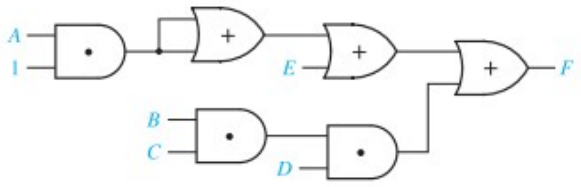
2.1 Prove the following theorems algebraically:
(a) $X(X' + Y) = XY$ (b) $X + XY = X$
(c) $XY + XY' = X$ (d) $(A + B)(A + B') = A$

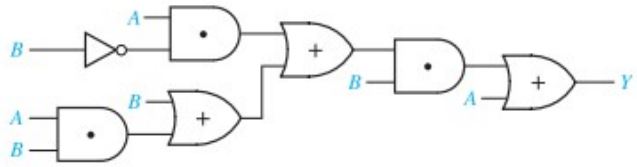
2.2 Illustrate the following theorems using circuits of switches:
(a) $X + XY = X$ (b) $X + YZ = (X + Y)(X + Z)$
In each case, explain why the circuits are equivalent.

2.3 Simplify each of the following expressions by applying *one* of the theorems. State the theorem used (see page 55).
(a) $X'Y'Z + (X'Y'Z)'$ (b) $(AB' + CD)(B'E + CD)$
(c) $ACF + AC'F$ (d) $A(C + D'B) + A'$
(e) $(A'B + C + D)(A'B + D)$ (f) $(A + BC) + (DE + F)(A + BC)'$

2.4 For each of the following circuits, find the output and design a simpler circuit having the same output. (*Hint:* Find the circuit output by first finding the output of each gate, going from left to right, and simplifying as you go.)



(a)



(b)

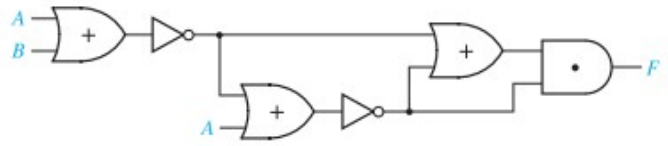
2.5 Multiply out and simplify to obtain a sum of products:
(a) $(A + B)(C + B)(D' + B)(ACD' + E)$
(b) $(A' + B + C')(A' + C' + D)(B' + D')$

2.6 Factor each of the following expressions to obtain a product of sums:
(a) $AB + C'D'$ (b) $WX + WY'X + ZYX$
(c) $A'BC + EF + DEF'$ (d) $XYZ + W'Z + XQ'Z$
(e) $ACD' + C'D' + A'C$ (f) $A + BC + DE$
(The answer to (f) should be the product of four terms, each a sum of three variables.)

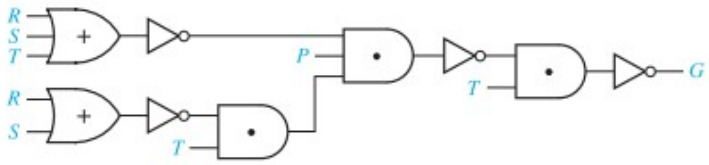
2.7 Draw a circuit that uses only one AND gate and one OR gate to realize each of the following functions:
(a) $(A + B + C + D)(A + B + C + E)(A + B + C + F)$
(b) $WXYZ + VXYZ + UXYZ$

2.8 Simplify the following expressions to a minimum sum of products.
(a) $[(AB)' + C'D]'$ (b) $[A + B(C' + D)]'$ (c) $((A + B')C)'(A + B)(C + A)'$

2.9 Find F and G and simplify:



(a)



(b)