Chance-Constrained Real-Time Volt/Var Optimization Using Simulated Annealing

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Abstract—Solar is the fastest growing source of renewable electricity in the U.S. The anticipated PV proliferation brings integration challenges on system volt/var performance at the utility scale. One of the greatest challenges is to maintain desirable feeder voltages in utility distribution networks. The intermittent PV generation causes more frequent operation of volt/var control (VVC) devices to alleviate voltage issues. This paper proposes a real-time volt/var optimization (VVO) strategy to control voltage regulators, switched capacitors, and PV inverters for minimizing the active power loss. Chance constrained programming is used to model solar uncertainty. Simulated annealing technique is applied to solve the developed optimization problem. The proposed VVO strategy is tested in the modified IEEE 37-bus system. Simulation results demonstrate that the coordination of VVC devices and reactive power support of PV inverters can help handle solar variability and uncertainty in real-time volt/var operation.

Index Terms—Chance constrained programming, distribution power flow, PV inverters, simulated annealing, and volt/var optimization.

I. INTRODUCTION

The integration of photovoltaic (PV) generators into distribution network is fast growing. This helps reduce the system loss from transporting electricity over long distance and reduce the carbon emission from fossil fuel units. However, the increasing penetration level of PV units also brings integration challenges to distribution utilities. One of challenges is how to maintain the voltage quality at customer’s terminal and minimize the operation cost of PV-integrated distribution system.

Distribution utilities control devices, including on-load tap changer (OLTC), voltage regulator (VR), and switched capacitor (SC) to manage voltage and reactive power (var) in the distribution system. The so-called volt/var control (VVC) ensures the efficiency, reliability and quality of the power delivered to customers. However, these traditional VVC devices have daily operational constraints, due to wear and tear and high maintenance cost. They are also slow in responding to quick variations from renewables. When a large number of distributed PV generators connected into distribution system, the inherent variability and uncertainty of solar energy may cause voltage rise and fluctuations, and increase power loss. It can also disturb the operations of OLTC, VR, and SC. High PV penetration has becoming a major impact on voltage regulation and VVC [1-2].

In the literature, different approaches have been proposed to address the issues of VVC in the PV-integrated distribution system. The supervisory control and optimization approach were proposed in [3-5] to overcome the shortcomings of traditional VVC devices. Authors in [6] proposed to use reactive power of PV inverters for fast VVC. A method to control PV active power injection was developed in [7]. Inverter var control technique [8] and battery storage system [9] have been proposed to mitigate voltage fluctuation due to PV power fluctuation. The coordinated operation of OLTC and SC was developed in [10] to decrease power loss and improve voltage regulation capability. The stochastic framework of VVC was modeled in [11-13]. However, limited work has considered the coordination of traditional devices and PV inverters to handle both variability and uncertainty of solar energy in real-time VVC.

This paper proposes an optimal real-time volt/var control strategy to control VR, SC, and PV inverters for minimizing the active power loss. Chance constrained programming (CCP) is used to model solar uncertainty. Simulated annealing technique is applied to solve the developed optimization problem. The proposed VVO strategy is tested in the modified IEEE 37-bus system. Simulation results demonstrate that the coordination of VVC devices and reactive power support of PV inverters can help handle solar variability and uncertainty in real-time operation. The proposed strategy is able to increase PV penetration level and reduce system loss. The rest of this paper is organized as follows: the overall model of proposed VVO strategy is introduced in Section II. Section III presents the problem formulation and solution algorithm. Section III describes different test cases and analyzes simulation results. Conclusions and future work are summarized in Section IV.

II. OVERVIEW OF VVO STRATEGY

Smart meters and communication technology have been applied to improve the infrastructure of distribution system.

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Utilizing these new techniques, it is feasible to design a centralized control structure to remotely control smart control agents (SCAs) and perform VVO in real-time. The structure of proposed real-time VVO is shown in Fig. 1. The SCAs, including smart meters, smart inverters, and smart controllers, capture and process the field data, and send required information to the VVO module located at substation/control center. The VVO module minimizes the system active power loss via the optimal scheduling of VVC devices, and sends commands to all control devices using communication protocols in the distribution system [14].

A. Coordination Strategy to Handle PV Variability

The objective of VVO strategy is to minimize active power loss and enable high PV penetration level into distribution system without violating system constraints. The PV power variability can cause voltage variation and increase switching operations of VR and SC, which decreases their lifetime and increases maintenance cost.

The voltage variation issue due to PV power variation can be mitigated by injecting or consuming reactive power from PV inverters to control the voltage level. PV inverters must be coordinated with other VVC devices to avoid undesirable switching operation and obtain optimal voltage profile. Therefore, the VVO module will use an optimization process to determine the coordinated scheduling of VR, SC, and PV inverters for the real-time operation.

B. CCP to Handle PV Uncertainty

PV power generation is a random variable depending on weather conditions, like solar irradiance, temperature, etc. Chance constrained programming is used to model PV uncertainty. An equivalent deterministic model will be achieved according to following steps.

Given deterministic decision variable \( D \) and random variable \( R_v \), the chance constraint with confidence level \( \delta \) can be expressed as:

\[
\text{Prob}(D \leq R_v) \geq \delta
\]  

The deterministic equivalent of the random variable can be determined from the cumulative distribution function (CDF) of \( R_v \), given as:

\[
D \leq F_{R_v}^{-1}(1 - \delta)
\]  

where, \( F_{R_v}^{-1}(.\) is the inverse CDF of \( R_v \).

In the proposed VVO, it is assumed that the 15-minute ahead forecasted PV power is available with the resolution of one second. For every 24-hour operation, there will be 96 sets of forecasted PV power generation, and every set contains 900 sample data. These forecasted data are assumed to follow the normal distribution [15]. The deterministic equivalent of PV power can be obtained using (2).

C. VVO Framework

The proposed VVO is designed for system operators to minimize the active power loss. The framework of VVO strategy is shown in Fig. 2. The optimization module takes the input of load and PV profile to determine the optimal PV active power injection and inverter power factor every 15-minutes, and the optimal position of VR and SC every hour throughout 24 hours of a day.

III. PROBLEM FORMULATION AND SOLUTION METHOD

A. Problem Formulation

1) Objective function

The objective of VVO is to minimize system active power loss, as given in the following:

\[
\text{Min } f(\bar{X}) = (p_{\text{loss},i})
\]  

where,

\[
p_{\text{system},i} = \text{real} \left\{ \sum_{p=(a,b,c)} \sum_{l=1}^{L} \Delta V_{l,i}^p \times \text{conj}(I_{l,i}^p) \right\}
\]  

\( X = [\text{Tap}, \text{Sc}, p_{\text{pv}}^{\text{inj}}, \text{PF}] \) is the decision variable, \( \text{Tap} \) is VR tap position, \( \text{Sc} \) is SC switch position, \( p_{\text{pv}}^{\text{inj}} \) is the active power injection of PV generators, \( \text{PF} \) is the power factor of PV inverters, \( p_{\text{system},i}^{\text{loss}} \) is the active power loss of the test system, \( \Delta V_{l,i}^p \) is the voltage drop, and \( I_{l,i}^p \) is the current flowing in line section \( l \) during period \( t \). There are total \( L \) line sections, and \( p \) denotes any phase \((a, b, c)\) of three phase system.

2) Constraints

a) PV active power injection limit

The chance constraint of PV active power injection with confidence level of \( \delta_{pv} \) can be expressed as:

\[
\text{Prob}(p_{\text{pv},i}^{\text{inj}} \leq p_{\text{pv},i}^{\text{me}}) \geq \delta_{pv}
\]  

where, \( p_{\text{pv},i}^{\text{inj}} \) is PV active power injection into distribution system, \( p_{\text{pv},i}^{\text{me}} \) is the random variable of available PV generation at bus \( i \) and time \( t \). The equivalent deterministic
constraint of injected active power is given by equation (6). It is assumed that $\delta_{pv} = 10\%$ in this paper.

$$ P_{\text{pv},i}^{\text{inj}} \leq F_{\text{gen}}^{-1} \left( 1 - \delta_{pv} \right) $$  \hspace{1cm} (6)

b) Voltage regulation constraint

The per unit limit of bus voltage magnitude is set according to ANSI C84.1 standard for medium voltage level.

$$ 0.975 \leq |V_{i,j}| \leq 1.05 $$ \hspace{1cm} (7)

c) VR operation limit

Total number of VR tap change must not exceed the maximum daily limit $T_{\text{C daily}}^\text{max}$, as given by:

$$ T_{\text{C daily}} = \sum_{k=1}^{96} \left| T_{\text{tap}_k} - T_{\text{tap}_{k-1}} \right| \leq T_{\text{C daily}}^\text{max} $$ \hspace{1cm} (8)

It is assumed that there are 17 possible taps ($T_{\text{tap}_k} = -8, -1, 0, 1, \ldots, 8$), and $T_{\text{C daily}}^\text{max} = 30$ in this paper.

d) SC operation limit

Total number of SC change must not exceed the maximum daily limit $S_{\text{C daily}}^\text{max}$, as given by:

$$ S_{\text{C daily}} = \sum_{k=1}^{96} \left| S_k - S_{k-1} \right| \leq S_{\text{C daily}}^\text{max} $$ \hspace{1cm} (9)

It is assumed that there are 4 possible switching steps ($S_k = 0, 1, 2, 3$), and $S_{\text{C daily}}^\text{max} = 6$ in this paper. The reactive power injection at time $t$ is given by $Q^e_t = S_t \times \Delta Q^e$, where $\Delta Q^e$ is reactive power in each switch step.

e) PV inverter power factor

The power factor of PV inverters $PF_{\text{pv},i}$ can change between 0.85 lagging and 0.85 leading.

$$ -0.85(\text{lagging}) \leq PF_{\text{pv},i} \leq 0.85(\text{leading}) $$ \hspace{1cm} (10)

f) Line thermal limit

Each line flow $P_{l,i}$ should be within its thermal limit $P_{l,i}^{\text{max}}$, as given by:

$$ |P_{l,i}| \leq P_{l,i}^{\text{max}} $$ \hspace{1cm} (11)

g) Distribution power flow equation

Three-phase unbalanced distribution power flow is used in this paper to model distributed integration of PV generators. Following the technique developed in [16], branch current and bus voltage are given in the following:

$$ T_{B,i,j} = \text{BIBC} \times T_i $$ \hspace{1cm} (12)

$$ V_{i} = \tilde{V}_i^s - \text{BCBV} \times T_{B,i} $$ \hspace{1cm} (13)

where, $I_{B,i,j}$ is branch current in line section $l$, $I_i$, and $V_i$ are bus current and voltage at bus $i$, $V_i^s$ is the secondary voltage of VR, which is considered as slack bus for power flow analysis. BIBC is the matrix of bus injected to branch current, and BCBV is the matrix of branch current to bus voltage. The details of BIBC and BCBV can be referred in [16].

B. Solution Algorithm

The formulated VVO is a non-linear, discrete, combinatorial optimization problem, due to the non-linear objective function and discrete control variables. In this paper, a heuristic optimization algorithm, simulated annealing (SA), is applied to solve the problem efficiently.

Simulated annealing process starts with a valid current solution (vector of control variables), and randomly generate potential solutions during the process. The objective function value of every potential solution is evaluated. If the current objective value is less than previous one, which satisfies equation (14), the current solution is updated as the best solution. Otherwise, the worse solution is accepted based on the swap probability given by equation (15). This process is able to avoid converging at local optima. As the process continues, system temperature decreases according to equation (16), and the probability of accepting worse solution also decreases, until the final best solution obtained.

$$ \Delta P_{\text{loss}} = (P_{\text{loss2}} - P_{\text{loss1}}) < 0 $$ \hspace{1cm} (14)

$$ \text{Prob.}(\Delta P_{\text{loss}}) = e^{-\frac{\Delta P_{\text{loss}}}{T}} \geq \text{rand}(0,1) $$ \hspace{1cm} (15)

$$ T_k = T_0 \left( T_f \right)^{\frac{k-1}{\text{max}-1}} $$ \hspace{1cm} (16)

where, $\Delta P_{\text{loss}}$ is the change of objective function value for two consecutive potential solutions, $T_k$ is system temperature at $k^{th}$ iteration for given initial temperature $T_0$ and final temperature $T_f$.

In this paper, control variables are VR, SC, PV inverter active and reactive power injection. Following steps are implemented to solve the proposed VVO problem. The time step is set as 15 minutes, and total time is 24 hours.

- Step 1: Compute base case power flow (with load set to 1 p.u. and PV power set to zero) to find objective value and save it as the best objective value.
- Step 2: Generate a random control vector and solve power flow for particular load and PV power profile to find current objective value.
- Step 3: If current objective value is less than the best objective value, update the current control vector and objective value as the best solution and objective value, respectively. Otherwise, test the swap probability in equation (15) to determine whether include this worse solution.
- Step 4: If stopping criteria is met, end process. Otherwise, decrease the system temperature in equation (16), and go to Step 2.

IV. SIMULATION RESULTS AND ANALYSIS

A. Test System

The proposed VVO strategy is tested in the modified IEEE 37-bus feeder test system, as shown in Fig. 3. The network topology and component data can be referred in [17-
The substation includes a 2,500 kVA, 230/4.8 kV VR. Three-phase delta-connected PV sources are connected at bus 11 and 29, with capacity of 0.5 and 1 MW respectively. It is assumed that PV inverters can inject or consume reactive power by changing power factor between 0.85 (lagging) and 0.85 (leading). Two three-phase delta-connected SCs are connected at bus 7 and 22, with same capacity of 150 kvar. PV and load profile are shown in Fig. 4. PV active power profile (the base of 1.5 MW) is obtained from SDSU Microgrid Lab, and load apparent power profile is referred from NorthWestern Energy website [19], with the base of 2,457 kW and 1,201 kvar (total 2,734.82 kVA). Load uncertainty isn’t considered in this paper.

B. VVO Test Results

In the base case, the voltage optimization is performed in the original IEEE 37-bus system, with the VR tap optimally adjusted every hour to minimize active power loss. The daily active power loss is 630.67 kWh/day, the active power loss is 59.82 kW at peak load, and the average power factor is 0.89. The proposed VVO is tested in three different cases. Case I uses proposed VVO, considering solar uncertainty and the reactive power support from PV inverters. Case II also considers solar uncertainty, but without reactive power support from PV inverters. Case III uses the deterministic VVO with reactive power support from PV inverters. The comparison of base case and three cases using proposed VVO will show the effectiveness of the reactive power support from PV inverters. And the comparison of case I and III will show the effectiveness of chance constrained modeling of uncertainty.

1) Case I – solar uncertainty and PV reactive power support

Solar power uncertainty is modeled through chance constraints. PV inverters can either inject or consume reactive power to participate in voltage regulation. The VVO module determines the optimal schedule of VVC devices and PV inverter power factor. Simulation results are shown in Fig. 5–7 and Table I. PV inverters provide the reactive power support throughout the day, as shown in Fig. 5. During the peak load period, PV inverter power factor decreases to provide more reactive power support locally. It helps improve the voltage profile and reduces system loss. Active power loss is reduced to 333 kWh/day, and the average power factor is improved to 0.90, benefiting from local reactive power supply from inverters. To accommodate more active power injection during peak PV generation and avoid overvoltage, VR is scheduled to operate at lower taps, as shown in Fig. 6. As the active power loss decreases at higher voltage level, the VVO module maintains voltage level close to the upper limit, as shown in Fig. 7.

2) Case II – solar uncertainty and PV unity power factor

In this case, PV inverters are operated at unity power factor, so that they don’t support reactive power or participate in voltage regulation. The reactive power demand from substation, active power loss, and system demand increase, as shown in Table I. To accommodate higher active power injection and maintain voltage profile, the VVO module increases VR operations compared to Case I, as shown in Fig. 6. This comparison shows that the reactive power support from PV inverters help improve voltage profile and reduce system loss.

3) Case III – deterministic solar power and PV reactive power support

This case is designed to compare the deterministic VVO approach with chance constrained VVO. The deterministic approach takes the mean value of forecasted PV active power, which has larger fluctuations compared to PV power profile in Case I and II, as shown in Fig. 4. PV inverter reactive power support cannot compensate large fluctuations. Therefore, VR tap operation increases, as shown in Fig. 6.

![Figure 3. One line diagram of test system.](image)

![Figure 4. Load profile and PV active power injection in three cases.](image)

![Figure 5. PV inverter power factor in three cases.](image)
The increased PV penetration brings operation challenges on system volt/var performance at the utility scale. In this paper, a stochastic optimization approach is developed to control traditional VVC devices and PV inverters for real-time VVO. A centralized control model is proposed for coordination of different devices. The chance constrained programming technique is used to model solar power uncertainty. And simulated annealing algorithm is applied to solve the non-linear and discrete optimization problem. The proposed strategy is tested in the modified IEEE 37-bus test system. Simulation results show that the proposed optimization approach is able to minimize active power loss, impact of variability and uncertainty of PV sources on VVC, and help increase the PV penetration level. It should be also noted that this VVO strategy relies on smart agents and communication infrastructures, which may not be available in some power utilities. In the future work, we will test the proposed VVO in a large-scale low voltage system, and three-phase PV inverters will be controlled to inject unbalance power to compensate the unbalanced load and enhance the phase balance of distribution system.

### V. Conclusions

The increased PV penetration brings operation challenges on system volt/var performance at the utility scale. In this paper, a stochastic optimization approach is developed to control traditional VVC devices and PV inverters for real-time VVO. A centralized control model is proposed for coordination of different devices. The chance constrained programming technique is used to model solar power uncertainty. And simulated annealing algorithm is applied to solve the non-linear and discrete optimization problem. The proposed strategy is tested in the modified IEEE 37-bus test system. Simulation results show that the proposed optimization approach is able to minimize active power loss, impact of variability and uncertainty of PV sources on VVC, and help increase the PV penetration level. It should be also noted that this VVO strategy relies on smart agents and communication infrastructures, which may not be available in some power utilities. In the future work, we will test the proposed VVO in a large-scale low voltage system, and three-phase PV inverters will be controlled to inject unbalance power to compensate the unbalanced load and enhance the phase balance of distribution system.

### REFERENCES


