An Entropy-based Clustering Scheme in Mobile Ad hoc Networks

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The distributiveness of mobile ad hoc networks makes resource allocation strategies very challenging since there is no central node to monitor and coordinate the activities of all the nodes in the network. Since a single node cannot be delegated to act as a centralized authority because of limitations in the transmission range, several delegated nodes may coordinate the activities in certain zones. This methodology is generally referred to as clustering and the nodes are called clusterheads. The clusterheads employ centralized algorithms in its cluster; however, the clusterheads themselves are distributive in nature.

In this paper, we propose a clustering scheme i.e., identify a subset of nodes among all the nodes that are best suited to be clusterheads. Though there are several clustering algorithms previously proposed; however, to the best of our knowledge, there is none that characterizes the different node parameters in terms of an information theoretic metric. We use entropy as a measure of local and mutual information available to every node. We considered three parameters in the selection procedure, namely, mobility, energy, and degree. Extensive simulations have been conducted and the performance of the proposed clustering scheme has been compared with the Highest Degree and Lowest ID heuristics in terms of the average number of clusters, the average number of cluster changes, and the average connectivity. The results demonstrate that the mutual information captured through entropy is very effective in determining the most suitable clusterheads.

PACS numbers: Valid PACS appear here

I. INTRODUCTION

Deployment of infra-structured networks are time consuming and therefore cannot be set up at times of utmost emergency. Therefore, mobile multi-hop radio networks, also called ad hoc or peer-to-peer networks, play a critical role in places where a wired (central) backbone is neither available nor economical to build, such as law enforcement operations, battle field communications, disaster recovery situations, and so on. Such situations demand a network where all the nodes including the base stations are potentially mobile, and communication must be supported untethered between any two nodes. However, maintaining such seamless connection is difficult because of the inherent characteristic of mobile ad hoc networks i.e., highly dynamic topology changes due to the mobility of the nodes. Also, the bandwidth is limited and the signal quality is unpredictable.

In spite of these constraints, ad hoc networks are designed such that they are able to dynamically adapt themselves with the changing network configurations. One of the ways to handle the topology changes and maintain a connected network can be brought about by entrusting certain nodes with more responsibility. These nodes are typically called clusterheads and are responsible for the formation of clusters, each consisting of a number of ordinary nodes. A clusterhead is responsible for resource allocation to all the nodes belonging to its cluster. Due to the dynamic nature of the mobile nodes, their association and dissociation to and from clusters perturb the stability of the network and thus reconfiguration of clusterheads is unavoidable. This is an important issue since frequent clusterhead changes adversely affect the performance of algorithms such as scheduling, routing, and end-to-end delay. Choosing clusterheads optimally is an NP-hard problem. Thus, existing solutions to this problem are based on heuristic (mostly greedy) approaches and none attempts to retain the stability of the network topology [12]. We believe a good clustering scheme should preserve its structure as much as possible when nodes are moving and/or the topology is changing. Otherwise, re-computation of clusterheads and frequent information exchange among the participating nodes will result in high computation overhead.

In this paper, we propose a distributed clustering algorithm which takes into consideration the local information available to all the nodes. This local information is measured in terms of entropy. We consider three parameters for the determination of the clusterheads – mobility of the nodes, their energy consumption, and the number of neighbors a node is connected to. More specifically, our contributions are the following.

• First, we demonstrate the motivation behind using entropy as the metric for capturing relative information. We also show how the mutual information can be calculated when two marginal distributions and the joint distribution are given.

• We calculate the entropy for three node parameters – its mobility, energy, and degree. These three en-
tropies are combined through a simple linear model. The proposed method of calculating the mutual information is generic enough and can easily be extended to include other node and network parameters.

- Through simulation experiments, we demonstrate the performance of our proposed scheme in terms of the average number of clusters, the average number of cluster changes, and the average connectivity.

- We also compare the performance of our schemes with the Lowest ID and the Highest Degree heuristics.

The rest of the paper is organized as follows. In Section II, we present a literature survey of the previous work. In section III, we first discuss why a relative measure is required and how entropy can be used to capture the mutual information. We then discuss in detail our proposed entropy-based clustering scheme considering mobility, energy, and degree of the nodes. The simulation model and results are presented in Section IV. Conclusions are drawn in the last section.

II. RELATED WORK

Several clustering algorithms and heuristics have been proposed for ad hoc networks [1, 10, 16, 17]. Many existing solutions take into account various parameters of clusterhead suitability. However, the most recognized ones are based on clusterhead selection which rely on random events such as node ID assignment (as in the Lowest ID algorithm) and the degree of connectivity (as in the Highest Degree algorithm). The Lowest ID [4, 5] heuristic assigns a unique ID to each node and chooses the node with the minimum ID as a clusterhead. Thus, the IDs of the neighbors of the clusterhead will be higher than that of the clusterhead. In Highest Degree [17, 22], each node broadcasts its ID to the nodes that are within its transmission range. A node $x$ is considered to be a neighbor of another node $y$ if $x$ lies within the transmission range of $y$. The node with maximum number of neighbors (i.e., maximum degree) is chosen as a clusterhead. If there is a tie, it can be broken arbitrarily by the nodes’ IDs. There are other clustering schemes that consider node and network parameters for deciding the nodes best suited to act as clusterheads. In the node weight heuristic [6], the nodes are assigned weights based on clusterhead suitability; neighbor with highest weight wins. This scheme has infrequent node updates but moderate computational overhead; however, it is not optimized for system throughput and power control. Uniform leader election [19] is an easy to implement scheme where a rotated binary tree is used. The non-uniform leader election [20] and the oblivious leader election [21] algorithms are similar in nature; however, based only on a ternary tree and transmit slots respectively. Once again, node suitability is not taken into consideration in neither of the three schemes. The least cluster change (LCC) [11] scheme is based on Lowest ID or highest connectivity. Re-election is only initiated when clusterhead moves into another cluster or when node becomes separated from a cluster. This scheme reduces cluster re-association and increases stability, but is potentially unfair in terms of load distribution. The mobility-based adaptive clustering (MBAC) [16] scheme is an event driven algorithm based on hybrid routing and nodal mobility. Two parameters control path availability and effective capacity of path as well as cluster size. It is capable of multipath transmission to increase capacity; however it has a high computational complexity. In access-based clustering protocol [18], a node receiving a clusterhead declaration from its neighbor prior to declaring itself as a clusterhead becomes a member node. Access to control channel is based on TDMA with short execution time and incurs low control message overhead. However, clusterhead suitability is not considered. In linked cluster algorithm (LCA) [5], the entire band is divided into $M$ sub-bands (epochs) and the algorithm is performed on each sub-band. The nodes are assumed to have precise synchronized clocks and the number of nodes are known priori. The max-min D-clustering [2] scheme uses two consecutive broadcasts that are sent in $N$ timeslots to each one-hop neighbor. The scheme is fault tolerant due to availability of multiple paths from gateway nodes; produces fewer clusterheads and is more stable than LCA. Comparative performance evaluation of various clustering protocols that help backbone formation in ad hoc networks is achieved in [7].

There are several clustering schemes that take into account the dynamic topology of the network. The topology adaptive spatial clustering (TASC) algorithm [25] is distributed in nature that partitions the network into a set of locally isotropic, non-overlapping clusters without prior knowledge of the number of clusters, cluster size, and node coordinates. This is achieved by deriving a set of weights that encode distance, connectivity, and density information within the locality of each node. The mobility-based $d$-hop clustering algorithm (MobDHop) that forms variable-diameter clusters based on node mobility was proposed in [14]. A new metric to measure the variation of distance between nodes over time to estimate the relative mobility of two nodes was introduced.

The hybrid energy-efficient distributed (HEED) clustering proposed in [28] periodically selects clusterheads according to a model that combines the the residual energy and a secondary parameter, such as node proximity to its neighbors or node degree. In [26, 27], the authors propose a novel energy efficient clustering scheme (EECS) in single-hop wireless sensor networks, particularly for periodic data gathering applications. This approach elects clusterheads with more residual energy in an autonomous manner through a local radio communication with no iteration while achieving clusterhead distribution. It also introduces a novel distance-based
method to balance the load among the clusterheads. The EECS protocol is further improved to address the problem of “hot spots” in [15]. The proposed scheme partitions the nodes into clusters of unequal size, and cluster closer to the base station have smaller sizes than those farther away. Thus, clusterheads closer to the base station can preserve energy for the inter-cluster data forwarding. Weighted clustering algorithm (WCA) [10] uses four parameters: battery life, mobility, total distances from neighbors, and degree of a node. These parameters are combined through a simple linear model; however, the computational overhead of the algorithm can be considered high since a single node can cause the re-invocation of the algorithm which effects the overall network rather than the nodes local to the area of incident. The performance of WCA has been further optimized using techniques such as genetic algorithm [23] and simulated annealing [24].

III. PROPOSED ALGORITHM

Different algorithms emphasize characteristics which may or may not be important based upon the architectural features of the individual network application. For example, a network that handles multimedia traffic, topological instability causes changes in the data transfer path threatening the timely transmission of streaming media. Thus, heavy-duty clusterhead election techniques may be favored if greater network stability is achieved. In mobile ad hoc networks consisting of heterogeneous mobility devices, more powerful devices may overburden smaller less capable ones, placing a demand in which cannot be met. These devices also may tend to transmit fewer data packets with less frequency, placing very little or no burden on the network. Clearly, an under-powered device would not be a good candidate as a clusterhead, regardless of its node ID; therefore, we argue that the Lowest ID election in general is not a suitable algorithm for heterogeneous networks. In highly mobile and dynamic networks, clusterhead elections are unavoidable, and the aim must be to minimize the impact of the election process.

Algorithms which guarantee leader election with a certain amount of deviation from the optimal solution can be considered suitable. That would also reduce the periodic instabilities brought on by the high rate of clusterhead changes, while minimizing the impact of the routing overhead associated with high levels of nodal re-affiliation.

Most algorithms work based on a pre-defined metric. The clustering decisions are based on the absolute values obtained by these metrics. Though it might seem to work, but at times the performance is misinterpreted. For example, consider an ad hoc network in operation. When the nodes are initialized, the performance of the network is expected to be at its best since the energy is at its maximum. However, with the lapse of time and energy depletion, there would be a performance degradation. So, we must consider, the parameters at that point of time. More importantly, compare the nodes’ suitability relative to each other.

We propose to take advantage of the mutual information; therefore, we use entropy-based measures. Entropy has been widely used to capture the information content within a system. A measure of statistical dependence or correlation is usually sought between two or more parameters, i.e., the random variables in a time series.

If X and Y are the random variables with joint distribution \( p(X,Y) \) and marginal distributions \( p(x) \) and \( p(y) \), then the mutual information \( I(X;Y) \) is the relative entropy between the joint distribution and the product distribution. Hence, \( I(X;Y) \) is given by

\[
I(X;Y) = \sum_{x} \sum_{y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}
\]

By analyzing the relative entropy of the nodes, we can derive values which help determine nodal suitability. The proposed algorithm consists of the weighted linear sum of three entropy measurements: mobility, battery expectancy, and degree. We use these parameters to demonstrate how the relative entropy can be calculated and the results combined to find the most suitable nodes to act as clusterheads. Though we use these three measurements, our algorithm is generic enough and can be extended to account for any other physical parameters.

A. Mobility entropy

Determination of mobility entropy is based upon mutual information, which is an appropriate measure of change based upon previous expectation values. Each node collects a history of the broadcast (beacon) signals received from its neighbors during a period of time. Every node maintains a list containing the IDs of each node heard within the hearing range of the node. A node whose mobility is stable relative to its virtual cluster would see fewer changes in its neighborhood list. Since the motion is relative, it is impossible to ascertain whether the node itself or the neighbors node moved. Comparing the change in the neighbor list and more importantly the rate of change of the neighbor list, it is possible to infer the relative mobility of the nodes with respect to each other and to the clusterhead. We make a few probabilistic statements.

1. Several and frequent changes in neighbor list are more likely due to the mobility of the node in question rather than a large number of neighbors moving at once.

2. A few changes in neighbors are more likely due to the neighbor’s movement either away from or towards the node rather than the node’s own movement.
3. Nodes lie close to periphery of the transmission are likely to have a ping-pong effect, i.e., in and out of the neighborhood list.

Observing the mobility of a node with respect to another, the probability that a node is moving, and the marginal probability that the node itself is moving can be calculated by the mutual information.

Let us observe the behavior of the neighbor list of a particular node \( i \) for a time interval of \( \Delta t \). Let us assume that node \( j \) appeared in the list at least once. We measure two quantities. First, the number of times node \( j \) appeared. It can be noted that for a node to appear multiple times, it must also disappear that many times. Second, the total amount of time node \( j \) stayed in the neighbor list of node \( i \) during the interval \( \Delta t \). The first quantity gives a measure of relative mobility and the second provides an intuition about the relative stationarity of nodes \( i \) and \( j \). If node \( i \) counts the number of appearances of other nodes \( j \), then it can compute the joint distribution for all the other nodes, i.e., \( p(i,j) \) for all \( j \). Also, \( p(i) \) is known to node \( i \), and it can gather information about \( p(j) \) from its neighboring nodes, or the nodes that visited \( i \). Thus, we obtain the mutual information as was given by equation (1).

### B. Energy entropy

Since clusterheads have the extra responsibility to forward packets on behalf of other nodes, they are prone to battery drainage. Therefore, a node with good residual battery power is a better candidate for being a clusterhead. Though, the remaining battery is easy to measure, the rate at which it will deplete is still uncertain. This uncertainty arises due to the fact that the energy spent by a forwarding node is proportional to the transmission power, i.e., the power at which a node transmits a packet so that the packet reaches the intended receiver. It is known that more power is required to communicate to a larger distance. Thus, transmit power depends on the relative distance between the transmitter and the receiver nodes. Note, that the maximum range \( (R_{\text{max}}) \) attainable by a node is limited by the maximum allowable transmit power, \( P^{\text{max}} \).

Let us now calculate the uncertainty in the relative distance between a transmitter and a receiver. Since the nodes are randomly scattered, the receiver lies anywhere in the circle with radius \( R_{\text{max}} \) with equal probability, with the transmitter node being at the center of the circle. If we use polar co-ordinates, the radial distance is assumed to be uniformly distributed between 0 and \( R_{\text{max}} \), and the angle uniformly distributed direction between 0 and \( 2\pi \).

The position of the receiver is characterized by \( f_R(r) \) and \( f_\Theta(\theta) \), denoting respectively the distance probability density function (pdf) and the directional pdf. The two pdfs are defined as follows:

\[
f_R(r) = \begin{cases} \frac{2r}{R_{\text{max}}^2}, & 0 \leq r \leq R_{\text{max}} \\ 0, & \text{elsewhere.} \end{cases}
\]

\[
f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta \leq 2\pi \\ 0, & \text{elsewhere.} \end{cases}
\]

The joint pdf is given by

\[
f_{R\Theta}(r, \theta) = \begin{cases} \frac{r}{\pi R_{\text{max}}^2}, & 0 \leq r \leq R_{\text{max}}, 0 \leq \theta \leq 2\pi \\ 0, & \text{elsewhere.} \end{cases}
\]

Given this pdf of the distance of the receiver from the transmitter, the transmission power distribution, and hence the energy dissipation can be obtained. For the joint pdf of distance as \( f_{R\Theta}(r, \theta) \), we calculate the pdf for the transmission power. We assume that the attenuation in the signal strength is inversely proportional to the square of the distance, i.e., if \( P_t \) and \( P_r \) are the transmit and receiver powers respectively,

\[
P_r = P_t \times d^{-\alpha}
\]

where \( \alpha \) is the path loss exponent and usually lies between 2 and 6. Therefore, the pdf for the transmission power, \( f_P(P_t) \), is given by

\[
f_P(P_t) = \begin{cases} f_{R\Theta}(P_t^{-\alpha}), & 0 \leq P_t \leq R_{\text{max}}, 0 \leq \theta \leq 2\pi \\ 0, & \text{elsewhere.} \end{cases}
\]

Since our assumption that the transmission power is directly proportional to the energy consumed, we use the transmission power pdf to calculate the energy entropy. We use Shannon’s entropy for this purpose. Shannon’s entropy for a random variable with \( Y \) with pdf \( f_Y(y) \) is

\[
H(Y) = \int_{-\infty}^{+\infty} f_Y(y) \log f_Y(y) dy
\]

Thus, the energy entropy is given by

\[
H(f_P) = \int_0^{R_{\text{max}}} \left( f_{R\Theta}(P_t^{-\alpha}) \log f_{R\Theta}(P_t^{-\alpha}) \right) dP_t
\]

### C. Degree entropy

A clusterhead is not only responsible for forwarding packets on behalf of its member nodes but also for coordinating their transmission. In other words, the clusterhead acts as the scheduler in that cluster and allocates resources. The resources could be time slices in TDMA, frequency bands in FDMA, or codes in CDMA systems.
Obviously, it becomes difficult for a clusterhead to manage the resources if there are too many member nodes associated with it. Ideally, each clusterhead can only handle a pre-defined number of nodes to ensure efficient medium access control (MAC) functioning. If the clusterhead tries to serve more nodes than it is capable of, the system efficiency suffers in the sense that the nodes will incur more delays because they have to wait longer for their turn (as in TDMA) to use their share of a resource. A high system throughput can be achieved by limiting or optimizing the degree of each clusterhead.

Due to the random position and movement of the nodes, the degree of a node is uncertain. Theoretically, the minimum degree could be 0, in which case the node is said to be isolated. The other extreme is that a node is connected to all the other nodes in the network. Ideally, the degree of a node is probabilistic; and we can always calculate the probability of a node having a certain degree. If we assume Poisson distribution of the nodes, then the probability that a node will have a degree \( \delta \) is given by [9]

\[
Prob(deg = \delta) = P_3 = \frac{N^{\delta} \cdot e^{-\frac{\pi R_{\text{max}}^2}{A}}}{\delta!}
\]

where \( N \) is the total number of nodes in the network confined within an area \( A \). Recall, \( R_{\text{max}} \) is the maximum range of a node. It is to be noted that equation (9) holds true for both large \( N \) and \( A \).

With the probabilities of nodes having all possible degrees known, we can calculate the entropy due to degree uncertainty as

\[
H_{\text{degree}} = \sum_{i=0}^{N} P_3 \log(P_3)
\]

**D. Total Entropy**

At all times, every node computes its instantaneous mobility entropy \( H_{\text{mobility}} \), energy entropy \( H_{\text{energy}} \), and degree entropy \( H_{\text{degree}} \), and announces these values to the current clusterhead (if it exists). The node with the lowest entropy wins the election; receives the node list for the virtual cluster and notifies each member of its new role. A node whose neighbor list never changes would have a total combined entropy of 0. A node with a significant amount of relative motion and a small residual energy would have significantly higher total entropy. We use a simple linear combination to find the total entropy, \( H_{\text{total}} \). We define the total entropy as

\[
H_{\text{total}} = w_1H_{\text{mobility}} + w_2H_{\text{energy}} + w_3H_{\text{degree}}
\]

where \( w_1, w_2, \) and \( w_3 \) are the weighing factors and \( w_1 + w_2 + w_3 = 1 \). The weighing factors can be adjusted as per the desired priority for the network i.e., how important are mobility, energy, and degree are with respect to each other.

**IV. SIMULATION MODEL AND RESULTS**

To study the performance of our proposed clustering scheme, we conducted extensive simulation experiments where \( N \) nodes were randomly distributed over an area of 100 \( \times \) 100 units. The mobility of nodes followed the random waypoint model [8] with the displacement varying uniformly between 0 to a maximum value per unit time. The other parameters for simulation are shown in Table I.

To measure the performance of our proposed entropy-based clustering scheme, we identify three metrics: (i) the average number of clusters, (ii) the average connectivity, and (iii) the average number of clusterhead changes. It can be noted that the average size of a cluster, i.e., average number of nodes in a cluster is calculated by the total number of nodes in the network divided by the number of clusters. We define connectivity as the number of nodes that are reachable by a node. These three metrics are studied for the varying number of nodes, transmission range, and maximum displacement.

![Table I: Simulation Parameters](image)

We studied the performance of the algorithm in two phases. First, we observed the behavior of the proposed algorithm as a function of the three performance metrics. Second, we compared the performance of our scheme with respect to the Lowest ID [4, 5] and Highest Degree [17, 22] heuristics.

**A. Performance of the proposed algorithm**

Figures 1 and 2 show the average number of clusters as a function of transmission range and maximum displacement respectively. For low transmission ranges, the number of clusters is much higher because the member nodes are likely to wander out of the range of the clusterhead. As the transmission range increases, nodes are more likely to remain within the radius of the clusterhead regardless of the speed of the movement. As the average node velocity increases (i.e., the maximum displacement increases), there is a small increase in the number of clusters due to the nature of the motion to disperse the nodes.
Figures 3 and 4 show the average clusterhead connectivity, or nodal degree during the course of the simula-
tion. As transmission ranges increase, there is almost a linear increase in the nodal degree, owing to the uniform random motion.

Figures 5 and 6 show the rate at which the clusterheads change. It is to be noted that a lower value of clusterhead changes is desirable since it reflects the stability of the topology.

B. Comparison of the proposed algorithm

To evaluate the effectiveness of the proposed entropy-based clustering scheme, we compare the performance to the Highest Degree and Lowest ID heuristics which are briefly discussed below.

• **Highest Degree Heuristic:**
  The Highest Degree, also as known as connectivity-based clustering, was originally proposed in [17, 22] in which the degree of a node is computed based on its distance from others. Each node broadcasts its ID to the nodes that are within its transmission range. A node is considered to be a neighbor of another node if it lies within the transmission range of the other. The node with maximum number of neighbors (i.e., maximum degree) is chosen as a clusterhead.

• **Lowest ID Heuristic:**
  The Lowest ID, also as known as identifier-based clustering, was originally proposed by Baker and Ephremides [4, 5, 13]. This heuristic assigns a unique ID to each node and chooses the node with the minimum ID as a clusterhead. Thus, the IDs of the neighbors of the clusterhead will be higher than that of the clusterhead. However, the clusterhead can delegate its responsibility to the next node with the minimum ID in its cluster.
FIG. 11: Avg. clusterhead changes vs. transmission range

FIG. 12: Avg. clusterhead changes vs. max displacement

Figures 7 and 8 demonstrate that the average number of clusters is lower than the other schemes. This suggests that unnecessary nodes are not selected as clusterheads. The number of clusterhead required to cover the entire area depends on how much a clusterhead is able to cover, i.e., its transmission range. As the transmission range increases, the number of clusterheads (or clusters) required to cover the entire area decreases. However, the average connectivity is compromised because of the small number of clusterheads as can be seen from Figures 9 and 10. The novelty of the proposed entropy-based scheme (particularly the mobility entropy) is manifested in the form of a relatively stable network which is in agreement with the work in [3]. This is demonstrated through Figures 11 and 12 where we see less number of clusterhead changes. However, the tendency of all the schemes is the same.

V. CONCLUSIONS

In this paper, we proposed a clustering scheme for ad hoc networks based on entropy measures. We considered three important aspects of ad hoc networks – mobility, energy consumption, and degree of the nodes. Through the exchange of beacon messages, the nodes gather information about their mutual mobility, energy, and degree. We use a generic linear combination model to consider all three entropies. We conducted simulations that show the performance of the proposed clustering scheme in terms of the average number of clusters, the average number of cluster changes, and the average connectivity. We also compared our results to Lowest ID and Highest Degree clustering heuristics. The results demonstrate that the mutual information captured through entropy is very effective in maintaining the stability of the network.


