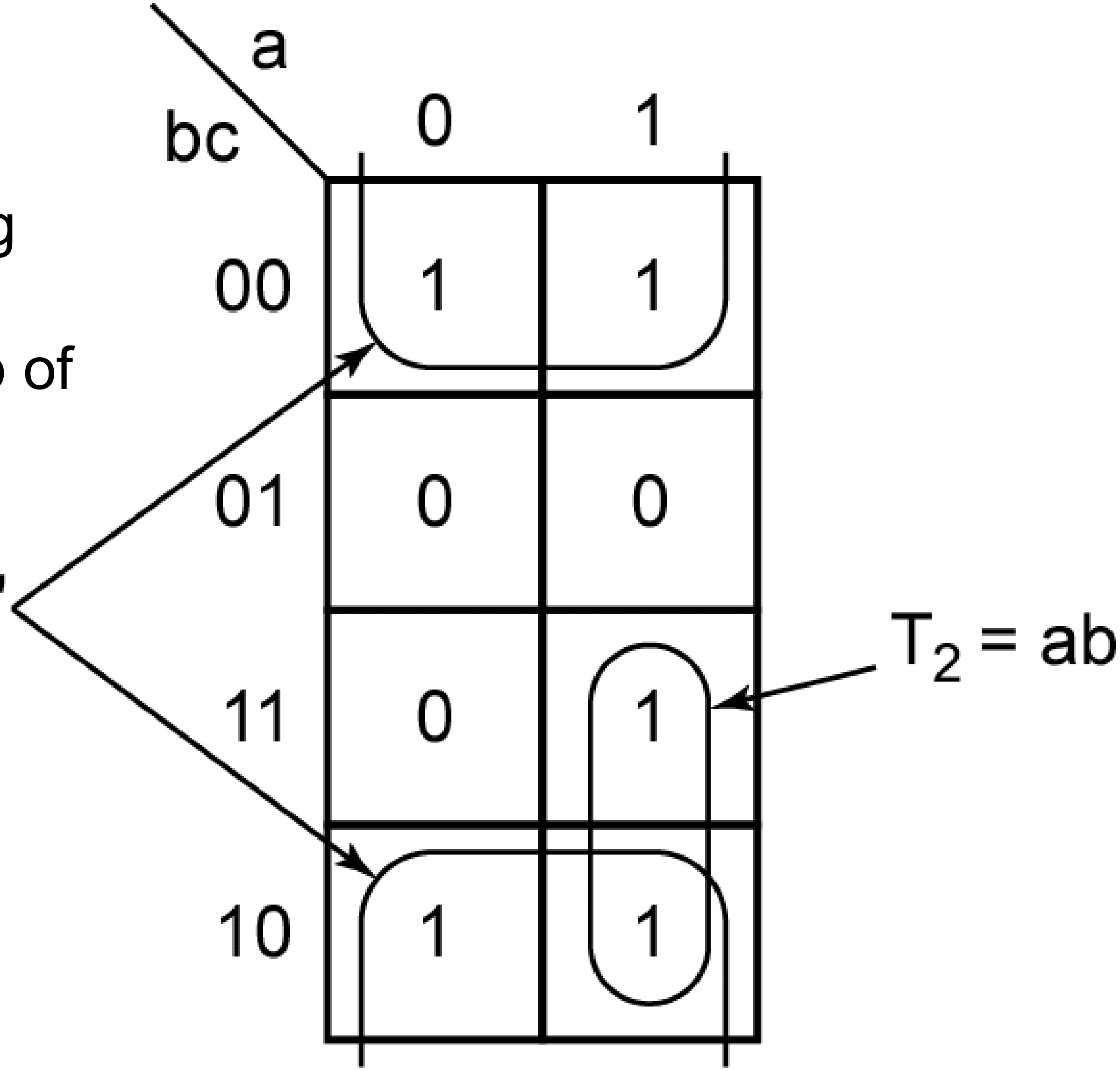


Figure 5-6: Simplification of a Three-Variable Function

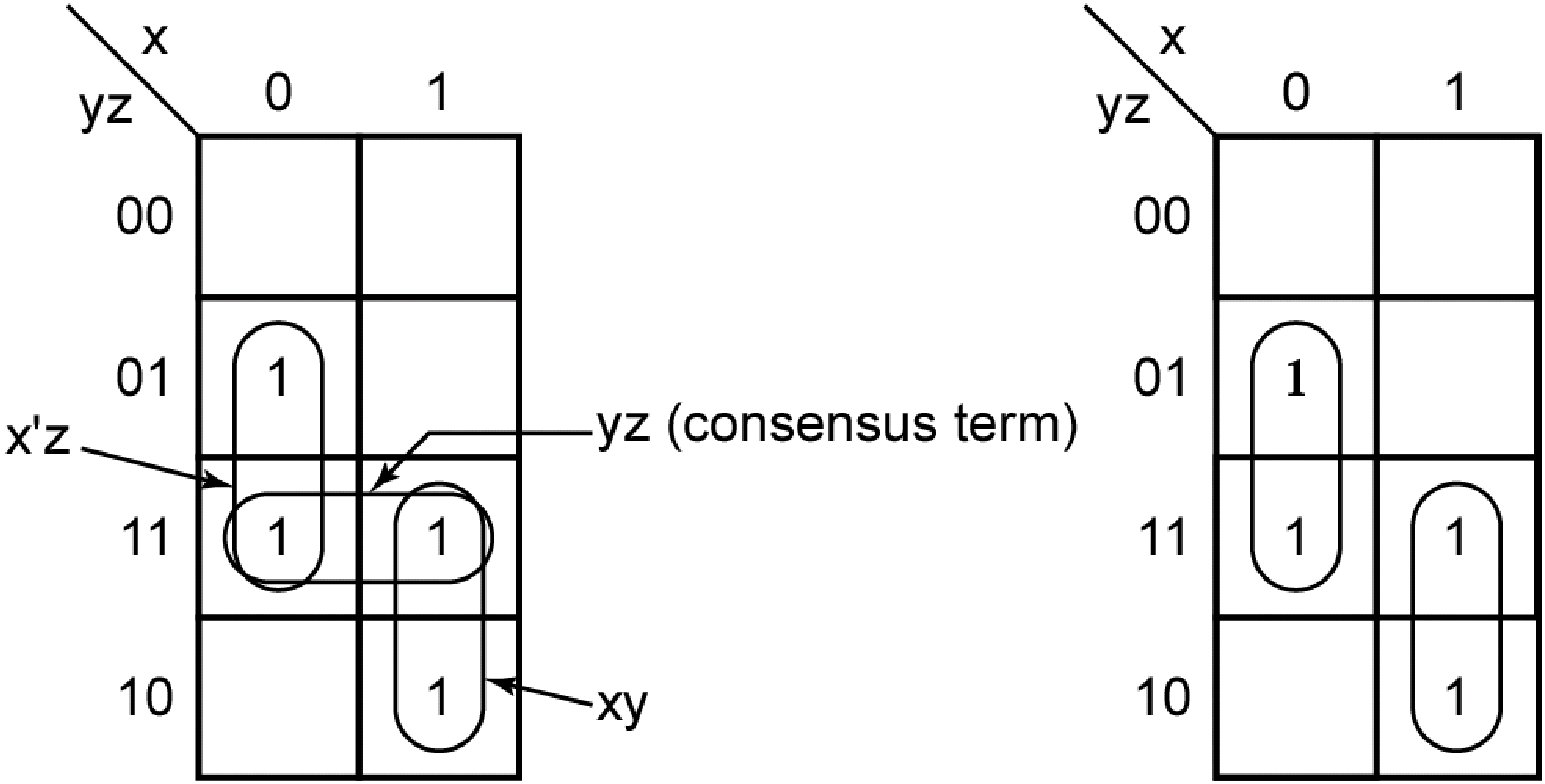
The map for the complement of *F* is formed by replacing 0’s with 1’s and 1’s with 9’s on the map of *F*.

$T_1 = b'c' + bc' = c'$



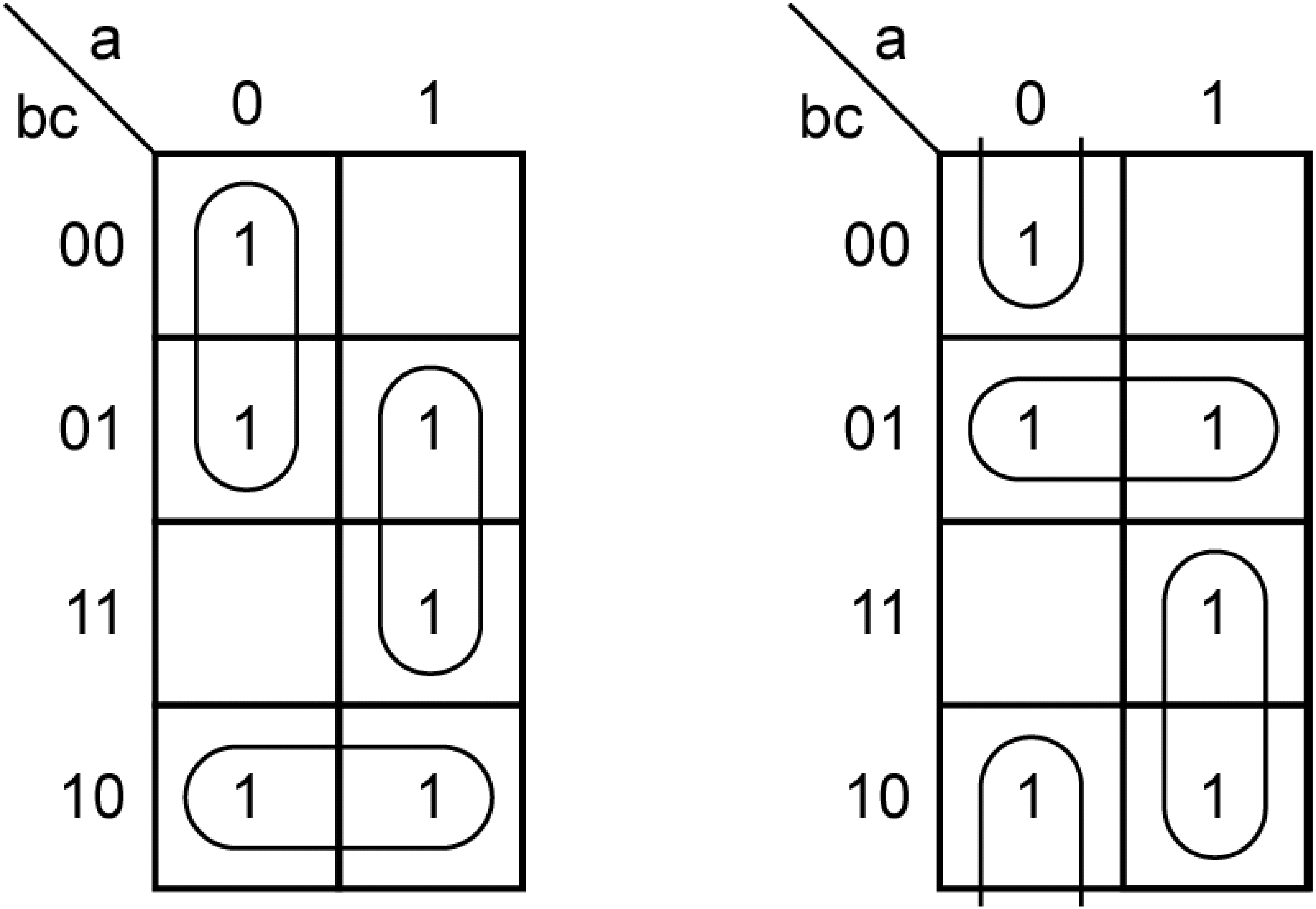
$F' = c' + ab$

Figure 5-7: **Complement of Map in Figure 5-6a**



$$xy + x'z + yz = xy + x'z$$

Figure 5-8: **Karnaugh Maps Which Illustrate the Consensus Theorem**



$F = a'b' + bc' + ac$ $F = a'c' + b'c + ab$

Figure 5-9: **Function with Two Minimal Forms**

Each minterm is located adjacent to the four terms with which it can combine. For example, m_5 (0101) could combine with m_1 (0001), m_4 (0100), m_7 (0111), or m_{13} (1101).

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Figure 5-10: **Location of Minterms on Four-Variable Karnaugh Map**

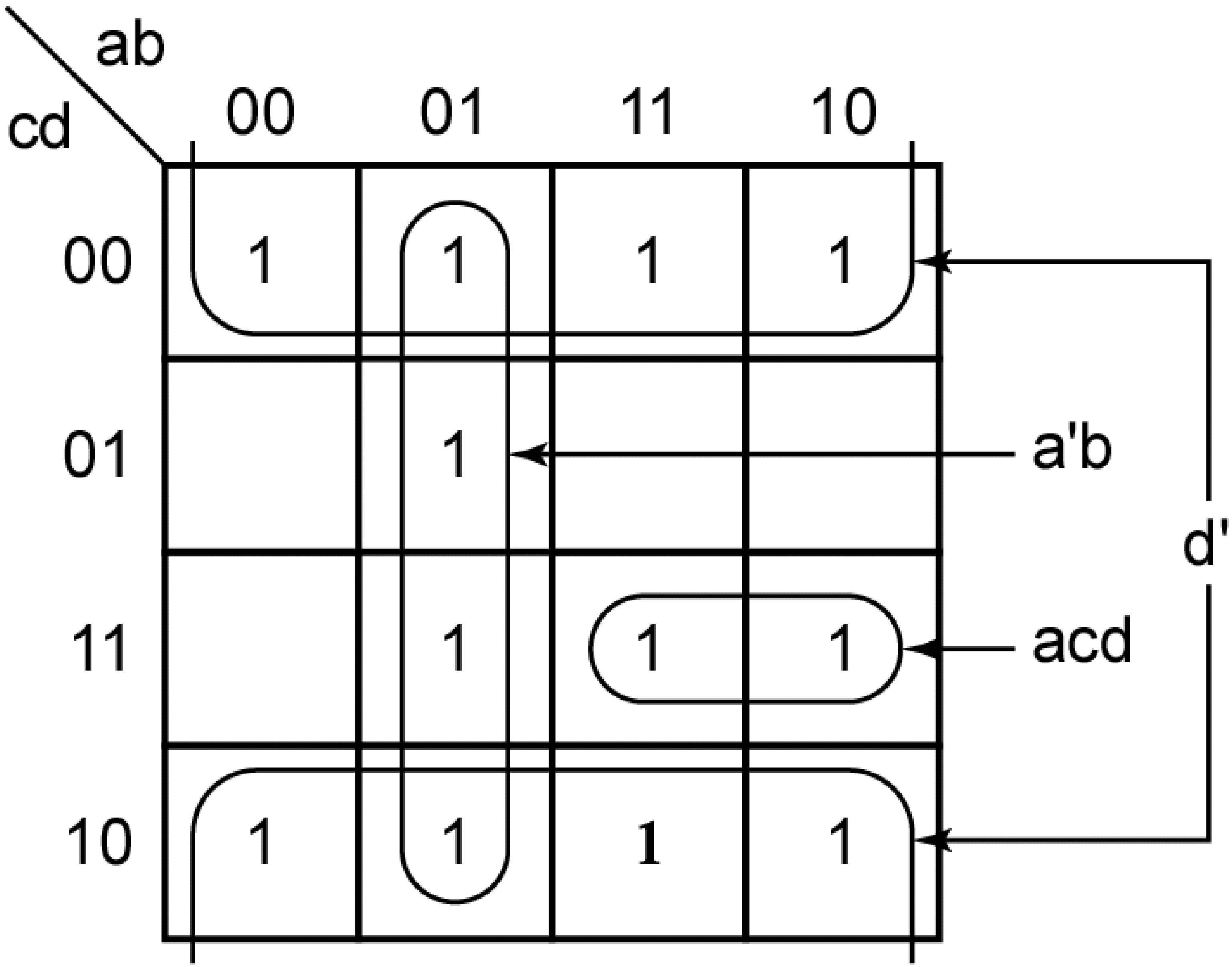
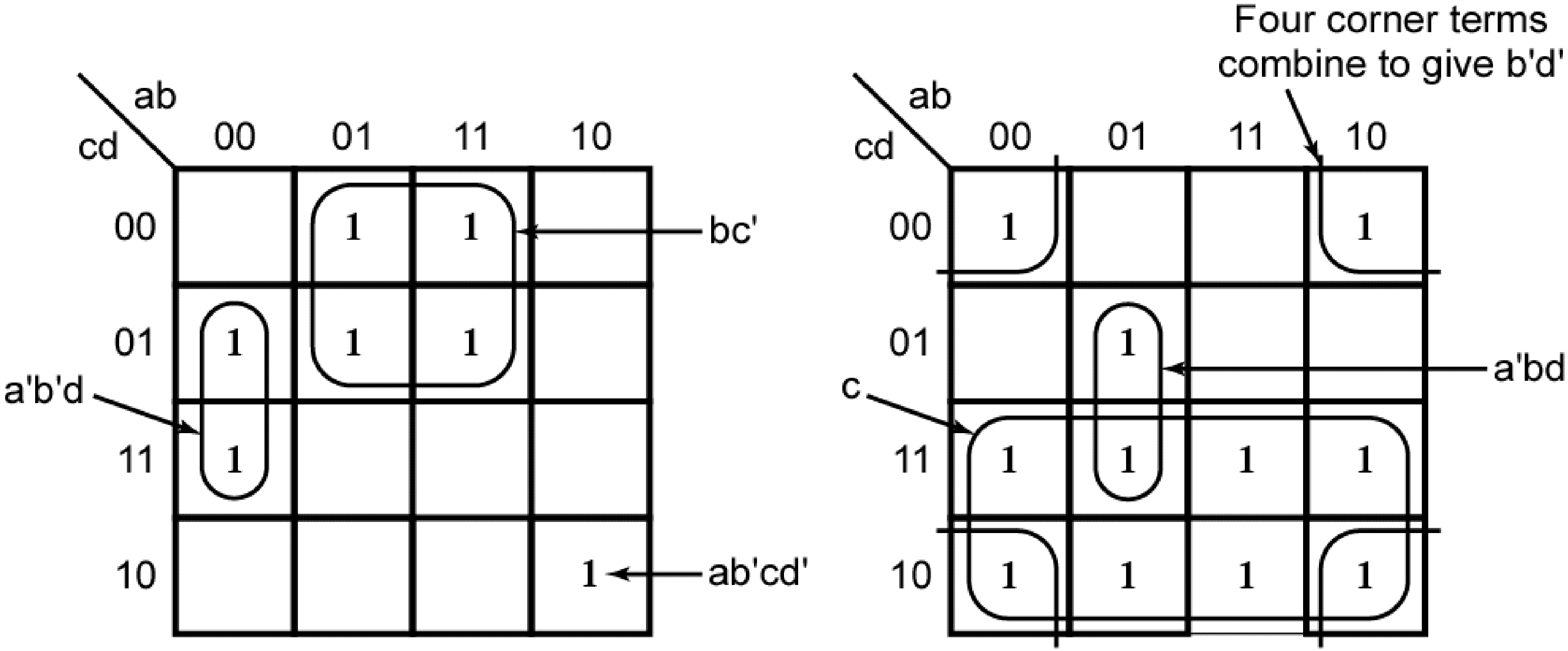


Figure 5-11: **Plot of *acd + a'b + d'***



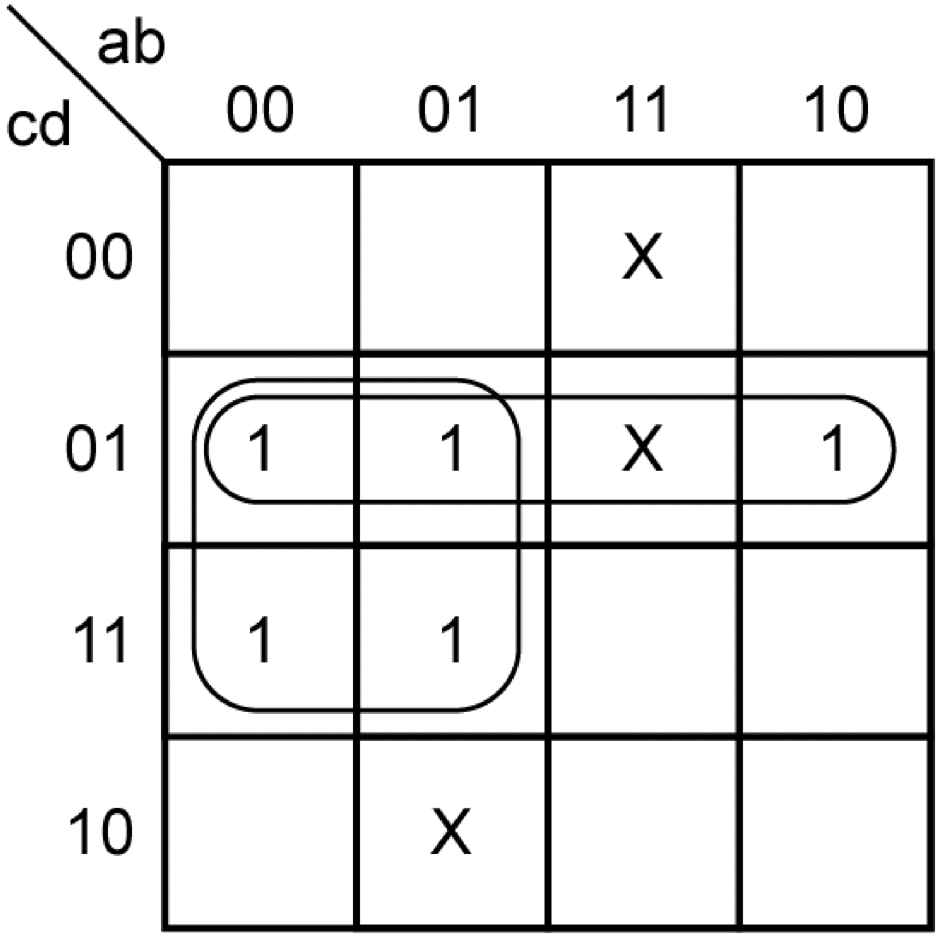
$f_1 = \Sigma m(1, 3, 4, 5, 10, 12, 13)$
$= bc' + a'b'd + ab'cd'$

(a)

$f_2 = \Sigma m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$
$= c + b'd' + a'bd$

(b)

Figure 5-12: **Simplification of Four-Variable Functions**



$$f = \Sigma m(1, 3, 5, 7, 9) + \Sigma d(6, 12, 13)$$
$$= a'd + c'd$$

Figure 5-13: **Simplification of an Incompletely Specified Function**

Example

Use a four variable Karnaugh map to find the minimum product of sums for f:

$f = x'z' + wyz + w'y'z' + x'y$

First, we will plot the 1's on a Karnaugh map.

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Then, from the 0's we get:

$f' = y'z + wxz' + w'xy$

Finally, we can complement f' to get a minimum product of sums of f:

$f = (y + z')(w' + x' + z)(w + x' + y')$

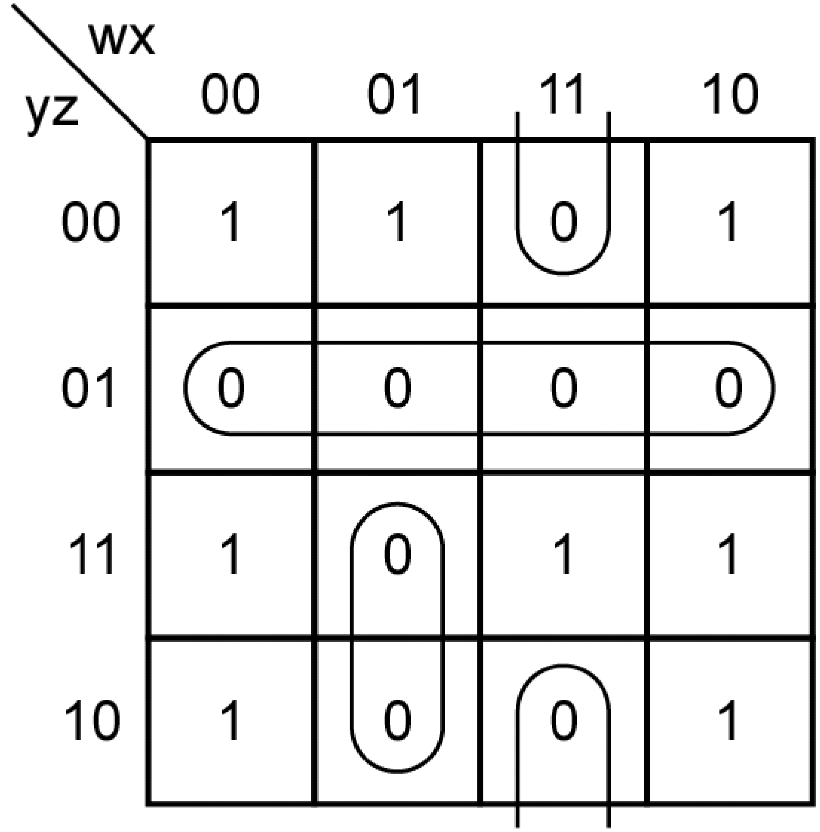

Figure 5-14

Implicants and Prime Implicants

Any single 1 or any group of 1's which can be combined together on a map of the function *F* represents a product term which is called an *implicant* of *F*.

A product term implicant is called a *prime implicant* if it cannot be combined with another term to eliminate a variable.

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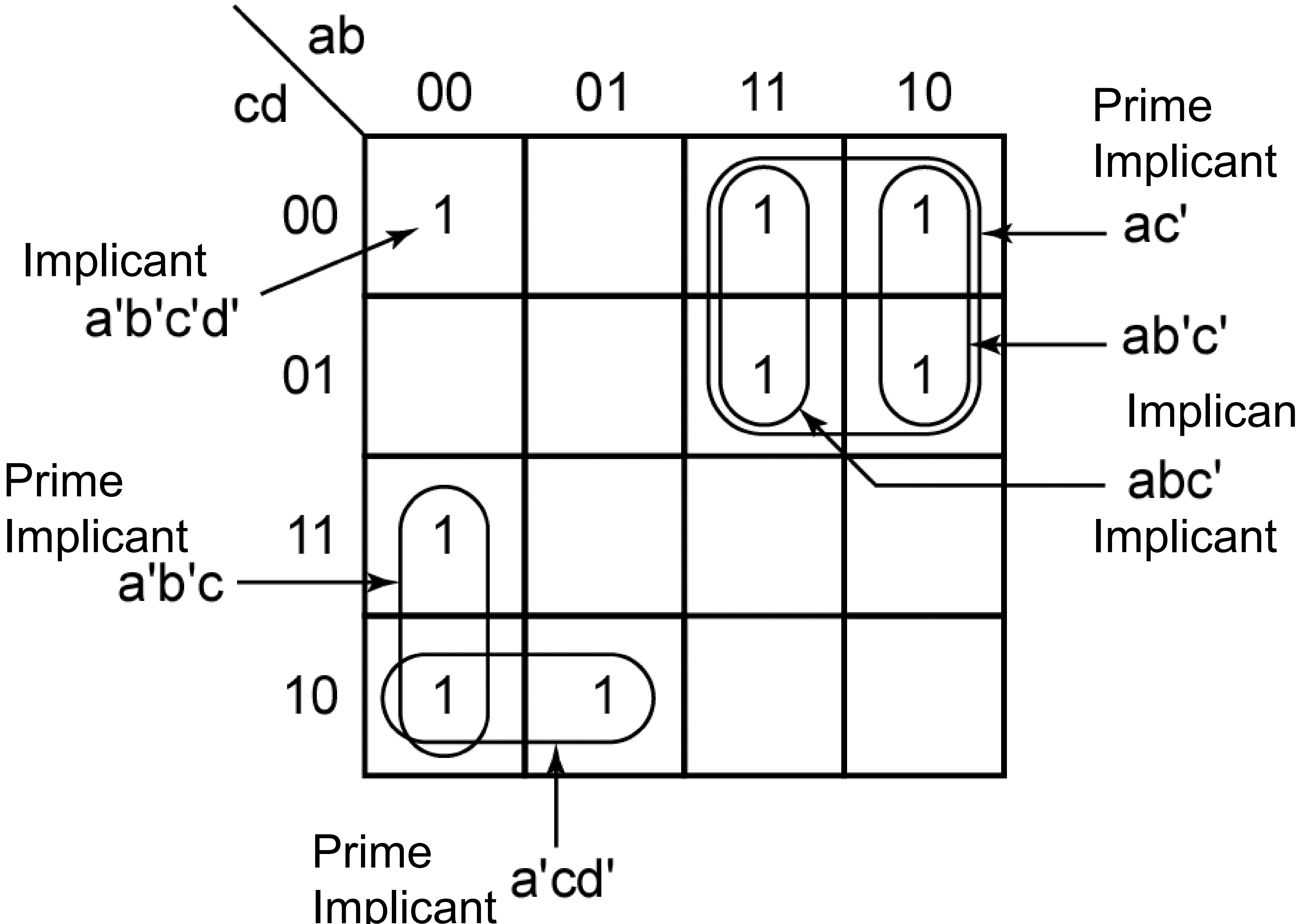
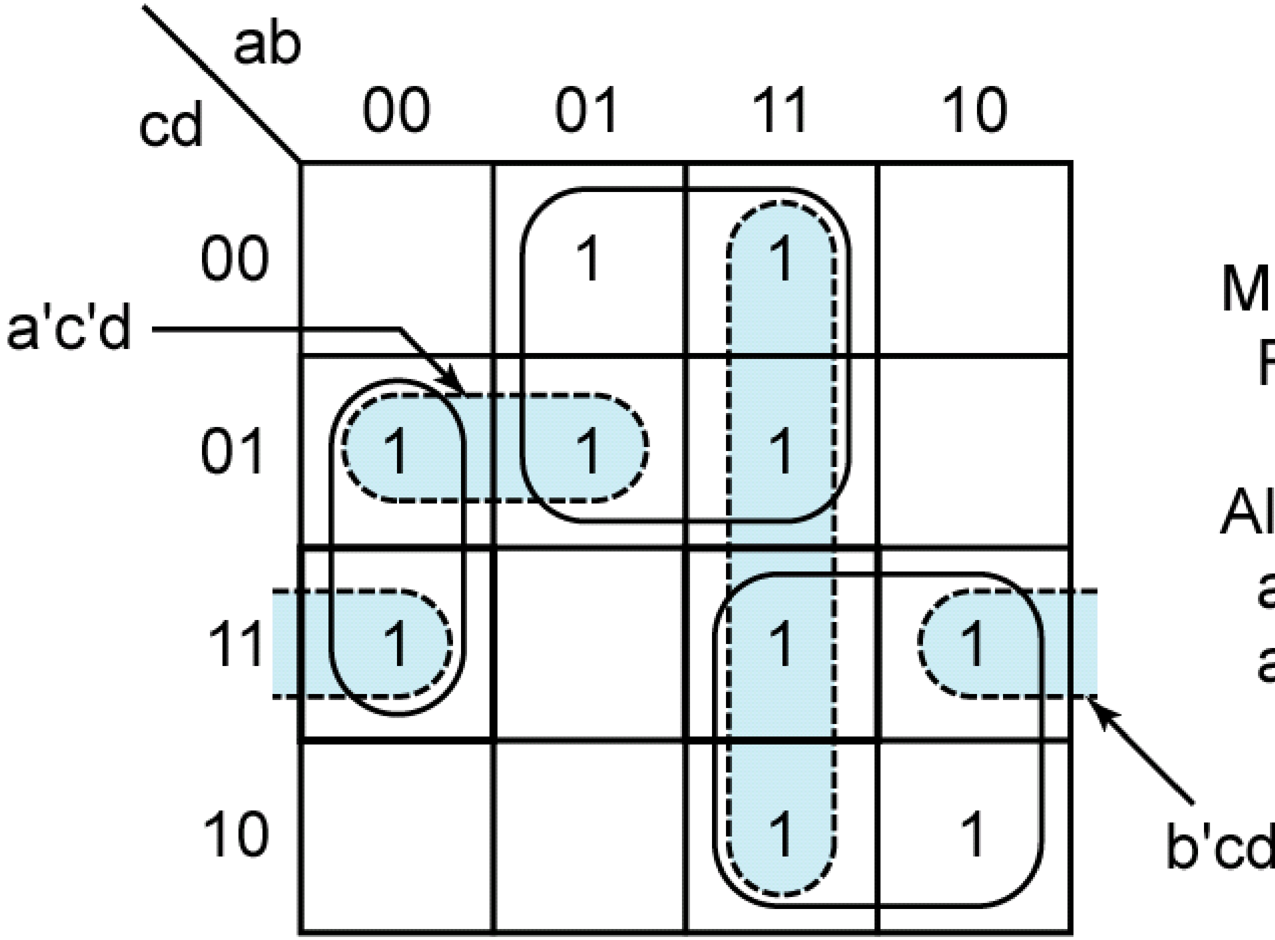

ab
cd
00
01
11
10
00
01
11
10
1
1
1
1
1
1
1
1
Implicant
a'b'c'd'
Prime
Implicant
ac'
ab'c'
Implicant
abc'
Implicant
Prime
Implicant
a'b'c
Prime
Implicant
a'cd'

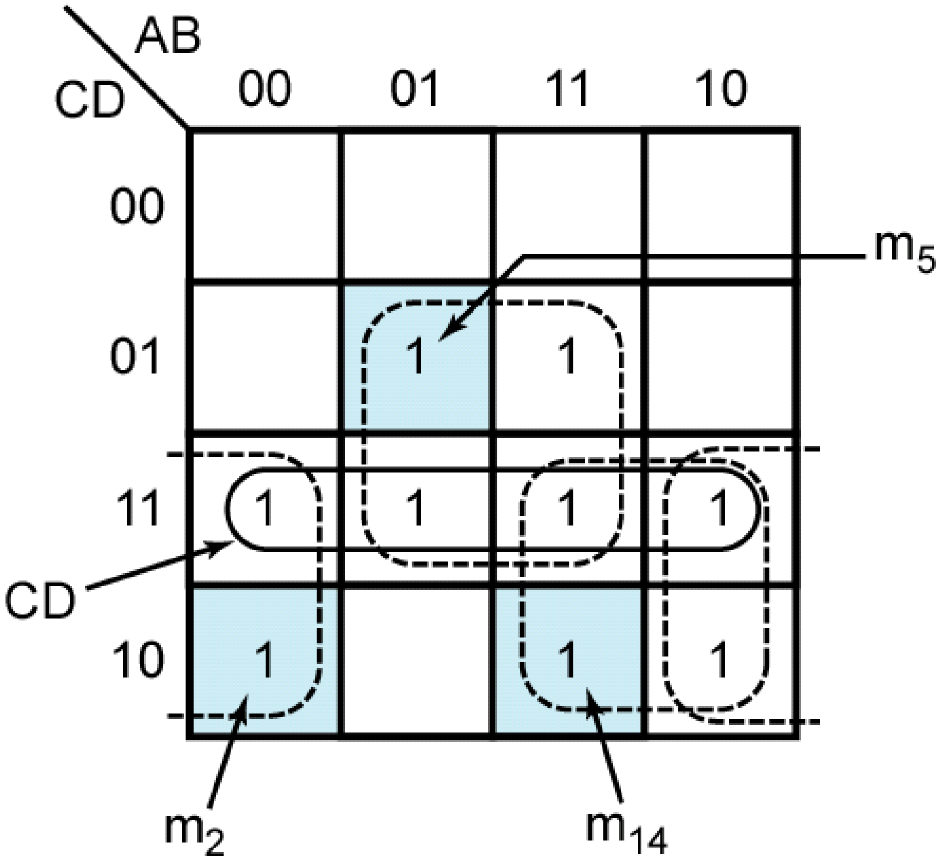
Figure 5-15



Minimum solution:
F = a'b'd + bc' + ac

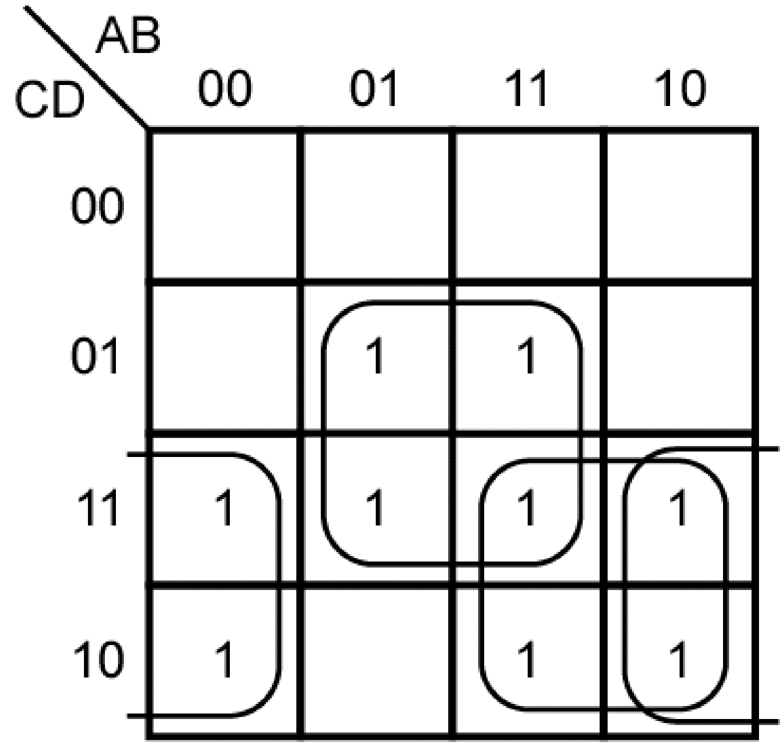
All prime implicants:
a'b'd, bc', ac,
a'c'd, ab, b'cd

Figure 5-16: **Determination of All Prime Implicants**



f = CD + BD + B'C + AC

(a)



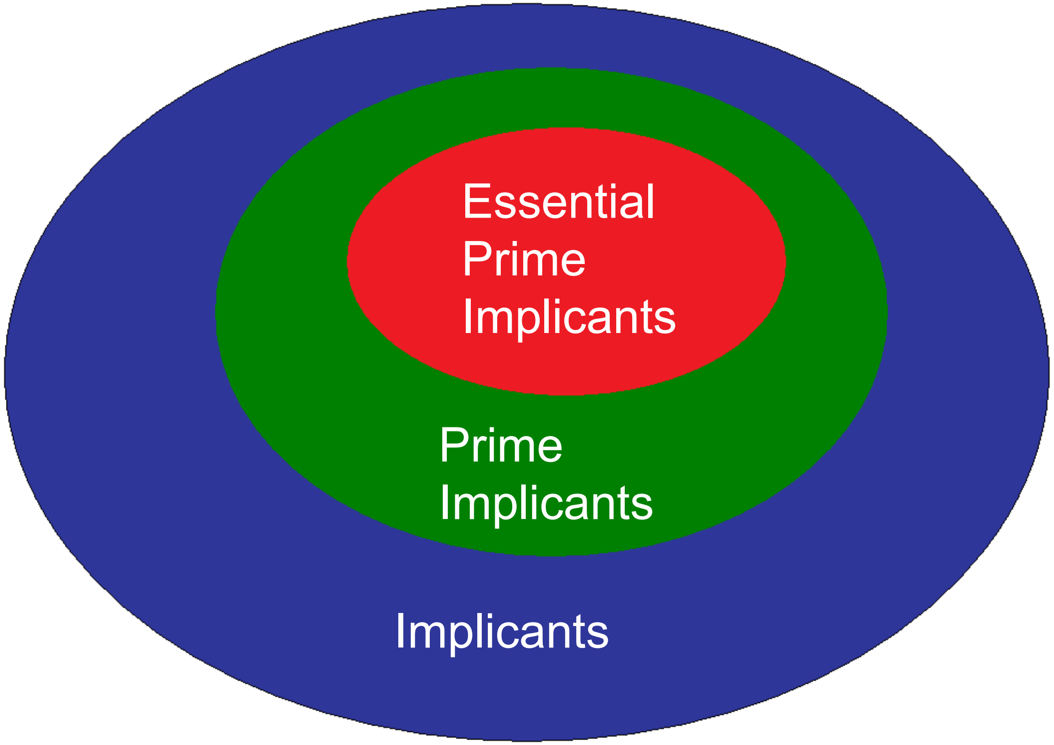
f = BD + B'C + AC

(b)

Figure 5-17

Essential Prime Implicants

If a minterm is covered by only one prime implicant, that prime implicant is said to be *essential*, and it must be included in the minimum sum of products.



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Note: 1’s shaded in blue are covered by only one prime implicant. All other 1’s are covered by at least two prime implicants.

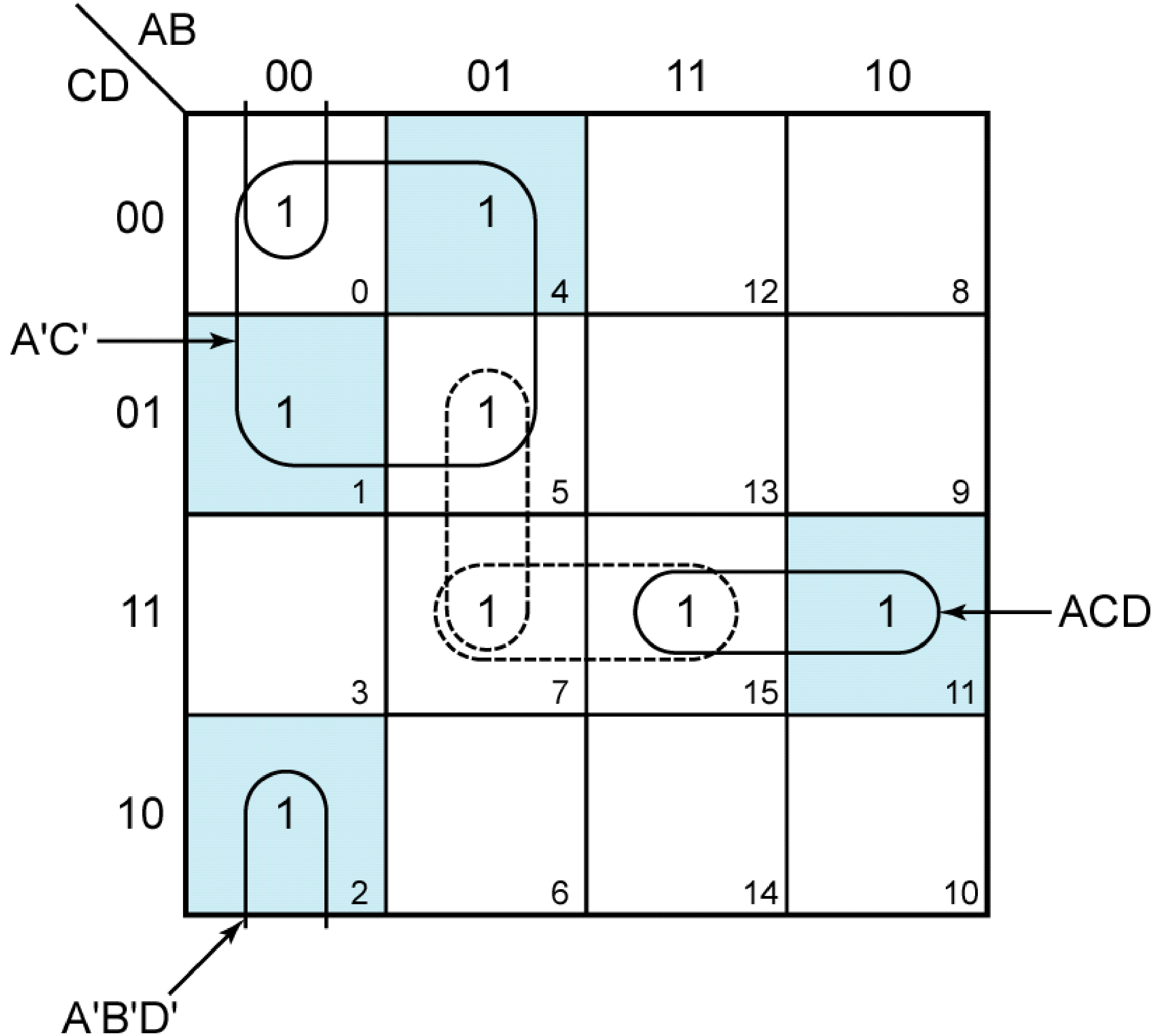

Figure 5-18

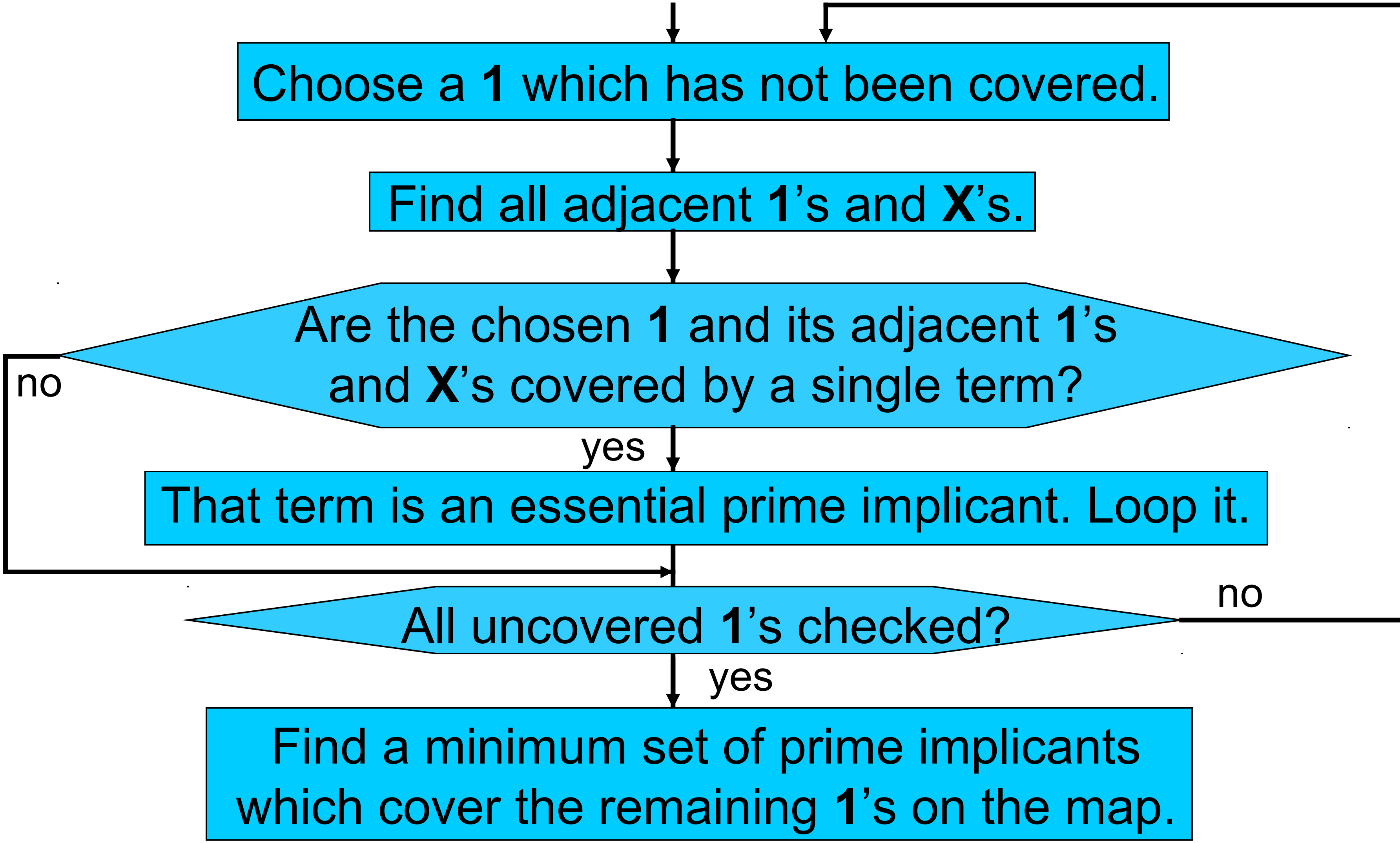

Figure 5-19:

Flowchart for Determining a Minimum Sum of Products Using a Karnaugh Map

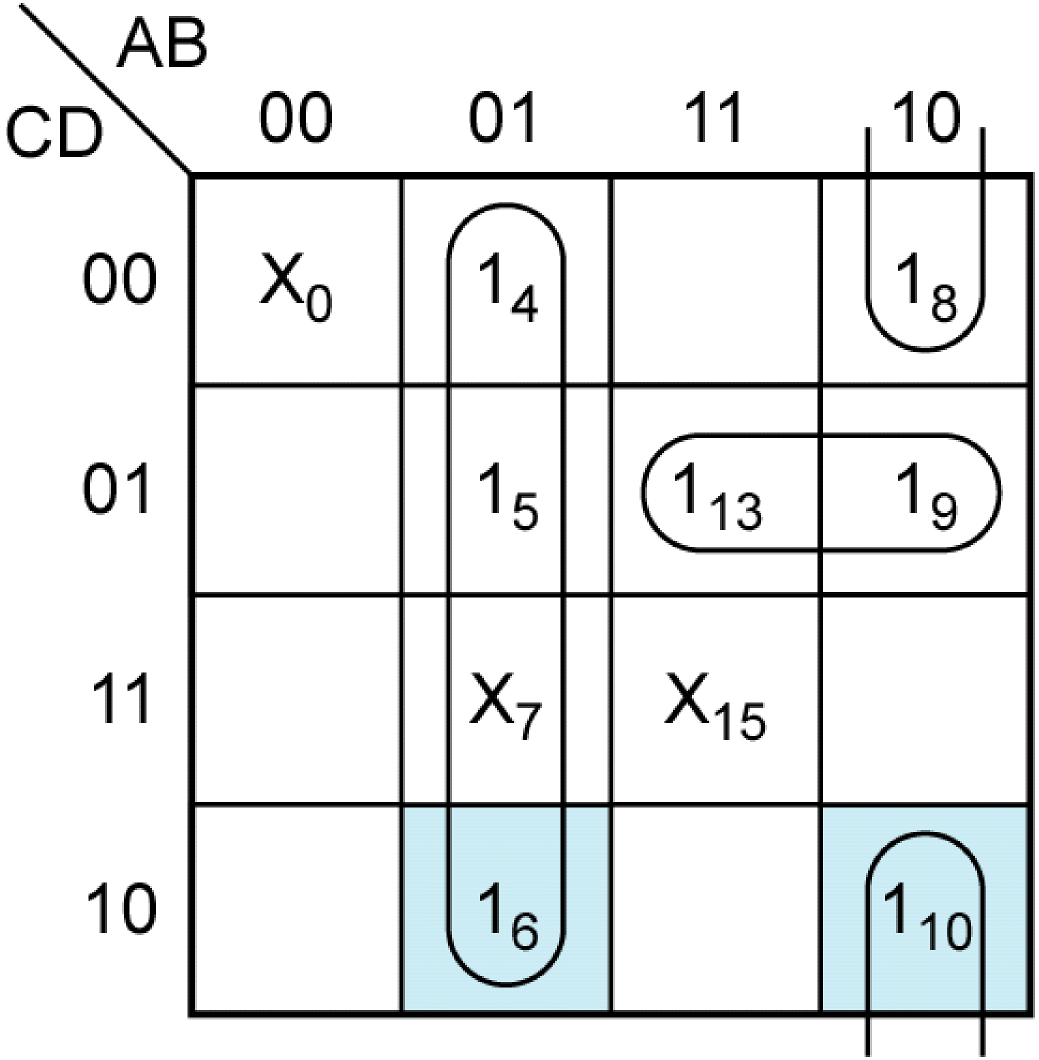

Figure 5-20

Shaded 1’s are covered by only one prime implicant.

Essential prime implicants:

A′B, AB′D′

Then *AC′D* covers the remaining 1’s.

Five-Variable Karnaugh Maps

A five-variable map can be constructed in three dimensions by placing one four-variable map on top of a second one. Terms in the bottom layer are numbered 0 through 15 and corresponding terms in the top layer are numbered 16 through 31, so that the terms in the bottom layer contain *A*' and those in the top layer contain *A*.

To represent the map in two dimensions, we will divide each square in a four-variable map by a diagonal line and place terms in the bottom layer below the line and terms in the top layer above the line.

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Note that terms m_0 and m_{20} do not combine because they are in different layers and different columns (they differ in two variables).

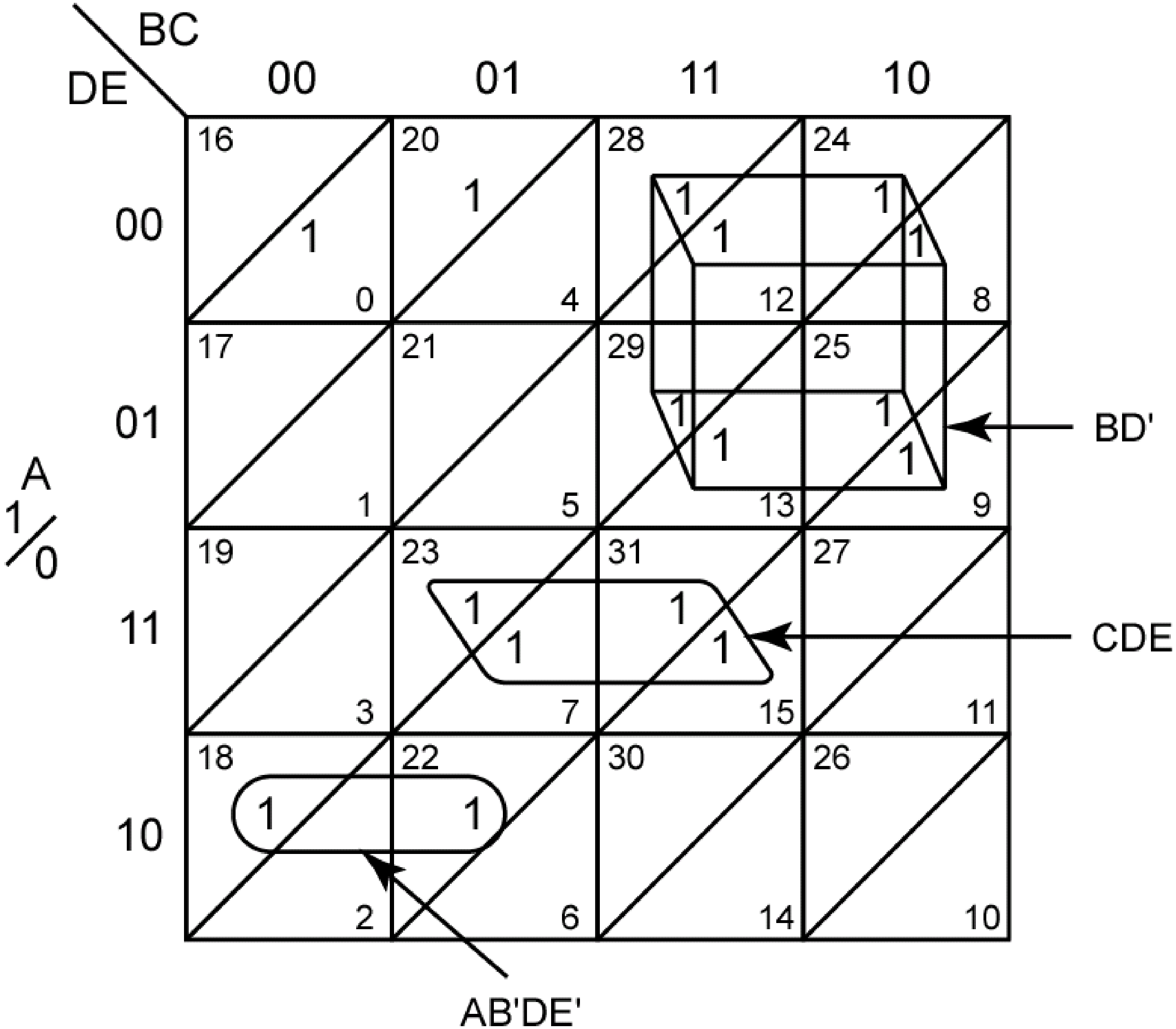


Figure 5-21: **A Five-Variable Karnaugh Map**

Each term can be adjacent to exactly five other terms: four in the same layer and one in the other layer.

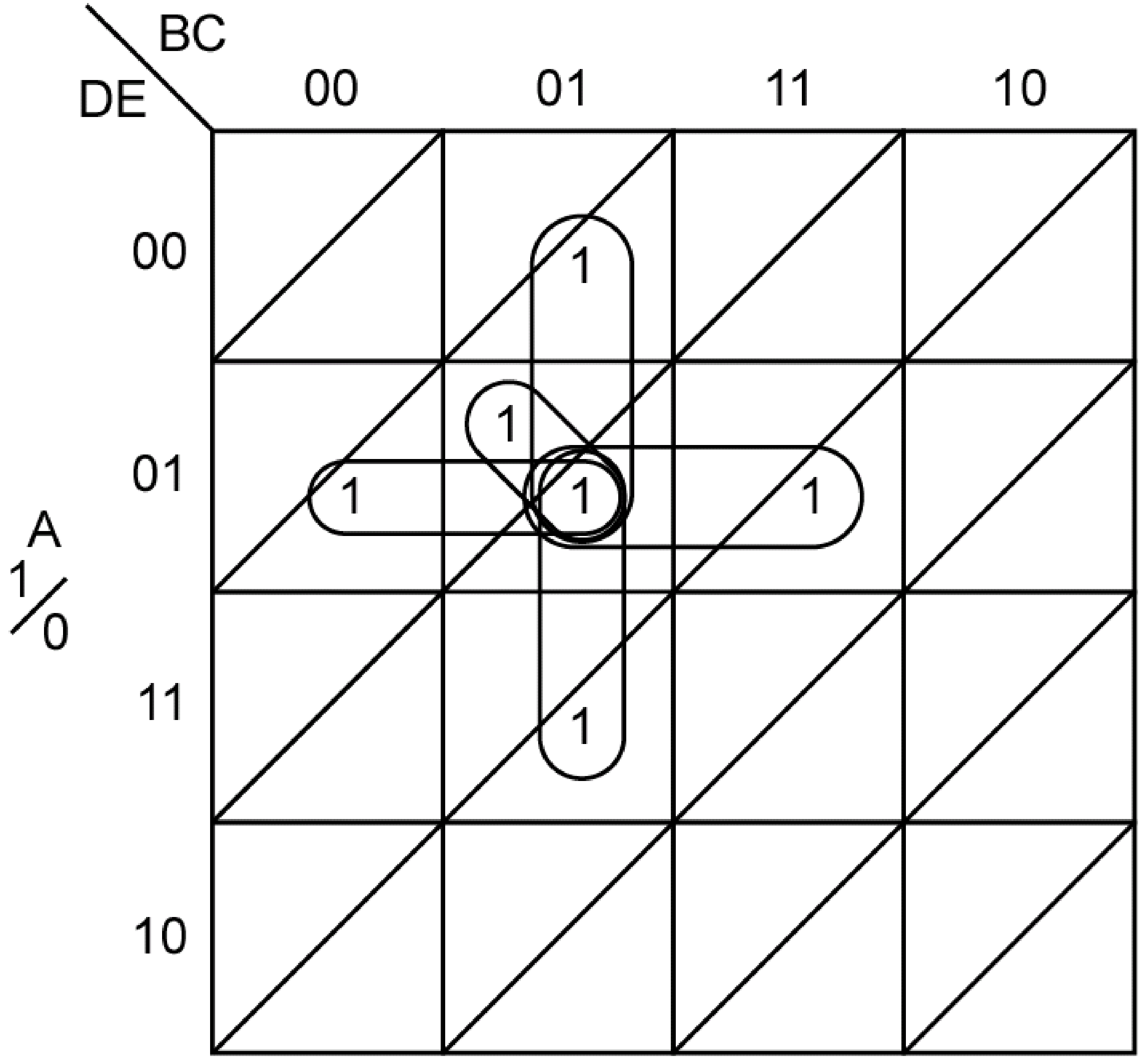


Figure 5-22

When checking for adjacencies, each term should be checked against the five possible adjacent squares.

P_1 and P_2 are essential prime implicants.

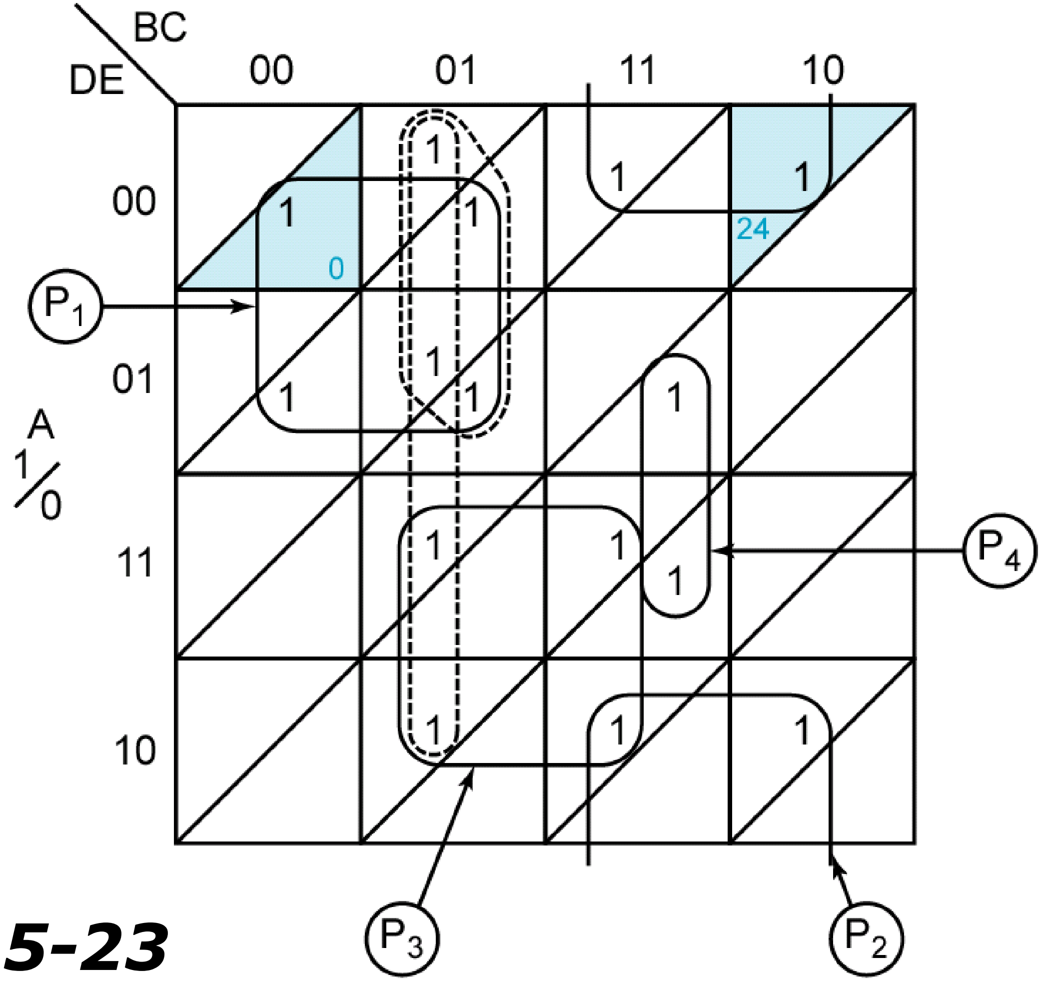
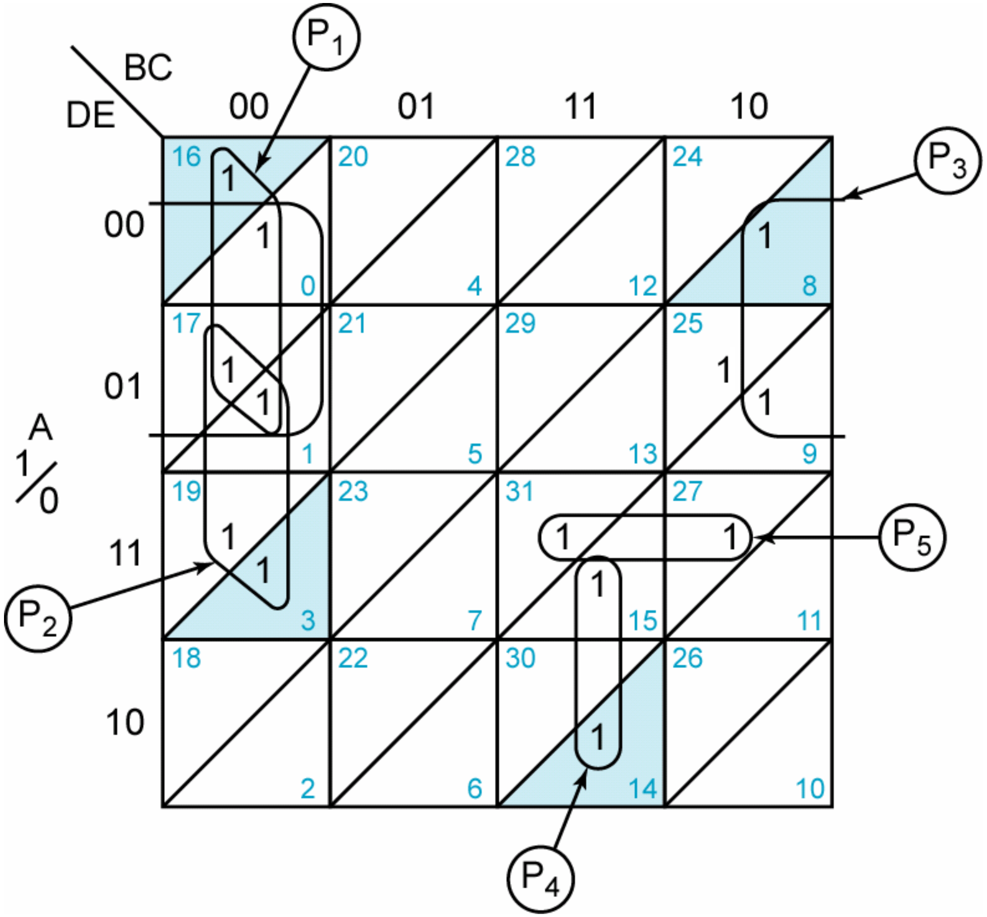


Figure 5-23

$$F = \underset{P_1}{A'B'D'} + \underset{P_2}{ABE'} + \underset{P_3}{ACD} + \underset{P_4}{A'BCE} + \begin{Bmatrix} AB'C \\ \text{or} \\ B'CD' \end{Bmatrix}$$



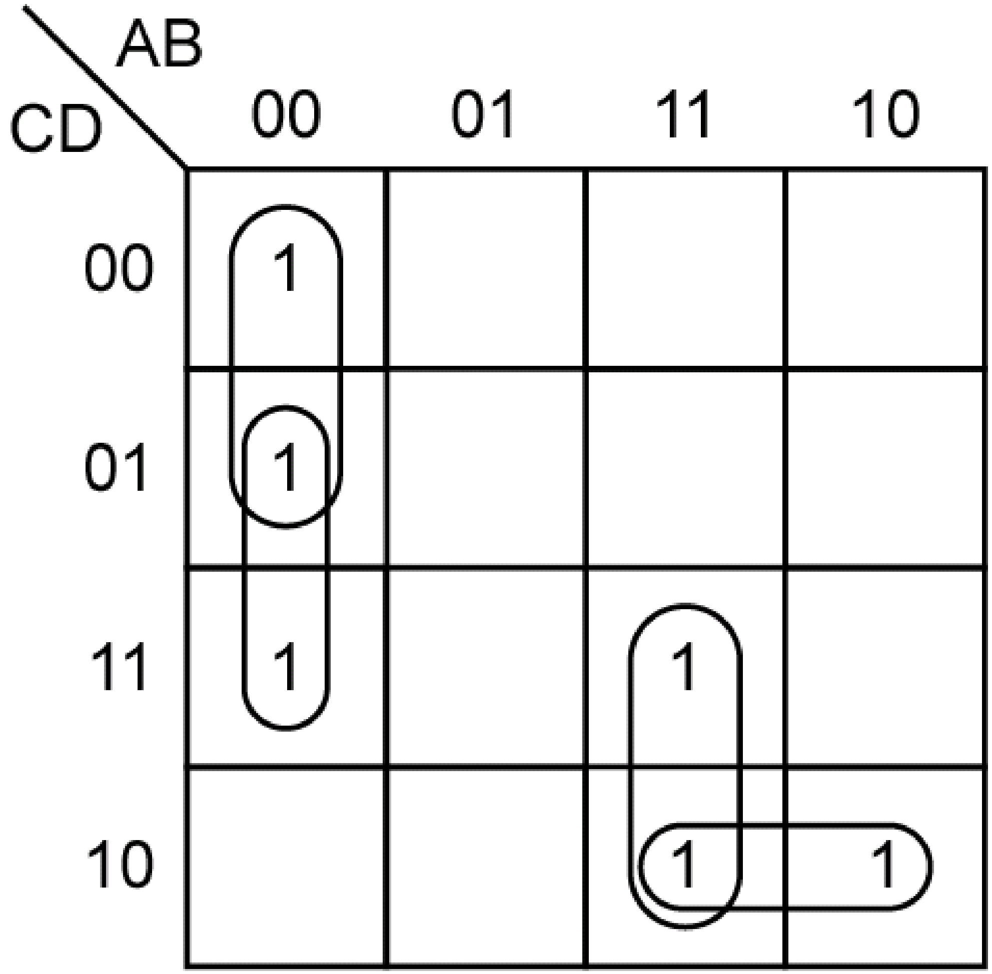
P_1, P_2, P_3, and P_4 are essential prime implicants.

$F(A, B, C, D, E) = \Sigma\, m(0, 1, 3, 8, 9, 14, 15, 16, 17, 19, 25, 27, 31)$

$$F = \underset{P_1}{B'C'D'} + \underset{P_2}{B'C'E} + \underset{P_3}{A'C'D'} + \underset{P_4}{A'BCD} + \underset{P_5}{ABDE} + \begin{Bmatrix} C'D'E \\ \text{or} \\ AC'E \end{Bmatrix}$$

Figure 5-24

Using a Karnaugh map to facilitate factoring, we see that the two terms in the first column have *A′B′* in common; the two terms in the lower right corner have *AC* in common.



$$F = A'B'(C' + D) + AC(B + D')$$

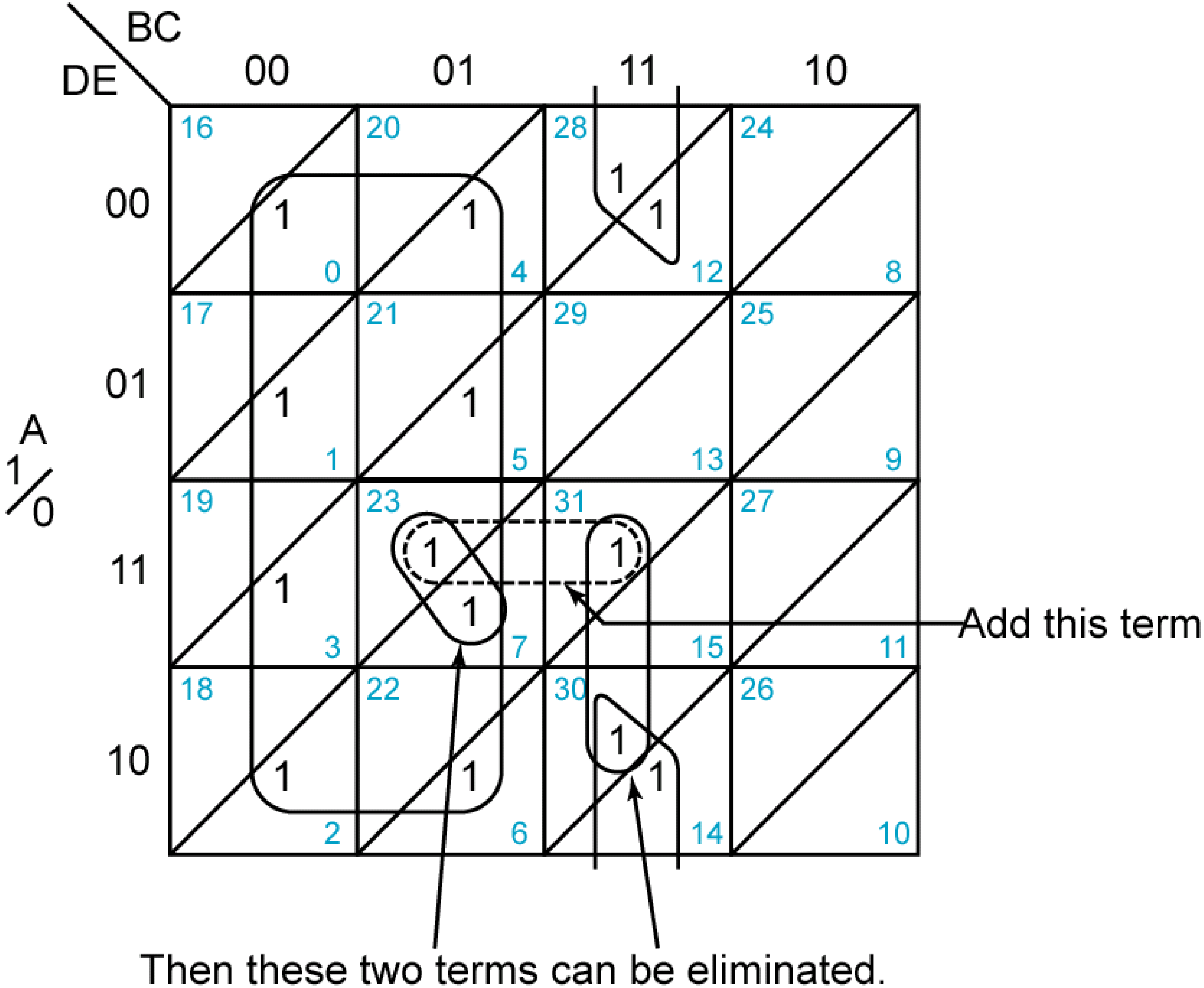
Figure 5-25

$$F = ABCD + B'CDE + A'B' + BCE'$$

We can use a Karnaugh map for guidance in algebraic simplification. From Figure 5-26, we can add the term ACDE by the consensus theorem and then eliminate ABCD and B'CDE.

$$F = ABCD + B'CDE + A'B' + BCE' + ACDE$$

$$F = A'B' + BCE' + ACDE$$



BC
DE
00
01
11
10
A
1/0
16
20
28
24
0
4
12
8
17
21
29
25
1
5
13
9
19
23
31
27
3
7
15
11
18
22
30
26
2
6
14
10
1
Add this term.
Then these two terms can be eliminated.

Figure 5-26

Other Forms of Karnaugh Maps

Instead of labeling the sides of a Karnaugh map with 0's and 1's, some people prefer to use Veitch diagrams. In Veitch diagrams, A = 1 for the half of the map labeled A, and A = 0 for the other half. The other variables have a similar interpretation.

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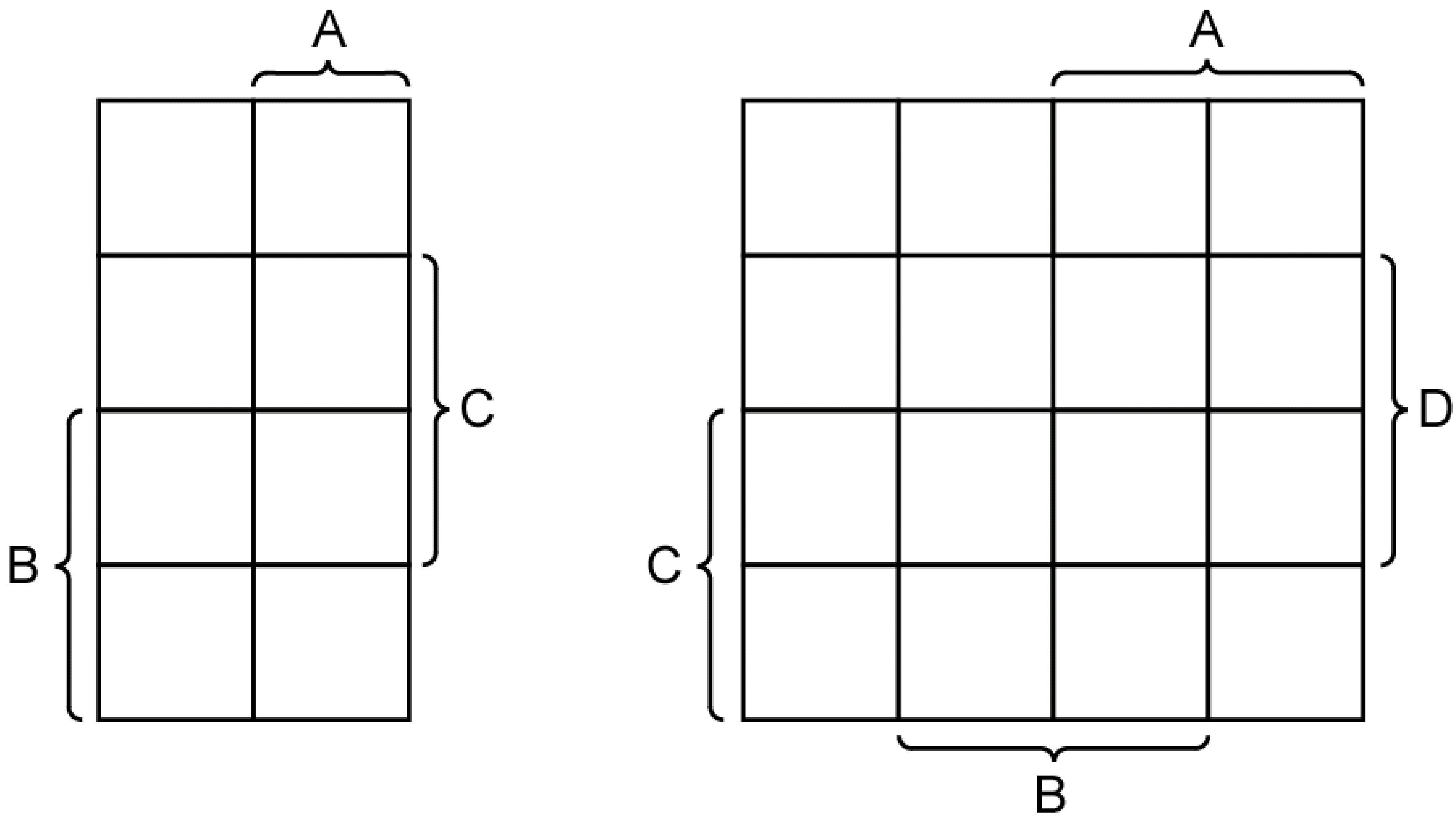
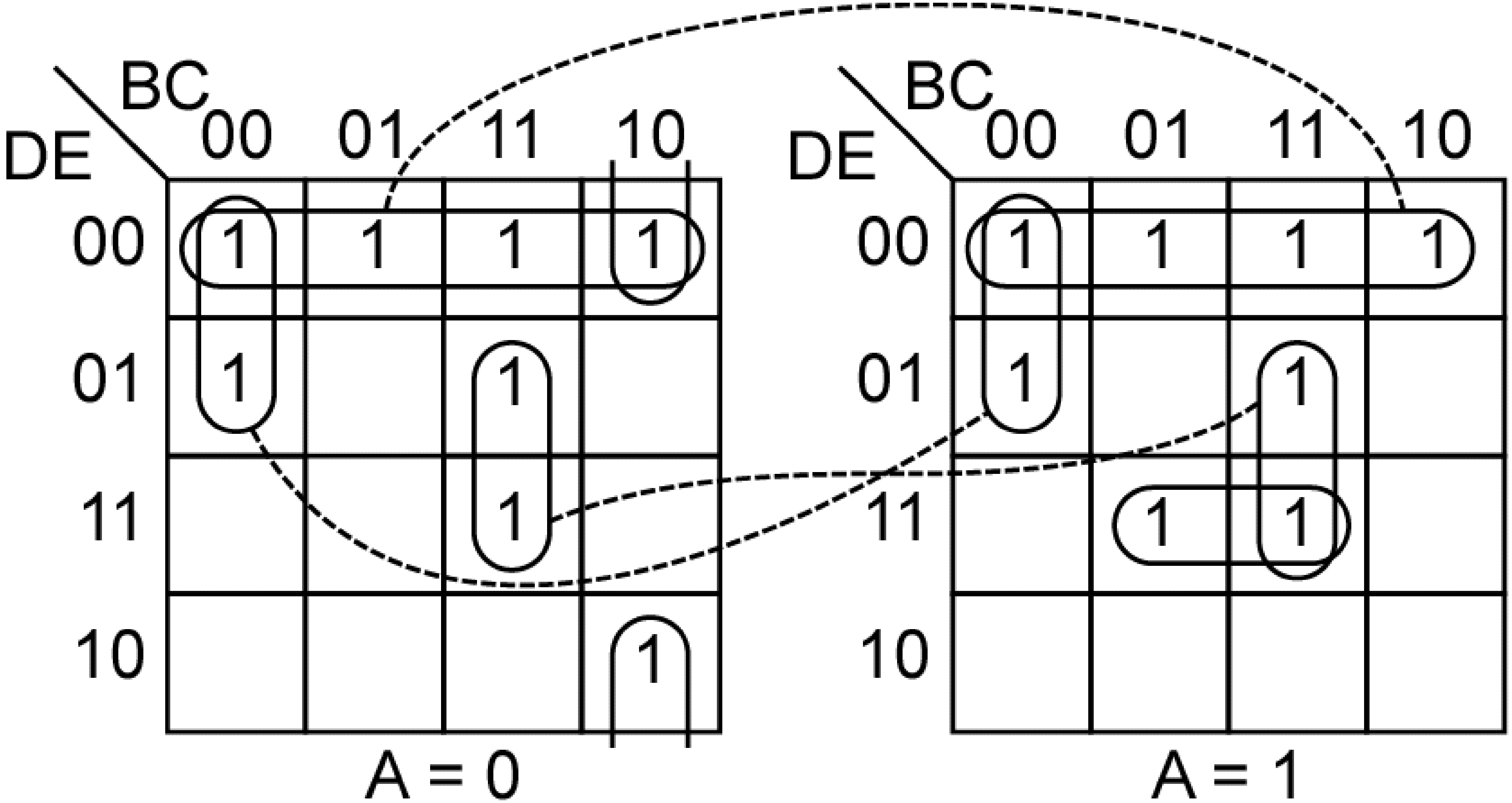


Figure 5-27: **Veitch Diagrams**

Two alternative forms of five-variable maps are also used. One form simply consists of two four-variable maps side-by-side.

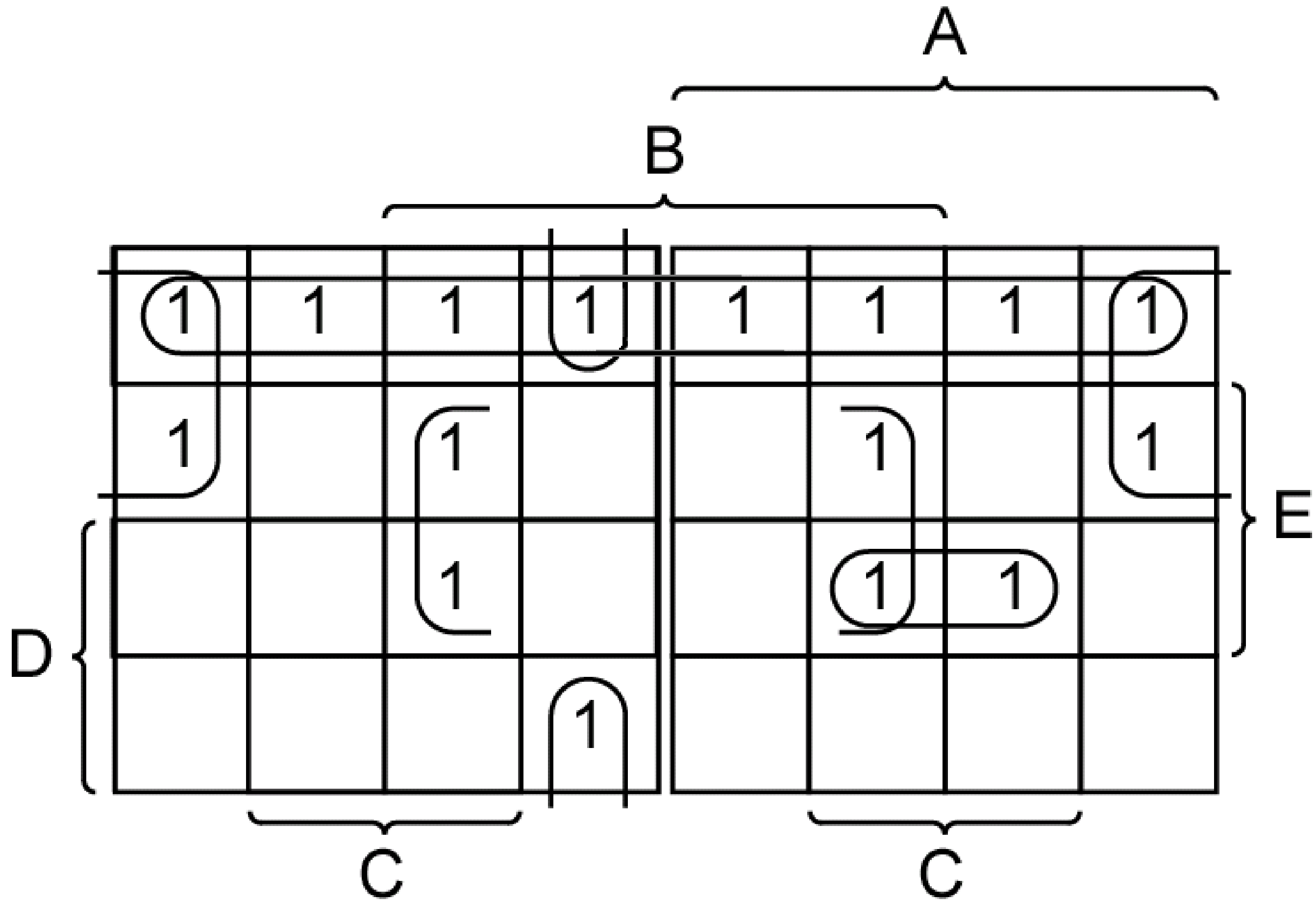
A modification of this uses a *mirror image* map. In this map, first and eighth columns are “adjacent” as are second and seventh columns, third and sixth columns, and fourth and fifth columns.



(a)

F = D'E' + B'C'D' + BCE + A'BC'E' + ACDE

Figure 5-28: **Side-by-side Form of Five-Variable Karnaugh Maps**



(b)

F = D'E' + B'C'D' + BCE + A'BC'E' + ACDE

Figure 5-28: **Mirror Image Form of Five-Variable Karnaugh Maps**