Coordinating generation and load pickup during load restoration with discrete load increments and reserve constraints

Zhijun Qin¹, Yunhe Hou¹, Chen-Ching Liu², Shanshan Liu³, Wei Sun⁴

¹Department of Electrical and Electronic Engineering, The University of Hong Kong, Hong Kong SAR, People’s Republic of China
²School of Electrical and Computer Science, Washington State University, Pullman, WA, USA
³Grid Operations Program, Electric Power Research Institute, Palo Alto, CA 94304, USA
⁴Electrical Engineering and Computer Science Department, South Dakota State University, Brookings, SD, USA
✉ E-mail: yhhou@eee.hku.hk

Abstract: After a major outage happens, the independent system operator, transmission owners (TOs), generation owners (GOs), and distribution owners (DOs) should coordinate control actions to restore the power system timely and reliably. This study proposes a methodology to establish load restoration plans for the coordination among these participants. This methodology models the load restoration as a multi-stage decision-making process. At each stage, a mixed-integer nonlinear load restoration model (MINLR) is formulated to maximise load pickup subject to AC power flow and reserve constraints. Comprehensive load characteristics are considered in this model. The solution of the MINLR model provides power set points for GOs/TOs and load pickup amount for DOs. A complete load restoration plan is obtained by solving a series of MINLR models. To solve MINLR models efficiently, a branch-and-cut solver is constructed by identifying efficient cutting planes and a reliable problem-specific branching method. The applicability of cutting planes is proven. This methodology is tested using a 24-bus system with 170 interrupted load increments, and a 118-bus system with 637 interrupted load increments. The simulation results show that the proposed methodology can be efficiently applied to aid restoration participants pickup load increments within the TO’s islands, while maintaining adequate reserve margins.

Nomenclature

Sets

\[ \Omega^u, \Omega^p \] set of unserved loads at stage \( m \)

\[ \Omega^u_b, \Omega^p_b \] set of unserved loads on bus \( k \) at stage \( m \)

\[ \Omega^G^u, \Omega^G^p \] set of synchronised generating units at stage \( m \)

\[ \Omega^B, \Omega^B_u \] set of energised branches and buses at stage \( m \)

Parameters

\[ P^m_{G_i}, Q^m_{G_i} \] real and reactive power set points of generating unit \( i \) at stage \( m \)

\[ P^m_{L_i}, Q^m_{L_i} \] aggregate load demand of load increments restored prior to stage \( m \) on bus \( i \)

\[ r_i^m \] effective ramping time of generating unit \( i \) for spinning reserve at stage \( m \)

\[ K^m_i \] effective responsive reserve of generating unit \( i \) at stage \( m \)

\[ u^m_i \] on/off state of load increment \( i \) at stage \( m \) (1 – restored at this stage and 0 – remains interrupted)

\[ \Delta P^m_{L_i}, \Delta Q^m_{L_i} \] load demand of load increment \( i \) at stage \( m \)

\[ l_i, \beta_i \] priority weighting and cold load effect coefficient of load increment \( i \)

\[ R_i, \rho_i, C_i \] ramping rate, load pickup factor, and capacity of generating unit \( i \)

\[ \bar{\tau} \] maximum ramping time for synchronous reserve of generating units

\[ P^m_{L_i} \] maximum restorable load level at stage \( m \)

\[ a_i, b_i, c_i \] voltage-dependent coefficients of load \( i \)

\[ \bar{V}_i, \bar{L}_i \] upper and lower bounds of voltage of bus \( i \)

Variables

1 Introduction

Power system restoration is highly challenging as it covers a wide spectrum of challenges [1, 2] and involves various independent entities [3]. Despite that different entities have specific major concerns and obligations to restore their equipment, control actions should be coordinated to maintain security and reliability. To ensure reliability during restoration, various regulatory authorities and independent system operators or regional transmission organisations (ISOs/RTOs) have prepared guidelines with highlights on the importance of coordination [4–9].

It is highly recommended that restoration plans should be prepared considering coordination issues among participants. Although restoration plans differ from each other due to power system characteristics, power system restoration procedures share some common phases, namely, preparation, network reconfiguration, and load restoration [3, 10]. After major generating units and the
transmission skeleton have been restored, restoring service to customers based on priorities is the major concern of the load restoration phase.

1.1 Related work and research motivation

Plenty of research work has been conducted on load restoration for microgrids [11], distribution systems [12–16], and transmission systems. In [17], an optimisation model was formulated to maximise load pickup for a given substation. With accurate phasor measurement units data, an analytical method to predict restorable load level was proposed in [18]. To overcome the cold load pickup issue, distributed generation units have been introduced in load restoration [19]. During the load restoration phase, network reconstructions are also necessary [20–24]. In [21], a DC optimal load shed recovery with transmission switching model was proposed to increase the operating capacity, considering load demands as continuous variables. A mixed-integer model considering discrete load pickup was proposed in [25] to reduce the standing phase angle. This model is solvable by a two-stage decoupled algorithm due to the relatively small load increments required as a control means.

However, the design and implementation of a methodology to establish load restoration plans for transmission systems is highly challenging. The difficulties may lie in the following aspects:

- Load restoration in the transmission systems requests coordinated control of restoration participants on a system-wide basis. For example, the transmission owner (TO) specifies the magnitude and location of load pickup for distribution owners (DOs) [7]. Generation owners (GOs) manage the capacity resource as directed by ISO to guard against contingencies [5]. Mathematically, the computation of the optimal load restoration plan combines non-convex (due to power flow constraints) and combinatorial nature (considering discrete load increments of feeders), and is computationally expensive.

- Security constraints need to be considered in various time scales [26]. A tradeoff should be made to keep the computational complexity tractable while providing feasible restoration strategies.

In short, the implementation of a coordinated load restoration plan represents a challenging technical problem blending non-convex, combinatorial, and sequential (multi-stage decision-making) features. To the best of our knowledge, little, if any, research work has been done to address this challenge.

In view of these technical challenges, this paper proposes a methodology and associated algorithms to aid restoration participants maximise load pickup based on load priorities, while maintaining frequency security, adequate reserve levels, and meeting comprehensive steady-state constraints. The proposed methodology can be applied for the restoration planning of transmission systems. We are also mindful to extend this methodology to consider dynamic security issues.

1.2 Research scope and basic assumptions

This paper focuses on the technical issues of the load restoration phase. As the entire restoration process is highly complicated, the following assumptions are made for a moderate research scope:

(i) The communication infrastructure works properly and enables a smooth collaboration among restoration participants.

(ii) The model of the entire power system and individual components can be established in a high confidence level with the historical data and operators’ experience.

(iii) The power system has been restored with sufficient strength, although some components may be unavailable, or put into operation gradually, due to physical or technical constraints.

(iv) The sequence to restart generating units and to switch transmission is given information. The determination of generating unit start-up sequence and transmission switching can be found in the reference focusing on these topics (e.g. [20, 22, 27]).

(v) The automatic generation control may be deferred prior to interconnection of TO islands [5, 6]. Thus, approximate real power balances should be maintained with appropriate power set points.

1.3 Paper contribution and organisation

The major contribution of this paper is two-fold:

(i) A methodology to aid the decision-making of restoration participants in the load restoration phase is proposed. As a key part of this methodology, a mixed-integer non-linear load restoration model (MINLR) is formulated to maximise the load pickup at each stage considering discrete load increments, cold load effect, reserve requirements, and other steady-state constraints. By solving a series of MINLR models, a load restoration plan can be established as a baseline for the operators.

(ii) To solve MINLR models efficiently, a branch-and-cut (B&C) solver is constructed in this paper by identifying valid cutting planes and a reliable problem-specific branching method. The applicability of cutting planes is proven.

The rest of this paper is organised as follows. The methodology and MINLR model to establish load restoration plans are proposed in Section 2. The construction of the B&C solver is introduced in Section 3. Case studies are presented in Section 4. This paper is concluded in Section 5.

2 Methodology to establish load restoration plans

Industry practices in load restoration suggest that the amount, timing, and location of load being restored should be controlled by restoration participants coordinately [4, 7, 8]. To mitigate unknown impacts, participants wait for voltage and frequency to stabilise prior to picking up the next block of load increments [7, 8].

The load restoration is therefore modelled as a multi-stage process with two-fold. First, the total restorable load level at each stage is determined by the frequency response capacity of the synchronised generating units. Second, each generating unit’s output at each stage is restricted by time-related operational constraints such as start-up requirement, ramping rate, minimal technical output etc.

The methodology to establish the load restoration plan consists of the following sub-tasks: (i) estimate output bounds of generating units and the total restorable load level at each stage; (ii) formulate a model at each stage to compute the maximum load pickup for DOs; and (iii) estimate the duration of each stage. These sub-tasks are described as follows.

2.1 Estimation of operational bounds of each generating unit and total restorable level

The generating unit’s steady-state model described in [10] is used to estimate the output of each generating unit. As shown in Fig. 1, for the generating unit $i$, $M_i$ is the MW start-up requirement, $a_i$ is the minimum technical output ratio, and $t_f$ is the remaining time from start-up to parallel operation.

For synchronised generating units, the output range is restricted by capacities and minimal technical outputs. For unsynchronised ones, the outputs are fixed as start-up requirements. Therefore, $P_{G_i}^M$ and $P_{G_i}^m$ are determined by

$$P_{G_i}^M = \begin{cases} P_{G_i}^{m-1}, & \text{if } P_{G_i}^{m-1} + \gamma \leq t_f \\ \min \{ P_{G_i}^{m-1} + (t_f - \gamma)R_i, C_i \}, & \text{otherwise} \end{cases}, \quad (1)$$

$$P_{G_i}^m = \begin{cases} P_{G_i}^{m-1}, & \text{if } P_{G_i}^{m-1} + \gamma \leq t_f \\ P_{G_i}^{m-1}, & \text{if } P_{G_i}^{m-1} + \gamma > t_f \text{ and } P_{G_i}^{m-1} \geq a_iC_i \\ \max \{ P_{G_i}^{m-1} - (t_f - \gamma)R_i, a_iC_i \}, & \text{otherwise} \end{cases}, \quad (2)$$


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where $t^{m-1}$ is the end time of the previous stage, $\gamma$ is the allowed ramping time for generating units. Particularly, as seen from the third condition of (2), once a generating unit has reached the minimum output, it cannot ramp down to the minimum output again.

Note that if for some generating unit $i$, $t^{m-1} < t_0$ and $t^{m-1} + \gamma - t_0 > 0$, this generating unit will be paralleled to the power system at the intermediate stage and must be reloaded to stabilise the unit operation. Equation (2) will enforce this unit only ramp up until it reaches the minimal technical output.

The load pickup should not be so large that the frequency deviation triggers the protective relay to trip generating units or until it reaches the minimal technical output.

Note that the load pickup factor $\rho_i$ is a system-specific parameter. For example, PJM Interconnection (PJM) uses 5% for fossil steam units, 15% for hydro units, and 25% for combustion turbines [28]. Midcontinent Independent System Operator, Inc. uses $\rho_i = 5\%$ for all units [6].

2.2 MINLR model formulation to maximise load pickup

The MINLR model is formulated here to maximise the load pickup at each stage subject to various constraints:

(i) **Objective function**: The objective of each stage is to maximise the load pickup for DOs through load priorities as

$$P_i^m = \sum_{l \in \Omega_G^m} \rho_i C_l^m, \quad \Omega_G^m = \{i | t_0 = 0\}.$$  

(ii) **Power balance constraints**: At the end of each stage, a steady state should be achieved with an approximate active/reactive power balance as in (5). The voltage-dependent load characteristics (i.e. the composition of constant impedance (Z), constant current (I), and constant power (P) components, or ZIP model) of both the already restored load increments and the load increments restored in the current stage are expressed in a quadratic form as in (6) and (7). For ease of notation, the load increments restored prior to stage $m$ on the same bus are modelled using aggregated coefficients $a_i$, $b_i$, $c_i$ in (6). Accordingly, $P_{Lk}^m$ denotes the aggregation of the restored load increments, whereas $\Delta P_{Lk}^m$ denotes the interrupted load increments at stage $m$. Here we assume the pre-disturbance load characteristic, such as $\Delta P_{Lk}^m$ and $P_{Lk}^m$, can be estimated from the normal operating point of the power system (see (5))

$$P_{Lk}^m = \left( a_i V_{lk}^m \right)^2 + b_i V_{lk}^m + c_i \right) P_{Lk}^m$$

$$Q_{Lk}^m = \left( a_i V_{lk}^m \right)^2 + b_i V_{lk}^m + c_i \right) Q_{Lk}^m, \forall k \in \Omega_{Bu}^m$$

$$\Delta P_{Lk}^m = \left( a_i V_{lk}^m \right)^2 + b_i V_{lk}^m + c_i \right) \left[ \Delta P_{Lk}^m \right], \forall k \in \Omega_{Bu}^m$$

(iii) **Operational constraints**: The steady state at the end of each stage should meet operational constraints as follows

$$P_{Gi}^m + \rho_i R_i \leq C_i$$

$$\rho_i \leq \tau,$$

$$P_{Gi}^m \leq \sum_{i \in \Omega_{Gi}^m} \left( \rho_i R_i \right), \forall \tau \in \Omega_{C}^m.$$  

The spinning reserve level is defined as the reserve associated with the largest loss of energy contingency according to

$$\min \left[ \sum_{i \in \Omega_{Gi}^m} \left( \rho_i R_i \right) - P_{Gi}^m, \quad i \in \Omega_{Gi}^m \right].$$

(v) **Responsive (dynamic) reserve constraints**: Responsive reserves may consist of reserves on automatic governor response (up to the total reserve), and system load with under-frequency load shedding (UFLS). The load with UFLS is not considered in this reserve, such that the power system relies merely on the governor response to restore frequency if any credible contingency, which causes frequency decay, would occur.

$$K_i^m \leq \rho_i C_i$$

$$P_{Gi}^m \leq \sum_{i \in \Omega_{Gi}^m} K_i^m, \forall \tau \in \Omega_{C}^m.$$ 

$$\sum_{i \in \Omega_{Gi}^m} \left( V_{lk}^m \right)^2 G_{lk}^m \sin \theta_{lk}^m + B_{lk}^m \sin \theta_{lk}^m) = \sum_{i \in \Omega_{Gi}^m} \left( V_{lk}^m \right)^2 G_{lk}^m \cos \theta_{lk}^m - B_{lk}^m \cos \theta_{lk}^m) = \sum_{i \in \Omega_{Gi}^m} \left( V_{lk}^m \right)^2 \Delta P_{Lk}^m = 0, \forall k \in \Omega_{Bu}^m.$$  

$$\sum_{i \in \Omega_{Gi}^m} \left( V_{lk}^m \right)^2 \Delta Q_{Lk}^m = 0, \forall k \in \Omega_{Bu}^m.$$
The responsive reserve level is defined by
\[
\min \left\{ \sum_{i \in \Omega_{G_i}^m} K_i^m - P_{G_i}^m, \quad i \in \Omega_{G_i}^m \right\}.
\]

(vi) Frequency security constraints: The total load pickup should be limited up to the total restorable level. Considering the cold load effect, which may last from seconds to minutes, the total load pickup is restricted due to the over-load than in normal conditions.

Estimation of the cold load effect after an extended outage is complicated [30]. In this paper, we assume that the load level over normal conditions is denoted by a cold load effect coefficient \( \beta_i \). This coefficient represents the maximum load demand over normal condition on the re-closure of feeder breakers. The total load pickup is restricted by 
\[
\sum_{i \in \Omega_{B_i}^m} \beta_i u_i^m \Delta P_{L,0}^m \leq \Delta P_{L}^m.
\]
(13)

Note that the identification of load model parameters as \( \beta_i \) will be the subject of future research independent of this model.

### 2.3 Estimation of duration of each stage

The duration of each stage is primarily determined by the time for the following dispatch actions and processes: (i) communications among restoration participants; (ii) re-energise the path to the load buses by closing breakers and switches; (iii) close breakers of feeders to pickup load increments; and (iv) generating units ramp to the power set points.

The methods for estimating the duration for (i) and (iii) can be found in [31]. Assuming the generation operators carry out the adjustments of generators in a parallel manner, the duration of (iv) at stage \( m \) is the maximum ramping time of the generating units given by
\[
\Delta t_m = \max \{t_{rG} - t_i, V, \forall i\},
\]
(14)

where \( [x] = \max\{x, 0\} \).

### 2.4 Procedure of the proposed methodology

The procedure of the proposed methodology is as follows:

1. **Step (1):** Let \( t = 0, m = 1 \). Identify the initial model of the power system, including \( P_{G_i}^0, Q_{G_i}^0, t_{rG} \), and the admittance matrix.
2. **Step (2):** Calculate \( \Omega_{B_i}^m, \Omega_{H_i}^m \), and \( \Delta P_{L,0}^m \) using (1)–(3).
3. **Step (3):** Establish paths to load buses, close transmission loops if necessary, update \( \Omega_{B_i}^m, \Omega_{H_i}^m \), and the admittance matrix.
4. **Step (4):** Build the MINLR models (4)–(13) and solve it.
5. **Step (5):** Calculate \( \Delta P_{L}^m \) with (14). Let \( t = t + \Delta t_m \) and \( t_{rG} = t_{rG} - \Delta t_m \), update \( \Omega_{B_i}^m, \Omega_{H_i}^m \), and \( \Delta P_{L,0}^m \).
6. **Step (6):** If all load increments are restored, stop; otherwise, set \( m = m + 1 \), and go to step (2).

### 3 Construction of B&C solver

The computation burden of the proposed methodology rests on the solution of a series of MINLR model. Therefore, efficient algorithms need to be identified. The construction of an efficient B&C solver for MINLR models is described in this section.

Note that relaxing the MINLR model converts it into an optimal power flow (OPF) model with continuous dispatchable loads under the ZIP model. It is therefore feasible to find a local optimum using interior point methods.

### 3.1 General B&C algorithm

The B&C algorithm is based on the branch-and-bound (B&B) algorithm. The basic idea of B&B is to solve the continuous-relaxation of an integer programming model and to branch on a selected integer variable. For a maximisation model, a relaxed model will provide an upper bound \( U \) for the objective function. If a node is integer-feasible, it provides a lower bound \( L \). The branching on some node can be stopped if no significant improvement can be found as \( |L - U|/U < \epsilon \), where \( \epsilon \) is the termination criterion. The B&C method is an extension of B&B by including cutting planes. The cutting planes achieve a tighter feasible set as well as preprocess sub-problems with heuristics [32].

### 3.2 Applicability of cutting planes

This section proves the applicability of cutting planes targeting on (13) to solve MINLR models.

The decision variables of MINLR models are collectively denoted as \( x \). Accordingly, the linear inequality (13) can be written as
\[
\Omega_{eq} = \{x | w^T x < P_{L,0}^m, x_i \in \{0, 1\}, \forall i \in \Omega_1\},
\]
where \( \Omega_1 \) is the set of binary variables.

The relaxation (or convexification) of \( \Omega_{eq} \) is as follows
\[
\text{conv}(\Omega_{eq}) = \{x | w^T x < P_{L,0}^m, x_i \in \{0, 1\}, \forall i \in \Omega_1\}.
\]
Using this result, the MINLR model (4)–(13) is recast as
\[
\Omega_{MINLR} \rightarrow \Omega_{MINLR-r},
\]
where \( \Omega_{MINLR-r} = \{x | x_i \in \Omega_1 \cap \text{conv}(\Omega_{eq})\} \) and \( \Omega_{MINLR-r} \) stand for a non-convex set defined by (3)–(12).

The relaxation of MINLR in the B&B framework is as
\[
\Omega_{MINLR-r} \rightarrow \Omega_{MINLR-r'},
\]
where \( \Omega_{MINLR-r'} = \{x | x_i \in \Omega_1 \cap \text{conv}(\Omega_{eq})\} \) and \( \Omega_{MINLR-r'} \) is the relaxed MINLR model.

**Definition:** (valid inequalities, [12] pages 114, 117–121): \( Ax \leq d \) is valid for \( \Omega \) iff \( x^* \in \Omega \) satisfies \( Ax^* \leq d \).

**Applicability of cutting planes to tighten \( \Omega_{MINLR-r'} \):** If \( Ax \leq d \) is valid for \( \Omega_{eq} \), it is valid for \( \Omega_{eq} \) where \( Ax^* \leq d \).

**Proof:** For \( \forall x_0 \in \Omega_{MINLR}, x_0 \in \Omega_{eq} \). As \( Ax \leq d \) is valid for \( \Omega_{eq} \), we have \( Ax_0 \leq d \).

By contrast, for \( \forall x \in (\Omega_{MINLR-r} \cap \Omega_{MINLR}), x' \in \Omega_{eq} \), thus \( Ax \leq d \). In other words, \( Ax \leq d \) cuts off part of \( (\Omega_{MINLR-r} \cap \Omega_{MINLR}) \), which may lead to a tighter relaxation without cutting any part of \( \Omega_{MINLR-r} \).

### 3.3 Incorporation of cutting planes for MINLR

The following cutting planes are identified as efficient and valid for (13). Note that, after each branching, (13) is converted into
\[
\sum_j u_j \beta_j \Delta P_{L,0}^m \leq P_{L}^m - \sum_j u_j \beta_j \Delta P_{L,0}^m,
\]
(15)

where \( u_j \in \{0, 1\} \) and \( \beta_j \in \{0, 1\} \).
(i) Fixing variable cut (FC): Fixing variables is a pre-solve technique for reducing the number of integer variables \[33\]. Intuitively, after branching on \(u_i\), if the model apparently becomes infeasible on setting some other \(u_j\) to one, \(u_j\) should be fixed as zero from the current node and its descendants. In other words, we mathematically add equalities for (15) using

\[
\hat{u}_j = 0, \quad \forall j \in F,
\]

\[
F_i = \{ j \beta_j \Delta P_{L,0}^m > \hat{P}_L^m - \sum u_i \beta_i \Delta P_{L,0}^m, \quad u_i \in \{0, 1\}\}. \tag{16}
\]

(ii) Gomory rounding cut (GC) ([32], page 119): The valid inequality for (15) is given by

\[
\sum_j \delta_j [\partial \beta_j \Delta P_{L,0}^m] \leq \partial (\hat{P}_L^m - \sum u_i \beta_i \Delta P_{L,0}^m), \tag{17}
\]

where \(\delta > 0\) and \([\bullet]\) is the biggest integer less than the input.

---

**Table 1** Aggregate characteristics of generators

<table>
<thead>
<tr>
<th>Bus</th>
<th>(b_{\mu} ) min</th>
<th>(a_{\mu} ) %</th>
<th>(P_{L,0}^m), pu</th>
<th>(Q_{L,0}^m), pu</th>
<th>(R_\mu), pu/h</th>
<th>(C_\mu), pu</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>0</td>
<td>0</td>
<td>1.020</td>
<td>-0.108</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>18</td>
<td>0</td>
<td>30</td>
<td>0.349</td>
<td>-0.116</td>
<td>1.5</td>
<td>4.0</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>50</td>
<td>0.270</td>
<td>0.184</td>
<td>1.0</td>
<td>1.55</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>50</td>
<td>0.063</td>
<td>0.087</td>
<td>1.0</td>
<td>1.92</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>50</td>
<td>-0.150</td>
<td>-0.110</td>
<td>1.0</td>
<td>1.92</td>
</tr>
<tr>
<td>15</td>
<td>14</td>
<td>50</td>
<td>-0.170</td>
<td>-0.130</td>
<td>1.0</td>
<td>2.15</td>
</tr>
<tr>
<td>13</td>
<td>29</td>
<td>60</td>
<td>-0.250</td>
<td>-0.190</td>
<td>1.5</td>
<td>5.91</td>
</tr>
<tr>
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<td>-0.210</td>
<td>1.0</td>
<td>4.00</td>
</tr>
<tr>
<td>7</td>
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<td>-0.225</td>
<td>1.0</td>
<td>3.00</td>
</tr>
<tr>
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<td>50</td>
<td>-0.500</td>
<td>-0.380</td>
<td>1.5</td>
<td>6.60</td>
</tr>
</tbody>
</table>

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**Fig. 2** Flowchart of the proposed B&C solver to solve MINLR

**Fig. 3** Initial state of RTS 24-bus test system
After solving the MINLR model, the solution is given with \( \hat{u}_j^* \). A new GC can be generated to cut this integer-infeasible solution by finding \( q^* \) such that
\[
\sum_j \hat{u}_j^* q^* b_j D P_{mL0}^\mu \leq \overline{P}_m L - \sum_i u_i b_i D P_{mL0}^\mu
\] (19)

(iii) Knapsack cover cut (KC) ([32], pages 147–151): The first step in generating KC of (15) is to find a positive integer set \( C \) such that
\[
\sum_k [C b_k D P_{mL0}^\mu] > \overline{P}_m L - \sum_i u_i b_i D P_{mL0}^\mu
\] (20)
Then the KC is given by
\[
\sum_{k \in C} \hat{u}_k \leq |C| - 1,
\] (21)
where \( |C| \) is the number of elements in \( C \).

3.4 Selection of branching method
The branching method is another key factor affecting the scale of B&C trees (see e.g. [34]). The following three branching methods are used for comparison in Section 4.

(i) Maximum fractional branching (MFB): This branching method selects the relaxed integer variable closest to 0.5, mathematically selecting the variable through
\[
\max_j \{ \min (1 - \hat{u}_j, \hat{u}_j) \},
\] (22)
where \( \hat{u}_j \) belongs to the optimal solution of the parent nodes.

(ii) Pseudo-cost branching (PCB): This branching method branches on the variable that most changes the objective function. For the MINLR model, the branching variable is selected by
\[
\max_j \{ \min (\Delta P_{L0}^\mu f_j, \Delta P_{L0}^\mu f_j) \}
\] (23)

(iii) Biggest-load-increment branching (BLB): A problem-specific branching method is proposed here for the MINLR model. We branch on the variable that is associated with the biggest (or most significant with priority ranking) interrupted load increment as in (24)
\[
\max_j \{ \Delta P_{L0}^\mu f_j | j \in F_2 \}, F_2 = \{ j | \hat{u}_j \in [0, 1] \}
\] (24)
To summarise the above process, the flowchart of the B&C solver for the MINLR model is given in Fig. 2.

4 Illustrative examples
The proposed methodology, including the B&C solver and the interior point method for OPF with dispatchable load under the
ZIP model, is implemented and tested using two illustrative examples in MATLAB 2013a on a personal computer with a 3.4 GHz i7 processor and 6 GB random access memory.

Example I is RTS 24-bus system with 17 load buses. It is assumed that the load in each load bus is fed equally by ten feeders (i.e. consists of ten identical load increments). The initial state of the load restoration stage, including the status, outputs, and $P_{ri}$ of generators, is given by the algorithm in [10]. This case shows that the computational complexity of the MINLR model is extremely high, even for tiny power systems.

Example II is IEEE 118-bus system with 91 load buses. It is assumed that the load in each load bus is fed equally by ten feeders. This case assumes that 637 load increments (70%) are interrupted. The initial state of this case is defined as a scale-down load level by 70% in each load bus with some offline generators and branches. Power flow calculation is applied to obtain this initial state. The BLB method succeeds in establishing a restoration plan considering the cold load effect, whereas the other two described in Section 3 fail to do so, even with all the aforementioned cutting planes.

4.1 Example I: RTS 24-bus system

The RTS 24-bus system includes 34 branches, 1 reactor, and 1 synchronous condenser. The hydro units on bus 22 serve as the blackstart resources. Other generating units are treated as non-blackstart units. The system data are taken from [35].

To clearly demonstrate the proposed methodology, it is assumed that each generation bus consists of three identical generating units. When the loss of energy contingencies are considered, the generator bus will lose 1/3 of the total capacity. The aggregate parameters and the initial states of the units, and the energised branches in the initial state, are shown in Table 1 and Fig. 3, respectively.

The computational settings are as follows. The bus voltages are restricted between 0.95 and 1.05 pu. Let $I_{i} = 1, \epsilon = 0.05, \beta = 1, a_{i} = 0, b_{i} = 0, c_{i} = 1.0, \tau = 10$ min, and $\rho_{i} = 5\%$.

We show the first stage as an example to investigate the computational complexity of the MINLR model. As shown in Table 2, the cut planes significantly reduce the computational complexity, enabling a reasonable central processing unit (CPU) time to establish a complete load restoration plan.

The complete restoration plan is achieved by using BLB and all the cuts described in Section 3. This plan consists of 23 stages. The estimated duration of the restoration process is 333.2 min. The total number of nodes in B&C trees of all these 23 stages reaches 886. The CPU time to compute this plan is 66 s. The real power output of generation buses, reserve levels, and the restored load level of selected load buses are shown in Figs. 4a, c, and 5, respectively. If not considering reserve constraints, the real power outputs and the reserve levels are as in Figs. 4b and d, respectively.

As in Fig. 4b, if the reserve constraints are not considered the outputs of generating units become uneven. Although load restoration duration without reserve constraints will drop to 326 min, this restoration process cannot survive the loss of energy contingency due to insufficient (negative) reserve, as in Fig. 4d. By contrast, with reserve constraints, the responsive reserve constraints become binding from 220 min on (see Fig. 4c). The power system can survive the largest loss of energy contingency even relying merely on the governor response.

Network operations are conducted as necessary when each stage begins, as listed in Table 3. The condenser on bus 10 is dispatched as a reactive source after it is put back on. The reactor on bus 6 is indispensable once the path 6–10 is energised. In stage 14 and thereafter, the reactor is modelled as a static shunt in the admittance matrices. The secure dispatch of network operations can be found in [20, 22]. The coordination among TO, DOs, and GOs, taking stage 14 as an example, is listed in Table 4.

### Table 3 Network operation at beginning of stages

<table>
<thead>
<tr>
<th>Stage</th>
<th>Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>energise path 9–11</td>
</tr>
<tr>
<td>9</td>
<td>energise path 1–2</td>
</tr>
<tr>
<td>10</td>
<td>energise path 16–19–20–23</td>
</tr>
<tr>
<td>12</td>
<td>energise path 1–5–10, put the condenser back on</td>
</tr>
<tr>
<td>14</td>
<td>energise path 2–6–10, put the reactor back on</td>
</tr>
<tr>
<td>15</td>
<td>energise path 9–12–13</td>
</tr>
<tr>
<td>21</td>
<td>energise path 10–12–23</td>
</tr>
<tr>
<td>22</td>
<td>energise path 15–21–22</td>
</tr>
</tbody>
</table>

### Table 4 Coordination of TO, DOs and GOs at stage 14

<table>
<thead>
<tr>
<th>Stage</th>
<th>TO actions</th>
<th>DO actions</th>
<th>GO active set point changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>energise path 2–6–10, put the reactor back on</td>
<td>restore to 40% load of bus 6</td>
<td>gen 22: 2.325 pu–2.235 pu</td>
</tr>
<tr>
<td></td>
<td></td>
<td>restore to 100% load of bus 8</td>
<td>gen 18: 3.310 pu–3.300 pu</td>
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<td></td>
<td></td>
<td>restore to 10% load of bus 16</td>
<td>gen 16: 1.517 pu–1.537 pu</td>
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<tr>
<td></td>
<td></td>
<td>restore to 20% load of bus 20</td>
<td>gen 1: 1.829 pu–1.906 pu</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>gen 2: 1.808 pu–1.905 pu</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>gen 15: 2.079 pu–2.139 pu</td>
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<td></td>
<td></td>
<td></td>
<td>gen 13: 3.411 pu–3.775 pu</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>gen 21: 1.828 pu–1.929 pu</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>gen 7: 1.521 pu–1.985 pu</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>gen 23: 2.435 pu–2.797 pu</td>
</tr>
</tbody>
</table>

Fig. 5 Restored loads on selected load bus with reserve constraints
Fig. 6  Initial state of IEEE 118-bus system

Fig. 7  Real power output and restored loads of IEEE 118-bus system

a Reserve level
b Aggregate restored load level
c Generator output curves
d Restored load levels
4.2 Example II: IEEE 118-bus system

The data of the IEEE 118-bus system are taken from [36, 37]. There are 22 units and 14 transmission paths are offline in the initial state, as shown in Fig. 6. The offline units will not be cranked and are therefore non-dispatchable during the load restoration in this case. Part of the offline transmission paths will be restored during the load restoration as necessary.

The computational settings are as follows. The bus voltages are restricted to 0.88–1.08 pu. Let $I_1 = 1$, $r = 0.05$, $b_f = 4$, $a_f = 0.4$, $b_i = 0.3$, $c_i = 0.3$, $\tau = 10$ min, $t_p = 0$ min, and $\rho_i = 5\%$. At each stage, the B&C solver is terminated if the number of nodes in the B&C tree at this stage reaches 1000.

A load restoration plan is successfully established using the BLB with the cutting planes mentioned in Section 3. This plan consists of 34 stages. The estimated duration of the restoration process is 236.6 min. The total number of nodes in B&C trees reaches 1451. The CPU time to figure out this plan is 17 min. The reserve levels, the aggregate restored load level, the real power output of selected generating units, and the restore load level on selected load buses, are shown in Fig. 7.

The computation is also carried out to establish a complete load restoration plan considering the load priorities. The load priorities are generated as random integers between 1 and 4. This new plan consists of 34 stages. The estimated duration of the restoration process is 239.1 min. The total number of nodes in B&C trees reaches 1460. The CPU time to figure out this plan is 28 min. The restored load curves of selected load buses are shown in Fig. 8.

4.3 Discussion of the above examples

(i) The restoration plan provides GOs with power set points for frequency control, and provides TOs with reactive power set points for voltage control. The load pickup amount will be the baseline for DOs to restore service.

(ii) To emphasise the ramping time of units, the time for communication/interaction, and for the execution within power plants/substations, are not counted in this paper. It can be considered by adding extra time duration between stages.

(iii) The merit of BLB results from the fact that it selects branching variables regardless of the value of $\tilde{u}_i$. By contrast, general-purpose branching methods fail to identify the important branching variable if some $\tilde{u}_i$ tends to 0 (it is the case in the early steps of load restoration) while others tend to 1.

(iv) The load pickup amounts on load buses are ready for dynamic simulation, which can be considered in future work. If the dynamic security constraint is violated, additional cutting planes can be generated by sensitivity analysis and incorporated into the MINLR model to mitigate this violation.

(v) The proposed methodology does not account for the optimality of the restoration plan over the entire load restoration phase. On one hand, the computational burden with such consideration is not affordable. On the other hand, the restoration process is difficult to be accurately predicted. Accordingly, the model parameters of MINLR should be updated to reflect the actual restoration process.

5 Conclusions

It is critical to coordinate generation and load pickup to ensure reliability during the load restoration process.

To facilitate this coordination, the work done in this paper is as follows. (i) This paper proposes a methodology associated with the MINLR model to establish load restoration plans considering reserve constraints, frequency security constraints, and steady-state constraints of power systems. This methodology can aid DOs to restore service to customers, and aid GOs/TOs to conduct frequency/voltage control. It also helps GOs maintain adequate reserve levels among generating units. (ii) An efficient B&C solver is constructed to establish complete load restoration plans within reasonable time. The applicability of cutting planes to solve MINLR models is proven.

Case studies with up to 637 interrupted load increments show that this methodology can be efficiently applied to aid the operators pickup large load increments within the TO’s islands, guarding against the loss of energy contingencies.

6 Acknowledgments

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7 References

36 IEEE 118-bus generator data. Available at http://www.motor.ece.iit.edu/data/JEAS_IEEE118.doc