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1.21 (a) 4 + 3 is 10 in base 7, i.e., the sum digit is 0 with a carry of 1 to the next column. 1 + 5 + 4 is 10 in base 7. 1 + 6 + 0 is 10 in base 7. This overflows since the correct sum is 1000_7.

(b) 4 + 3 + 3 + 3 = 13 in base 10 and 23 in base 5. Try base 10. 1 + 2 + 4 + 1 + 3 = 11 in base 10 so base 10 does not produce a sum digit of 2. Try base 5. 2 + 2 + 4 + 1 + 3 = 22 in base 5 so base 5 works.

(c) 4 + 3 + 3 + 3 = 31 in base 4, 21 in base 6, and 11 in base 12. Try base 12. 1 + 2 + 4 + 1 + 3 = B in base 12 so base 12 does not work. Try base 4. 3 + 2 + 4 + 1 + 3 = 31 in base 4 so base 4 does not work. Try base 6. 2 + 2 + 4 + 1 + 3 = 20 so base 6 is correct.

1.24 (a) Expand the base b number into a power series

$N = d_{3k-1}b^{3k-1} + d_{3k-2}b^{3k-2} + d_{3k-3}b^{3k-3} + \ldots + d_5b^5 + d_4b^4 + d_3b^3 + d_2b^2 + d_1b^1 + d_0b^0 + d_{-1}b^{-1} + d_{-2}b^{-2} + d_{-3}b^{-3} + \ldots + d_{-3m+2}b^{-3m+2} + d_{-3m+1}b^{-3m+1} + d_{-3m}b^{-3m}$ where each d_i has a value from 0 to (b-1). (Note that 0's can be appended to the number so that it has a multiple of 3 digits to the left and right of the radix point.) Factor b^3 from each group of 3 consecutive digits of the number to obtain

$N = (d_{3k-1}b^2 + d_{3k-2}b^1 + d_{3k-3}b^0)(b^3)^{(k-1)} + \ldots + (d_5b^2 + d_4b^1 + d_3b^0)(b^3)^1 + (d_2b^2 + d_1b^1 + d_0b^0)(b^3)^0 + (d_{-1}b^2 + d_{-2}b^1 + d_{-3}b^0)(b^3)^{-1} + \ldots + (d_{-3m+2}b^2 + d_{-3m+1}b^1 + d_{-3m}b^0)(b^3)^{-m}$

Each $(d_{3i-1}b^2 + d_{3i-2}b^1 + d_{3i-3}b^0)$ has a value from 0 to $[(b-1)b^2 + (b-1)b^1 + (b-1)b^0]$

$= (b-1)(b^2 + b^1 + b^0) = (b^3-1)$

so it is a valid digit in a base b^3 number. Consequently, the last expression is the power series expansion for a base b^3 number.

1.22 If the binary number has n bits (to the right of the radix point), then its precision is $(1/2^{n+1})$. So to have the same precision, n must satisfy

$(1/2^{n+1}) < (1/2)(1/10^4)$ or $n > 4/(\log 2) = 13.28$ so n must be 14.

1.23

.363636....

$= (36/10^2)(1 + 1/10^2 + 1/10^4 + 1/10^6 + \ldots)$

$= (36/10^2)[1/(1 - 1/10^2)] = (36/10^2)[10^2/99]$

$= 36/99 = 4/11$

8(4/11) = 2 + 10/11
8(10/11) = 7 + 3/11
8(3/11) = 2 + 2/11
8(2/11) = 1 + 5/11
8(5/11) =3 + 7/11
8(7/11) = 5+ 1/11
8(1/11) = 0 + 8/11
8(8/11) = 5 + 9/11
8(9/11) = 6 + 6/11
8(6/11) = 4 + 4/11
8(4/11) = 2 + 10/11

Repeats: .27213505642…….

1.24 (b) Expand the base b^3 number into a power series

$N = d_k(b^3)^k + d_{k-1}(b^3)^{k-1} + \ldots + d_1(b^3)^1 + d_0(b^3)^0 + d_{-1}(b^3)^{-1} + \ldots + d_{-m}(b^3)^{-m}$

where each d_i has a value from 0 to $(b^3 -1)$. Consequently, d_i can be represented as a base b number in the form

$(e_{3i-1}b^2 + e_{3i-2}b^1 + e_{3i-3}b^0)$

Where each e_j has a value from 0 to (b-1). Substituting these expressions for the d_i produces a power series expansion for a base b number.

1.33 (a)

In 2's complement	In 1's complement
(–10) + (–11)	(–10) + (–11)
110110	110101
110101	110100
(1)101011 (–21)	(1)101001
	└→1
	101010 (–21)

1.33 (b)

In 2's complement	In 1's complement
(–10) + (–6)	(–10) + (–6)
110110	110101
111010	111001
(1)110000 (–16)	(1)101110
	└→1
	101111 (–16)

1.33 (c)

In 2's complement	In 1's complement
(–8) + (–11)	(–8) + (–11)
111000	110111
110101	110100
(1)101101 (–19)	(1)101011
	└→1
	101100 (–19)

1.33(d)

In 2's complement	In 1's complement
11 + 9	11 + 9
001011	001011
001001	001001
010100 (20)	010100 (20)

1.33 (e)

In 2's complement	In 1's complement
(–11) + (–4)	(–11) + (–4)
110101	110100
111100	111011
(1)110001 (–15)	(1)101111
	└→1
	110000 (–15)

1.34 (a) 01001-11010

In 2's complement	In 1's complement
01001	01001
+ 00110	+ 00101
01111	01110

1.34 (b)

In 2's complement	In 1's complement
11010	11010
+ 00111	+ 00110
(1)00001	(1)00000
	└→1
	00001

1.34 (c)

In 2's complement	In 1's complement
10110	10110
+ 10011	+ 10010
(1)01001	(1)01000
overflow	└→1
	01001
	overflow

1.34 (d)

In 2's complement	In 1's complement
11011	11011
+11001	+11000
(1)10100	(1)10011
	└→1
	10100

1.34 (e)

In 2's complement	In 1's complement
11100	11100
+ 01011	+ 01010
(1)00111	(1)00110
	└→1
	00111

1.35 (a)

In 2's complement	In 1's complement
11010	11010
+ 01100	+ 01011
(1)00110	(1)00101
	└→1
	00110

1.35 (b)

In 2's complement	In 1's complement
01011	01011
+ 01000	+ 00111
10011	10010

1.35 (c)

In 2's complement	In 1's complement
10001	10001
+ 10110	+ 10101
(1)00111	(1)00110
overflow	└→1
	00111
	overflow

1.35 (d)

In 2's complement	In 1's complement
10101	10101
+ 00110	+ 00101
11011	11010