Abstract—The large-scale power system outage is one of the most catastrophic disasters in modern society that results in enormous damage of billions per year for US economy. So, system operators are required to maintain plans for any unforeseen event that forces power system to operate without reliability. Contingency analysis is one of the well-known methods to paint the future scenarios in the power system. However, large numbers of possible $N - k$ combinations make its assessment computationally prohibitive. This paper proposes a new method to search most vulnerable transmission lines efficiently based on electrical distance. Specifically, a new electrical network is first built based on the impedance matrix (by inverting admittance matrix). Then, we will prune this impedance matrix based on the number of connections in topology network. Next, the common connections in two different structures (i.e., electrical network and topology network) will be observed for contingency experiments. Our results verify that violations of transmission lines limit due to contingencies are mostly associated with those common branches. In addition, voltage profiles are studied to validate that the vulnerable transmission lines found above are critical in power system stability.

Index Terms—Vulnerability analysis, electrical distance, $N - k$ contingency, power system stability, admittance matrix, impedance matrix.

I. INTRODUCTION

The electric power grid is regarded as one of the critical infrastructures in the world. It is a complex network consisting of numerous components like generators, transformers, transmission lines, circuit breakers, etc. As the power systems continue to increase in size and complexity because of grid modernization, cascading events leading to blackout are more likely to happen. U.S. electricity consumers incur a loss of almost $80 billion annually because of power interruptions [1]. The operation of the power grid has also changed dramatically due to change in the way the system is being owned and operated because of deregulation. With the aim of increasing reliability and efficiency of the power grid, the use of advanced information and communication technology is incorporated in addition to the physical infrastructure [2], [3]. However, security is a critical issue in the operation of such cyber-physical systems because of increasing malware from cyber side to compromise the physical components. Since many large scale electrical disruptions have occurred in the past [4]–[6], it has become necessary to ensure the operation of power systems economically and reliably. A detailed security assessment is essential in dealing with all credible outages in the system, its consequences and the remedial actions for them. For a power system to be secure, it must have a continuous supply of power without loss of load. With this aim, security analysis is performed to develop various control strategies to guarantee the avoidance and survival of any emergency conditions. Whenever the imposed limits of the power system get violated, the system is said to be in emergency condition. Thus an important part of the security analysis revolves around the power system ability to withstand the effect of any contingencies. An important factor in the operation of the power grid is the desire to operate it robustly because any kind of unplanned outages could lead to cascading events or even costly blackouts. One of the major agenda of power system planning and its operation is to study the effect of outage in terms of severity.

Contingency analysis (CA) is a well-known function in modern energy management system (EMS) to ensure power system security during equipment failure. It assists engineers to operate power system at a secure operating point, where transmission lines are loaded within their safe limit and consumers are provided power with acceptable quality standards [7]. In general, an outage of one transmission line or transformer or combination of different outages may lead to overload in other branches and/or sudden system voltage rise or drop. CA is used to calculate violations and analyze those violations for maintaining system security. It executes a power flow analysis for each credible contingency event defined on the contingency list [2], which in turn helps to identify the thermal and voltage violations. Results after each power flow study are compared with the limits of each element in the power system to identify the violations. For instance, a transmission line that was loaded at 80% of its MVA rating before any event might be overloaded above 100% after some outages in the system.

The introduction of new North American Reliability Corporation (NERC) Standards necessitates the system operators to ensure that the performance of power system is within the operating limits such that single and multiple contingencies do not result in cascading outages [8]. While these standards mandate the power industry to consider multiple contingencies, it is still challenging to solve the problem due to a high number of possible events. Multiple approaches have been proposed previously to address the complexity problem of $N - k$ contingency. Because of the way the power system is designed and operated, not all the outages will actually cause trouble. Hence, most of the time and effort spent while running power flow experiment will go for solutions which discover

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that there are no any violations in the system. In fact, only a few of the power flow solutions will conclude an overload or voltage violation in the system. The solution to this situation is to find an efficient way to select only those contingencies that are likely to result in an overload of branches or voltage limit violations.

Contingency screening or contingency selection is an essential task in contingency analysis which helps to reduce the numerous computations. Contingency selection criterion based on the calculation of performance indices has been first introduced by Ejebe and Wollenberg [9], where the contingencies are sorted in descending order of the values of performance index (PI) reflecting the severity. However, the results were not reliable, and to improve the reliability, authors in [10] proposed the use of higher-order sensitivities. One way to gain speed of solution in a contingency analysis procedure is to use an approximate model of the power system. Linear sensitivities like power transfer distribution factor (PTDF) using the DC power flow solution in contingency analysis is computationally much faster, however, this approach will not catch all of the contingency violations due to the underlying assumptions [11]. The 1P−1Q (one P−θ calculation and one Q−V calculation) method for contingency selection using fast decoupled load flow has been presented in [12] where solution is interrupted after one iteration. The application of Genetic Algorithm for contingency ranking has been studied in [13] where the ranking problem is formulated as an optimization problem with an objective of finding the critical cases. Ranking of contingency based on risk index along with the likelihood of each contingency and severity is computationally efficient and could be used for power system security assessment [14]. Inspired from [15], multi-element contingency screening algorithms were able to detect nearly all contingencies which result in violations by solving small number fraction of possible contingencies [16]. The screening algorithms considered linear sensitivities like power flow solutions in contingency analysis are performed to analyze the real and reactive power flowing in branches are tested for overloads and all conclusions are presented in Section V.

Despite these facts, most of the aforementioned literature still require conducting power flow experiments for various combinations of contingencies to identify the vulnerable branches for security assessment. With this consideration, we propose an electrical distance approach for searching of vulnerable branches efficiently during single and multiple outages in the system. A new connection of power system is defined based on the electrical distance between nodes or buses using the inverse of an admittance matrix (i.e., an impedance matrix). The entries of non-sparse impedance matrix are pruned in order to obtain a comparable structure consistent with original topological structure. \(N−1\), \(N−2\), and \(N−3\) contingency analysis are performed to analyze the post-contingency results in vulnerable branches. This method is more efficient to search vulnerable branches and reduce computational complexity during contingency screening. Also, it is not required to conduct power flow experiments for each possible event. In addition, the effect of loss of vulnerable branches on bus voltage profile is studied to validate the proposed approach.

The rest of this paper is organized as follows: Section II discusses the mathematical background of contingency analysis. Section III describes our proposed electrical distance method. Simulation results are shown in Section IV. Finally, conclusions are presented in Section V.

II. MATHEMATICAL BACKGROUND OF CONTINGENCY ANALYSIS

A well-known Full-Newton method is used for analyzing the system behavior during pre and post contingencies. This method will determine the overloads and voltage limit violations more accurately than approximate DC power flow. The relationship between node current \(I\) and node voltage \(V\) for a particular node \(i\) in a network of \(n\) nodes is given by the linear equation,

\[
I_i = \sum_{k=1}^{n} Y_{ik} V_k
\]  

where \(Y_{ik}\) is an element of the admittance matrix connecting nodes \(i\) and \(k\). Following equation (1), complex power at node \(i\) is given by,

\[
S_i = V_i I_i^* \tag{2}
\]

\[
P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^* \tag{3}
\]

Equation (3) represents real and reactive power flowing in \(i_{th}\) branch. Tripping of some sets of transmission lines changes the topology of grid which is reflected in admittance matrix, \(Y_{bus}\) matrix. The updated \(Y_{bus}\) matrix is then used to redistribute the branch flows after any outage of transmission line/s. The algorithm for AC power flow security analysis with contingency case selection is shown in Algorithm I. Each of the possible events is simulated by removing the elements defined in contingency list by updating the model. The post contingency flow in branches are tested for overloads and all the limit violations are reported. The impact of line outages is illustrated in Fig. 1.

\[
\begin{align*}
\text{Line 1} & \quad \text{Line 2} \\
\text{Line 3} & \quad \text{Line 4} \\
\text{Line 5} & \quad \text{Line 6} \\
\text{Line 7} & \quad \text{Line 8} \\
\text{Line 9} & \quad \text{Line 10} \\
\end{align*}
\]

Fig. 1. Contingency result for outage in power system. The lines identified in circle are overloaded due to outage of line/s identified in rectangle.
This figure illustrates the post contingency flow due to outage of particular line/s which are identified in rectangle. Only those lines which are carrying power beyond their limit are presented in the list (shown in circle) and their percentage loading due to outage is represented as $f_{a,b}$, where $f$ is the percentage loading of line $a$ due to outage of line $b$. For example, during $N-1$ contingency shown in first row, $f_{3,1}$ represents the percentage loading of line 3 due to outage of line 1. Similarly, during $N-2$ contingency shown in second row, $f_{3,1}k2$ represents the percentage loading of line 3 due to combined outage of lines 1 and 2.

Algorithm 1: Traditional contingency analysis

| Input: List of possible outages, Thermal limits |
| Result: Alarm list containing overload branches |
| for all list of possible outages do |
| 1. pick outage $i$ from the list and remove that component from the model |
| 2. run AC power flow in the updated model |
| if post contingency flow $>$ thermal limit, then |
| create list for vulnerable branch; |
| else |
| identify system as secure for that outage; |
| end |

Although NERC maintains requiring power grid security against $N-1$ contingency, they are still vulnerable to events which involve multiple component failures, i.e., $N-k$ contingency. So, as multiple outages are taken into considerations the number of events to be simulated grows rapidly and screening is intractable when $k > 1$. The number of total contingencies to handle for $k$ outages with $N$ number of branches in a system is given by,

$$Total = \binom{N}{k} = \frac{N!}{k!(N-k)!} (4)$$

For $k = 1$ the total number is simply $N$ which corresponds to $N-1$ contingency and for $k = 2$, the total number of combination is given as,

$$\binom{N}{2} = \frac{2!}{2!(N-2)!} = \frac{N(N-1)}{2} = \frac{(N^2 - N)}{2} (5)$$

Equation (5) suggests that for maintaining system security against $N-2$ contingency requires analyzing events in the order of $N^2$. In general, for simulating a $k$ number of outages, $O(N^k)$ power flow solutions are required to process. The number of transmission lines in the power system can be linearized with a number of buses ($n$) in the system, i.e., $N \approx 1.5n$ [16]. Based on this the computational complexity can be expressed as function of $n$ as,

$$O(N^k) = O((1.5n)^k) = O((1.5)^k(n))^k = O(n^k) (6)$$

The use of Newton’s method for power flow adds some computational effort in solving multiple outage contingency analysis. The elements of Jacobian matrix are $2 \times 2$ blocks of real numbers and computational effort for required factorization for $n$ number of buses is $O(n^{1.4})$ [21]. We will consider triple outage as the highest order in our case, and following equation (6), the computational complexity for $k = 3$ is given as

$$CE = O(n^{1.4})O(n^k) = O(n^{k+1.4}) = O(n^{4.4}). (7)$$

Equation (7) suggests that finding the critical combinations of contingency is challenging even for modest values for $N$ and $k$. So, to determine the vulnerable branches in the system for unseen events without any qualitative assumptions can be the remedy of this complexity. We will discuss the proposed approach in the following section.

III. PROPOSED ELECTRICAL DISTANCE METHOD

This section discusses the proposed method for finding vulnerable branches for any probable outages in the system. A new network structure is proposed based on electrical distance which is referred as electrical structure of the network. It will have the same number of the node to node connections as that of topological structure. The topological structure is an electrical power network formed from the admittance matrix ($Y_{bus}$). This new electrical structure is then compared with an original topological structure in order to explore the vulnerable branches. Here, “compare” means to find the branches that are common in both topological and electrical structure. In order to study the structure of power grids from a complex networks perspective, the electrical structure, as well as its topology, needs to be studied. Since flow in electrical networks is governed by Kirchoff’s law, this results in unique patterns of interaction between nodes in a network.

The bus admittance matrix defined by equation (8) captures the topological structure of power system network.

$$Y_{bus}^{ls} = \begin{cases} 
G_{ls} + jB_{ls}, & \text{if } l \neq s \\
- \sum(G_{ls} + jB_{ls}), & \text{if } l = s \\
0, & \text{if no connection exists}
\end{cases} (8)$$

Per definition, the $Y_{bus}$ matrix tends to be sparse as the value of some entries is 0 if nodes $l$ and $s$ do not have a direct physical connection. The admittance matrix, $Y_{bus}$ for W&W 6 bus network [22] in Fig. 2 is shown in Table I. This matrix shows all 11 branches of the system which can be represented by non-zero entries in the upper or lower triangular matrix.

**TABLE I.** $Y_{bus}$ for W&W 6 bus system.

<table>
<thead>
<tr>
<th>$Y_{bus}$</th>
<th>bus 1</th>
<th>bus 2</th>
<th>bus 3</th>
<th>bus 4</th>
<th>bus 5</th>
<th>bus 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>bus 1</td>
<td>4.0 + j18</td>
<td>-2.0 + j0</td>
<td>0.0</td>
<td>-1.2 + j4.7</td>
<td>-0.8 + j3.1</td>
<td>0.0</td>
</tr>
<tr>
<td>bus 2</td>
<td>-2.0 + j0</td>
<td>9.3 - j3.2</td>
<td>-0.7 + j0.8</td>
<td>-4.0 + j0.6</td>
<td>-1.0 + j0.0</td>
<td>-1.5 + j4.5</td>
</tr>
<tr>
<td>bus 3</td>
<td>0.0</td>
<td>-0.7 + j0.8</td>
<td>4.2 - j0.6</td>
<td>0.0</td>
<td>-1.5 + j2.2</td>
<td>-1.9 + j0.6</td>
</tr>
<tr>
<td>bus 4</td>
<td>-1.2 + j3.7</td>
<td>-4.0 + j0.0</td>
<td>0.0</td>
<td>6.2 - j4.7</td>
<td>-1.0 + j0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>bus 5</td>
<td>-0.8 + j3.1</td>
<td>-1.0 + j0.0</td>
<td>-1.5 + j2.2</td>
<td>-1.0 + j2.0</td>
<td>5.3 - j0.4</td>
<td>-1.0 + j0.0</td>
</tr>
<tr>
<td>bus 6</td>
<td>0.0</td>
<td>-1.6 + j3.5</td>
<td>-1.9 + j0.6</td>
<td>0.0</td>
<td>-2.0 + j0.0</td>
<td>4.5 - j17.0</td>
</tr>
</tbody>
</table>

There are variant measures of electrical distance for a power network ([23], [24]), but one of the efficient ways is the absolute value of the inverse of the system admittance matrix. To define the electrical distance in our work, we use the absolute value of the inverse of the $Y_{bus}$ matrix which is a non-sparse (dense) matrix, i.e., $Z_{bus} = |Y_{bus}^{-1}|$. The power system network is coupled by the following equation,
\[ YV = I \]  \hfill (9)

where \( V \) and \( I \) represent the bus voltage and injected current vectors respectively; and \( Y \) is the admittance matrix. Suppose a network has \( n \) nodes and \( m \) branches or links, each link has an impedance \( z = r + jx \) where \( r \) is resistance and \( x \) is reactance. The line admittance can be written as,

\[ y = g + jb = 1/z \]  \hfill (10)

where \( g \) is conductance and \( b \) is susceptance of any branch. Assume that a unit current flows along the link from node \( l \) to \( s \) which causes the voltage difference between ends of link equal to \( \delta v = V(l) - V(s) = Z_{ls} I \). Therefore \( Z_{ls} \) can be interpreted as the electrical distance between two nodes. It is also important to note that, electrical distance \( (Z_{bus} = |Y^{-1}|) \) does not perfectly represent all of the ways in which components in a grid connect, it is a useful starting point for structural analysis [25]. More mathematical justification of the proposed approach can be found in [26] and [27]. The distance matrix, \( Z_{bus} \) is a full matrix where each element \( Z_{ls} \) reflect the propagation of the voltage variation following a current injection in a given node pair throughout the system [28]. Since Kirchhoff’s and Ohm’s laws provide connectivity among all nodes pair in the system, the graph defined from electrical distance matrix \( Z_{bus} \) is fully connected. For the system shown in Fig. 2, there will be 15 links in the system as defined by,

\[ p = \frac{n^2 - n}{2} \]  \hfill (11)

where \( n \) is number of nodes in the network and \( p \) is the total number of connecting links between each node pair. These connections are represented by entries of electrical distance matrix, \( Z_{bus} \) given in Table II. Each term of the matrix \( (Z_{ls} \) or \( e_{ls} \)) describes the amount of connectivity between node pairs in the system.

The equivalent electrical distance between nodes \( s \) and \( l \) is thus given by the magnitude of the relevant entry of the \( Z_{bus} \). A small value of \( e_{ls} \) corresponds to a shorter electrical distance but a stronger coupling between these nodes (\( s \) and \( l \)). This reflects a larger propensity for power to flow between these nodes. A graph representation of the electrical structure of the system from the electrical distance matrix, \( Z_{bus} \) can be generated. The algorithm for representation of electrical structure graphically is given in Algorithm 2.

**Algorithm 2: Graphical representation of \( Z_{bus} \) matrix**

*Input: \( Y_{bus} \), Threshold (\( t \))*

*Result: Adjacency matrix with elements \( a_{ls} \)*

**Graphical representation of electrical network**

\[ Z_{bus} = |Y^{-1}|; \]

for every element of \( (Z_{bus}) \) do

if \( e_{ls} < t \), then

\[ a_{ls} = 1; \]

| draw connection between node \( l \) and \( s \); |

else

\[ a_{ls} = 0; \]

end

if \( a_{ls} \in Y_{bus}^t \), then

| branch \( a_{ls} \) exist; |

else

| branch \( a_{ls} \) does not exist; |

end

end

Since \( Z_{bus} \) is a non-sparse matrix, it will reflect all possible links between the nodes present in the network. In the **electrical structure**, the number of nodes \( n \) will be same as a topological network. But the existing \( m \) links in the topological network will be replaced by \( m \) smallest entries from the upper or lower triangle of \( Z_{bus} \) matrix (since \( Z_{bus} \) is symmetric). This means the new electrical structure will have a same number of the node to node connections as that of topological structure. Hence, the total \( p \) connection links of \( Z_{bus} \) matrix will be reduced to \( m \) by calculating the proper threshold value. With these newly selected connection links, we will have a different topology than the original network which is **electrical structure** of the system. The elements of the adjacency matrix, \( A \) of this new network will be defined as follows,

\[ a_{ls} = \begin{cases} 
1, & \text{if } e_{ls} < t \\
0, & \text{if } e_{ls} \geq t 
\end{cases} \]  \hfill (12)

where \( e_{ls} \) is any term of \( Z_{bus} \) matrix and \( t \) is the calculated electrical distance threshold in order to capture the same number of links as the previous network. Next, the branches in an electrical network are compared with topological connections and it is observed that some branches are common in both

<table>
<thead>
<tr>
<th>( Z_{bus} )</th>
<th>bus 1</th>
<th>bus 2</th>
<th>bus 3</th>
<th>bus 4</th>
<th>bus 5</th>
<th>bus 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>bus 1</td>
<td>3.6404</td>
<td>3.7041</td>
<td>3.7235</td>
<td>3.6963</td>
<td>3.7149</td>
<td>3.7236</td>
</tr>
<tr>
<td>bus 2</td>
<td>3.7041</td>
<td>3.6446</td>
<td>3.6970</td>
<td>3.7146</td>
<td>3.7066</td>
<td>3.7066</td>
</tr>
<tr>
<td>bus 4</td>
<td>3.6963</td>
<td>3.7146</td>
<td>3.6970</td>
<td>3.6515</td>
<td>3.7166</td>
<td>3.7203</td>
</tr>
<tr>
<td>bus 5</td>
<td>3.7149</td>
<td>3.7146</td>
<td>3.7029</td>
<td>3.7166</td>
<td>3.7203</td>
<td>3.6444</td>
</tr>
<tr>
<td>bus 6</td>
<td>3.7236</td>
<td>3.7066</td>
<td>3.6963</td>
<td>3.7203</td>
<td>3.7092</td>
<td>3.6444</td>
</tr>
</tbody>
</table>

Fig. 2. Single line diagram of W&W 6 bus system [22].
can be observed from the line impedance directly. However, it is vulnerable lines are those with short electrical distance and physical connections between the nodes. It is also true that important according to our proposed model and also have or vulnerable branches, we look into those branches that are power system test cases. With aim of finding the most critical electrical distance, contingency analysis is performed in power system test cases. With aim of finding the most critical or vulnerable branches, we look into those branches that are important according to our proposed model and also have physical connections between the nodes. It is also true that vulnerable lines are those with short electrical distance and can be observed from the line impedance directly. However, it will be difficult to find the set of vulnerable branches by just looking into their impedance directly since there is no any predefined number of branches that we are going to screen or search. So, forming the electrical structure and comparing it with the original topological structure will give the set of vulnerable branches.

The computational complexity for \( N - 2 \) contingency as given by equation (7) is \( O(n^{3.4}) \) and this complexity increases to \( O(n^{4.4}) \) if we consider \( N - 3 \) contingency (\( k = 3 \)). In our proposed work, the only computational cost is the calculation of inverse of \( n \times n \) matrix and this cost is independent of multiple outages taken into account. According to [29] and [30], the computational cost for obtaining inverse of matrix is \( O(n^{2.373}) \) which is less than traditional contingency analysis.

In this section, we described our proposed approach in finding the critical branches based on impedance matrix. In our approach, we follow the formulation as in references [25] and [31] such that smaller value of electrical distance corresponds to the larger propensity for power flow between the nodes. It is also true that loading will impact the location of critical branches. However, during the early stage of power system planning and operation, transmission line parameters (\( r, x, \) and \( b \)) are adjusted and its power transfer capability is determined based on the expected power that it is supposed to carry. This means, a line which has to carry higher MW has different line parameter (impedance) than the line which needs to carry less MW. So, the line criticality is closely related to its parameters which are reflected in terms of electrical distance. For example, having parallel transmission lines can make the line stronger as the power flowing through it gets divided according to the Kirchhoff law, given that loading does not change. In this case, the electrical distance of line will be reduced. Hence, loading will impact the location of critical branches and this will be somehow reflected in terms of electric connectedness between the nodes. And, we have not considered the dynamics of the system in our experiments, so the changing operating points can’t be reflected in terms of electrical distance.

IV. SIMULATION RESULTS

In this section, we use the IEEE-24 bus system as a test case to illustrate the electrical distance approach for selecting vulnerable branches. This system is obtained from Illinois Center for Smarter Electric Grid (ICSEG) [32]. It has 38 transmission lines with 6 transformers, which are given unique numbers as shown in Fig. 3. The transmission lines are numbered so that it helps identify the failed and overloaded branches during contingency analysis. The node positions and system admittance matrix, \( Y_{bus} \) is obtained from PowerWorld. The dense impedance matrix (\( Z_{bus} \)) is then calculated by inverting \( Y_{bus} \) matrix. Following equation (11), the total number of distinct node to node connections for 24 bus system is calculated as \( [(24^2 - 24)/2] = 276 \).

A. Case I: Contingency analysis based on electrical distance

In addition to the one-line diagram, Fig. 3 also shows the topological structure (node-branch representation) of the IEEE-24 bus system using the bus (node) position and connections between branches according to admittance matrix. Algorithm 2 presented in Section III is applied to IEEE 24 bus test case system to generate the equivalent electrical structure of the network. Initially, the \( Z_{bus} \) matrix is made triangular since the node pair \( i - j \) and \( j - i \) represents the same connection between the nodes. Then each entry is selected...
defined by adjacency matrix. It is not necessary that all the electrical connections as nodes with "high betweenness" or "high centrality" language of social networks, these nodes are often referred to pass through these nodes than the remaining other nodes. In the network as governed by Kirchhoff's law is more likely to connectivity to the rest of the network. Power flowing through electrical hubs. That is, those buses have a high electrical network seems to have a distinct group of nodes that are of the network, not simply the physical structure), 24 bus From an electrical perspective (which captures the behavior of the 38 different node-node connections and hence it is size-compatible with the topological structure. The two representations of the 38 strongest electrical connections (Fig. 4a), only 38 strongest electrical connections are presented. The links shown in Fig. 4b represents the 38 different node-node connections and hence it is size-compatible with the topological structure. The two representations of the 24 bus network suggests different structure. From an electrical perspective (which captures the behavior of the network, not simply the physical structure), 24 bus network seems to have a distinct group of nodes that are electrical hubs. That is, those buses have a high electrical connectivity to the rest of the network. Power flowing through the network as governed by Kirchhoff’s law is more likely to pass through these nodes than the remaining other nodes. In language of social networks, these nodes are often referred to as nodes with “high betweenness” or “information centrality” [31]. It is not necessary that all the electrical connections defined by adjacency matrix A will be physically present in the original system. So, we search those connections which are electrically important according to our approach and exist in the topological structure as well. Fig. 4c shows particular branches in the system which have the shortest electrical distance (electrically important connections) and are common between topological structure and electrical structure. The set of branches belonging to a different structure is also represented in Venn-diagram in Fig. 5 where the color of marks represent branches from different network structure. For the branches belonging to $n(E)$, they should have lower electrical distance among the impedance matrix. And the branches belonging to $n(T \cap E)$ are physically connected branches in the benchmark. But when we look into the $n(E)$ set only, the branches belonging to $n(T \cap E)$ are those branches which are among the shortest electrical distance and have a physical connection in the network as well. However, when we compare the impedance of branches in $n(T \cap E)$ with the remaining branches in $n(E)$ (i.e., $n(E) - n(T \cap E)$) it is not necessary that they have higher or lower impedance values compared to $n(E)$.

With the calculated threshold value, out of 276 total electrical connections (Fig. 4a), only 38 strongest electrical connections are presented. The links shown in Fig. 4b represents the 38 different node-node connections and hence it is size-compatible with the topological structure. The two representations of the 24 bus network suggests different structure. From an electrical perspective (which captures the behavior of the network, not simply the physical structure), 24 bus network seems to have a distinct group of nodes that are electrical hubs. That is, those buses have a high electrical connectivity to the rest of the network. Power flowing through the network as governed by Kirchhoff’s law is more likely to pass through these nodes than the remaining other nodes. In language of social networks, these nodes are often referred to as nodes with “high betweenness” or “information centrality” [31]. It is not necessary that all the electrical connections defined by adjacency matrix A will be physically present in the original system. So, we search those connections which are electrically important according to our approach and exist in the topological structure as well. Fig. 4c shows particular branches in the system which have the shortest electrical distance (electrically important connections) and are common between topological structure and electrical structure. The set of branches belonging to a different structure is also represented in Venn-diagram in Fig. 5 where the color of marks represent branches from different network structure. For the branches belonging to $n(E)$, they should have lower electrical distance among the impedance matrix. And the branches belonging to $n(T \cap E)$ are physically connected branches in the benchmark. But when we look into the $n(E)$ set only, the branches belonging to $n(T \cap E)$ are those branches which are among the shortest electrical distance and have a physical connection in the network as well. However, when we compare the impedance of branches in $n(T \cap E)$ with the remaining branches in $n(E)$ (i.e., $n(E) - n(T \cap E)$) it is not necessary that they have higher or lower impedance values compared to $n(E)$.

After finding the electrical structure of 24 bus system, it is simulated for $N - 1$, $N - 2$, and $N - 3$ contingency analysis to observe different possible scenarios in the post-contingency analysis. With the loss of single ($N - 1$) or a combination of transmission lines ($N - 2$ and $N - 3$), there occurs power flow redistribution in the network. This forces some of the remaining transmission lines to carry power beyond their line limit. During contingency analysis, only real power flow is considered for checking the line limits, and voltage limit violations are not accounted in this paper.

For example, loss of line 3 causes lines 1, 2, 4, 5, 8, and 10 to carry power beyond their capacity and forces them to
Fig. 7. \( N - 2 \) contingency analysis results for lines 3&6 and lines 6&8. The line identified in circles are overloaded lines due to outage of line identified in rectangle. The number in circle represents the percentage loading of lines.

Table III. Contingency analysis for IEEE-24 bus system showing line index and number of times it is violated for \( N - 1 \), \( N - 2 \), and \( N - 3 \) contingencies

<table>
<thead>
<tr>
<th>S.No</th>
<th>Line Index</th>
<th>Impedance ( Z_i )</th>
<th>Number of violations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>0.3570</td>
<td>31  453  4167</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.4085</td>
<td>28  372  3218</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.3802</td>
<td>30  429  3903</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.3862</td>
<td>29  402  3541</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0.3979</td>
<td>6   149  1767</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>0.3997</td>
<td>3   75   898</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>0.3962</td>
<td>6   155  1923</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>0.3906</td>
<td>1   25   365</td>
</tr>
<tr>
<td>9</td>
<td>6*</td>
<td>0.4111</td>
<td>-   -   -</td>
</tr>
<tr>
<td></td>
<td>Total line limit violations</td>
<td>134  2063  19783</td>
<td></td>
</tr>
</tbody>
</table>

* → line not captured by electrical distance approach

Algorithm 2 is applied and outage for different contingency combinations are simulated for several power system benchmarks. A 6-bus system is built in PowerWorld using all branch and loading information from [22]. A design project case with 37 bus and 57 transmission lines is taken from [33]. Similarly, remaining IEEE test cases are directly accessed from Illinois Center for Smarter Electric Grid (ICSEG) [32]. Table IV summarizes the contingency analysis results. The total number of violations and line identifier of the violated lines that contributed for those recorded violations are observed. The total number of violations due to \( N - 1 \), \( N - 2 \), and \( N - 3 \) contingencies captured by important electrical branches are expressed in percentages and a total number of sensitive lines for each contingency are reported. The total number of contingencies for \( N - 1 \), \( N - 2 \) and \( N - 3 \) can be calculated using equation 4. The contingency experiment is conducted for each combination for analyzing the line limit violations. A number of line limit violations means the total violations that occurred in the benchmark for a particular contingency experiment. For example, in IEEE-24 bus test case where \( n = 38 \) (number of transmission lines), the total combination of contingencies for second order outage \( (k = 2) \) is calculated using equation (4); \( N = 38(38 - 1)/2 = 703 \). The total number of line limit violations for these combinations are 2063 as shown in Table IV. Similarly, a total number of sensitive lines means the total number of branches which are overloaded during to \( N - 1 \), \( N - 2 \), and \( N - 3 \) contingency experiments. It is observed that “most of the violations” are associated with those branches which are obtained from electrical distance approach and have a physical existence in the power system network as well. Here, the term “most of the violations” is used for those benchmarks where all the violations are not captured by electrical distance approach. As shown in Table IV, in some test cases, the violations captured under electrical distance is not 100%. For example, in IEEE-39 bus system during \( N - 1 \) contingency, the total number of violations captured by electrical distance approach is 85.0% which means out of total 20 violations, 17 violations are from the lines which have strongest electrical connections. Similarly, during \( N - 2 \) contingency analysis, 86.5% of total 557 violations are captured by electrical distance approach. Remaining 75 line limit violations are from those transmission lines which are not identified as vulnerable based on our approach.

The results for contingency analysis in different power system test cases suggest that electrical distance approach for searching the vulnerable branches is more efficient and does not require power flow experiment for each probable event in the power system. With this proposed method, the computational effort in solving thousands of possible outages during higher order contingency to predict the effect of outages can be reduced.

B. Case II: Loss of sensitive branches and voltage profile

All the simulations set up for this case study are the same as that for Case I. Here, we study the effect in voltage profile of system with loss of transmission line. In any power system, transmission lines have a limited capability for power transfer, as well known from circuit theory [34]. This limit marks the...
TABLE IV. Contingency analysis in different benchmark system

<table>
<thead>
<tr>
<th>System</th>
<th>Number of Tr. Lines</th>
<th>Number of line limit violations</th>
<th>Violations captured under electrical distance</th>
<th>Number of sensitive lines</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N-1</td>
<td>N-2</td>
<td>N-3</td>
<td>N-1</td>
</tr>
<tr>
<td>W&amp;W-6</td>
<td>11</td>
<td>3</td>
<td>105</td>
<td>211</td>
</tr>
<tr>
<td>IEEE-14</td>
<td>20</td>
<td>1</td>
<td>23</td>
<td>200</td>
</tr>
<tr>
<td>IEEE-24</td>
<td>38</td>
<td>1</td>
<td>134</td>
<td>2063</td>
</tr>
<tr>
<td>IEEE-30</td>
<td>41</td>
<td>1</td>
<td>1005</td>
<td>12131</td>
</tr>
<tr>
<td>G&amp;S-37</td>
<td>17</td>
<td>3</td>
<td>103</td>
<td>2352</td>
</tr>
<tr>
<td>IEEE-39</td>
<td>46</td>
<td>20</td>
<td>557</td>
<td>9321</td>
</tr>
</tbody>
</table>

onset of voltage instability where the system is not able to maintain the steady voltage at all buses in the system after being subjected to a disturbance from a given initial condition.

Based on our proposed approach of electrical distance, we have some branches in a system that are identified as critical during contingency analysis and those branches are among having small electrical distance. The different set of branches belonging to the electrical and topological structure are shown in Fig. 5. To verify the proposed approach for selection of critical branches, the bus voltage profile is studied for loss of a particular branch in the system. Two branches are selected from a set of critical lines which are under the set \( n(T \cap E) \). And, two other branches are selected which are not captured in electrical connections and belong to set \( n(T) \setminus n(T \cap E) \). These branches are opened one by one at \( t = 1\text{sec} \) during normal operation condition.

![Fig. 8. Voltage profiles for loss of most sensitive branches.](image)

Transient stability analysis is done for these disturbances to see the change in voltage level of all 24 buses. The transient stability experiment is performed in PowerWorld software, which conducts transient stability in a quasi-steady-state operating condition for a power system. The contingency elements are loaded into the system (which is a loss of branches in our case) and the experiment is simulated for 5 seconds.

During normal operating condition, the voltage profile of the system is between 0.92 p.u. and 1 p.u. as shown in Fig. 8. First, line 1 is removed and the change in voltage level of all the buses is observed. This caused the voltage of several buses to decrease around 0.8 p.u. and increase above 1.1 p.u. which are beyond the normal operating range as shown in Fig. 8a. Again, line 4 is opened which is also identified as critical line according to our approach. Fig. 8b shows the degradation in voltage profile because of loss of another critical line. This decline in voltage level is not desired in power system and can damage the load or eventually lead to voltage collapse.

![Fig. 9. Voltage profiles for loss of least sensitive branches.](image)
Similarly, two different branches from set $n(T)-n(T \cap E)$ are selected. Lines 19 and 22 are opened at $t = 1$ sec and voltage profile is observed for both the cases. Fig. 9 suggests that these disturbances have a minimal effect on the system voltage profile and only few bus voltages are found to be declined below a normal operating point of 0.92 p.u.

In addition, we also studied the voltage distribution of all 24 buses for loss of each branch in the system. Fig. 10 shows the upper and lower voltage level of buses for an outage of each branch set. The blue mark represents the upper voltage level and black mark represents the lower voltage level for a particular loss. The red marks show the voltage level of particular buses which are beyond the range specified by the green line, and the x-axis shows the corresponding index of the branch. The line index suggests that the line captured in $n(T \cap E)$ caused the voltage to distribute beyond ±10% limit (i.e., below 0.9 p.u. or above 1.1 p.u.). For example, when line-1 is removed from the system, the bus voltage of all 24 buses ranges from 1.1308 to 0.8094. Our result shows that the deviation of voltage level from the defined boundary is because of loss of branch which belongs to $n(T \cap E)$ set. This suggests that branches with stronger electrical connections are more critical and hence their outage has a severe impact in the voltage.

V. DISCUSSION AND CONCLUSION

This paper presents a method of searching vulnerable or critical branches to avoid the computational expense of processing higher order outage event. In our proposed approach, the vulnerable branches are screened through the electrical distance between node pairs, so that the candidate search in contingency analysis for all possible events can be avoided. This significantly reduces the computational cost while having the ability to identify almost all vulnerable branches in the system. Our experimental results show that branches with small electrical distance are identified as vulnerable for $N-1$, $N-2$ and $N-3$ contingency. Study of the voltage profile of system for transmission line loss validated our proposed approach for searching the vulnerable branches. Based on vulnerable branches initial flow, loading limit, and the way power system is being operated, the identification alone can help in decision making during power system outage. Correspondingly, power system security can be enhanced by making those branches more rigid or re-scheduling of generator output in order to decrease the burden on these particular branches.

Along this direction, we are working on the following major tasks. First, we are working on large grid-connected power systems to quantify the improvement of this work in terms of loss of accuracy in capturing the vulnerable branches. Second, we didn’t include the dynamics of the system in our experiments, and the changing operating conditions of the power system cannot be reflected in terms of electrical distance. We will further investigate the power system dynamics and demonstrate the improved screening method in the future publications.

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