**5.5** A switching circuit has two control inputs ($C_1$ and $C_2$), two data inputs ($X_1$ and $X_2$), and one output ($Z$). The circuit performs one of the logic operations AND, OR, EQU (equivalence), or XOR (exclusive OR) on the two data inputs. The function performed depends on the control inputs:

| $C_1$ | $C_2$ | Function Performed by Circuit |
|---|---|---|
| 0 | 0 | OR |
| 0 | 1 | XOR |
| 1 | 0 | AND |
| 1 | 1 | EQU |

(a) Derive a truth table for $Z$.
(b) Use a Karnaugh map to find a minimum AND-OR gate circuit to realize $Z$.

**5.6** Find the minimum sum-of-products expression for each function. Underline the essential prime implicants in your answer and tell which minterm makes each one essential.
(a) $f(a, b, c, d) = \Sigma\, m(0, 1, 3, 5, 6, 7, 11, 12, 14)$
(b) $f(a, b, c, d) = \Pi\, M(1, 9, 11, 12, 14)$
(c) $f(a, b, c, d) = \Pi\, M(5, 7, 13, 14, 15) \cdot \Pi\, D(1, 2, 3, 9)$

**5.9** Find the minimum sum of products and the minimum product of sums for each function:
(a) $F(A, B, C, D, E) = \Sigma\, m(0, 1, 2, 6, 7, 9, 10, 15, 16, 18, 20, 21, 27, 30) + \Sigma\, d(3, 4, 11, 12, 19)$
(b) $F(A, B, C, D, E) = \Pi\, M(0, 3, 6, 9, 11, 19, 20, 24, 25, 26, 27, 28, 29, 30) \cdot \Pi\, D(1, 2, 12, 13)$

**5.10** $F(a, b, c, d, e) = \Sigma\, m(0, 3, 4, 5, 6, 7, 8, 12, 13, 14, 16, 21, 23, 24, 29, 31)$
(a) Find the essential prime implicants using a Karnaugh map, and indicate why each one of the chosen prime implicants is essential (there are four essential prime implicants).
(b) Find all of the prime implicants by using the Karnaugh map. (There are nine in all.)

**5.11** Find a minimum product-of-sums solution for $f$. Underline the essential prime implicants.

$$f(a, b, c, d, e) = \Sigma\, m(2, 4, 5, 6, 7, 8, 10, 12, 14, 16, 19, 27, 28, 29, 31) + \Sigma\, d(1, 30)$$

**5.17** (a) Plot the following function on a Karnaugh map. (Do not expand to minterm form before plotting.)

$$F(A,B,C,D) = A'B' + CD' + ABC + A'B'CD' + ABCD'$$

(b) Find the minimum sum of products.
(c) Find the minimum product of sums.

**5.18** Work Problem 5.17 for the following:

$$f(A,B,C,D) = A'B' + A'B'C' + A'BD' + AC'D + A'BD + AB'CD'$$

**5.19** A switching circuit has two control inputs ($C_1$ and $C_2$), two data inputs ($X_1$ and $X_2$), and one output ($Z$). The circuit performs logic operations on the two data inputs, as shown in this table:

| $C_1$ | $C_2$ | Function Performed by Circuit |
|---|---|---|
| 0 | 0 | $X_1X_2$ |
| 0 | 1 | $X_1 \oplus X_2$ |
| 1 | 0 | $X_1' + X_2$ |
| 1 | 1 | $X_1 \equiv X_2$ |

(a) Derive a truth table for $Z$.
(b) Use a Karnaugh map to find a minimum OR-AND gate circuit to realize $Z$.

**5.20** Use Karnaugh maps to find all possible minimum sum-of-products expressions for each function.
(a) $F(a, b, c) = \Pi\, M(3, 4)$
(b) $g(d, e, f) = \Sigma\, m(1, 4, 6) + \Sigma\, d(0, 2, 7)$
(c) $F(p, q, r) = (p + q' + r)(p' + q + r')$
(d) $F(s, t, u) = \Sigma\, m(1, 2, 3) + \Sigma\, d(0, 5, 7)$
(e) $f(a, b, c) = \Pi\, M(2, 3, 4)$
(f) $G(D, E, F) = \Sigma\, m(1, 6) + \Sigma\, d(0, 3, 5)$

**5.35** The decimal digits 0 though 9 are represented using five bits $A$, $B$, $C$, $D$, and $E$. The bits $A$, $B$, $C$, and $D$ are the BCD representation of the decimal digit, and bit $E$ is a parity bit that makes the five bits have odd parity. The function $F(A, B, C, D, E)$ has value 1 if the decimal digit represented by $A$, $B$, $C$, $D$, and $E$ is divisible by either 3 or 4. (Zero is divisible by 3 and 4.)

(a) Draw a Karnaugh map for $f$.
(b) Find all prime implicants of $f$. (Prime implicants containing only don't-cares need not be included.)
(c) Find all minimum sum of products for $f$.
(d) Find all prime implicants of $f'$.
(e) Find all minimum product of sums for $f$.

**5.36** Rework Problem 5.35 assuming the decimal digits are represented in excess-3 rather than BCD.

**5.37** The function $F(A, B, C, D, E) = \Sigma\, m(1, 7, 8, 13, 16, 19) + \Sigma\, d(0, 3, 5, 6, 9, 10, 12, 15, 17, 18, 20, 23, 24, 27, 29, 30)$.

(a) Draw a Karnaugh map for $f$.
(b) Find all prime implicants of $f$. (Prime implicants containing only don't-cares need not be included.)
(c) Find all minimum sum of products for $f$.
(d) Find all prime implicants of $f'$.
(e) Find all minimum product of sums for $f$.

**5.38** $F(a, b, c, d, e) = \Sigma\, m(0, 1, 4, 5, 9, 10, 11, 12, 14, 18, 20, 21, 22, 25, 26, 28)$

(a) Find the essential prime implicants using a Karnaugh map, and indicate why each one of the chosen prime implicants is essential (there are four essential prime implicants).
(b) Find all of the prime implicants by using the Karnaugh map (there are 13 in all).