Advanced power system partitioning method for fast and reliable restoration: toward a self-healing power grid

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Abstract: The recovery of power system after a large area blackout is a critical task. To speed up the recovery process in a power grid with multiple black-start units, it would be beneficial to partition the system into several islands and initiate the parallel self-healing process independently. This study presents an effective network partitioning algorithm based on the mixed-integer programming technique and considering the restoration process within each island. The proposed approach incorporates several criteria such as self-healing time, network observability, load pickup capability, and voltage stability limits. It can quickly provide multiple partitioning schemes for system operators to choose based on different requirements. Experimental results are provided to demonstrate the effectiveness of the proposed approach for IEEE 39-bus and IEEE 118-bus standard test systems. Also, the sensitivity of the partitioning solution with respect to the various parameters is presented and discussed. Ultimately, the advantage of the proposed method is demonstrated through the comparison with other references.

1 Introduction

Power systems have been operated under stressed conditions due to the rapid growth of electricity demand. This makes the system more vulnerable to cascading failures, which could lead to a widespread blackout. Despite all efforts to enhance power grids’ resilience through various preventive and corrective actions, the occurrence of large area blackout is still inevitable. Indeed, power industries around the world have witnessed several blackouts as a consequence of natural disasters [1]. As a key component in a self-healing smart grid, efficient recovery actions are critical in both planning and implementation phases to reduce the social and economic costs of power outages.

The restoration process brings the system back to its normal state. In the bottom-up restoration approach [2], restorative actions can be initiated after the blackout incidence. The first stage is to assess the post-outage conditions of system components, as well as the availability of generation sources and transmission paths. On the basis of the system topology and locations of black-start and non-black-start units (BSUs and NBSUs), system operators may prefer to partition the bulk power system into smaller islands for parallel restoration. After the partition, the recovery process is executed in each island independently and simultaneously, which will remarkably shorten the overall recovery time. Power system partitioning can be applied to either prevent a cascading failure leading to a wide-area blackout or expedite the recovery process by enabling parallel restoration actions. The first case is also referred as controlled islanding, when the preventive control actions failed to avert the power system from entering to an emergency state, whereas the second case is applied after the occurrence of a widespread blackout, which is the main concern of this paper.

The partitioning problem can be modelled as a multi-objective optimisation problem, which determines appropriate partitioning points to divide the large area into multiple islands with a similar restoration capability. Each island has sufficient generators, loads, and measurement devices while satisfying a set of operational constraints. Various partitioning methods have been proposed in the literature. A BS zone partitioning algorithm based on the fuzzy clustering approach [3] and tabu search [4] have been developed. In [5], a recovery time index was defined to quantify the disparity of each subsystems’ restoration time. It considers the electrical distance as a characteristic indicator to describe the strength of electrical connection between two nodes. Graph-theory and mathematical programming have been applied in developing efficient partitioning methods.

First, graph-theory-based techniques have been used to model the topological and electrical characteristics of power systems. A novel sectionalising strategy based on the un-normalised spectral clustering method was introduced in [6]. A two-step network partitioning strategy for parallel system restoration was proposed in [7]. To assign each NBSU to a proper island, a unit grouping model was developed to find the shortest path between BSUs and NBSUs in each island. A graph-theory-based sectionalising method was proposed in [8] based on the cut-set matrix. It provides the short list of sectionalising strategies for system operators to deploy. A constrained spectral clustering-based network partitioning approach was introduced in [9]. In this approach, a weighted graph was constructed using the electrical distance of power network. The objective is to maximise electrical cohesiveness within the islands, or equivalently, creating islands with the strongly connected lines inside and weak external connections.

Second, mathematical programming methods provide advanced modelling and solution algorithms in the network partitioning problem. In [10], a bi-level programming approach was proposed to solve the sectionalising problem with the objective of minimising the outage duration of the critical loads. A wide-area measurement system-based sectionalising method was proposed in [11]. They addressed the problem of observability of each formed island by integrating the observability constraints in the proposed algorithm. A sectionalising strategy based on the ordered binary decision diagrams (OBDDs) for parallel power system restoration was proposed [12]. They introduced a three-step OBDD search method to improve solution efficiency. Voltage stability was also checked by simulation of critical contingencies.

The network partitioning methods have been extensively studied in the aforementioned literature and the proposed approaches can generate various partitioning solutions. However, more comprehensive and effective approach is needed which can offer a deeper insight for decision makers to prioritise the solutions based on their qualities. To this end, a new network partitioning problem formulation accounting for the restoration sequence is proposed whose objective function reflects the restoration time of NBSUs and loads. Also, a number of linear constraints will be introduced, so as to ensure the quality and feasibility of the resulting islands. Specifically, the observability, load pickup...
capability, and voltage stability constraints are derived and integrated into the network partitioning optimisation problem. The proposed approach can generate a list of solutions from which the best feasible solution can be derived and implemented in practise.

2 Parallel restoration concepts

The recovery process after a blackout is consisted of several stages [13–15]: preparation and planning, BSU start-up, transmission lines energisation, supplying cranking power to start NBSUs, and load pickup. This work focuses on the first stage of the recovery process. At this stage, system operators obtain the current status of grid including the availability of transmission lines, buses, BSUs, and NBSUs to prepare an effective restoration plan. For instance in [8], Quiros-Tortos et al. emphasised on collecting the information related to the system topology as well as the availability of its elements right after blackout including availability of BSUs and interconnection assistance, the status of the non-BSU, the status of the lines and circuit breakers, and load levels. Moreover, the Pennsylvania–New Jersey–Maryland interconnection restoration manual [16] discussed the complete assessment of post-blackout system for determining the system status. When multiple BSUs are distributed in different geographical locations, system operators may initiate the parallel restoration strategy to speed up the power system recovery process. In this approach, the bulk power system can be split into smaller islands in which a bottom-up restoration strategy can be performed concurrently and independently.

Fig. 1 highlights the network partitioning problem for parallel restoration. First, the boundary transmission lines will be determined to isolate the islands. Each island incorporates at least one BSU, one or multiple NBSUs, and loads with various priorities. Then, BSUs provide the cranking power for NBSUs through energising the shortest transmission lines between them. Next, load buses should be energised in a priority order. Loads with high priority must be restored first to mitigate the impact of power outage on hospitals, data-centres etc. Ultimately, different islands can be re-connected and synchronised through a set of tie-line circuit breakers to form a bulk power grid.

One critical requirement for a successful parallel restoration is to ensure system observability before and after separation. Phasor measurement units (PMUs) are placed in a bulk power system to render the whole system observable. The PMU placement problem has been extensively investigated in [17–23] to find the minimum number and optimal locations of PMUs. The integer programming [17] and simulated annealing [18] techniques have been implemented to find the minimum number of PMUs to make the system observable. These methods only guarantee the system observability during the normal operation of power grid or under a specific network topology. Moreover, optimal PMU placements under the loss of communication channels, PMUs, and branch outages have been discussed in [19–21]. Optimal PMUs placement for power system restoration has been discussed in [22]. Optimal PMU placement considering controlled islanding and normal operation condition was proposed in [23]. After the formation of each island, a proper placement of PMUs can lead to a secure operation of islands by providing synchronised measurement signals for state estimators in control centres. Particularly, boundary buses through which transmission lines interconnecting two islands must be observable to guarantee a secure re-connection. This paper takes the optimal PMU placement as input, and also compares the impact of different PMU placement methods on the parallel restoration.

To achieve a self-dependent island, the following criteria must be fulfilled within each island:

• Each island should include at least one BSU together with one or several NBSUs [8, 9, 11].
• In each island, the total generation capability should be more than total load [7, 11, 12]. In other words, the maximum available generation should be greater than the maximum restorable load; therefore, the remaining capacity can be assumed as reserve.
• Generators and loads should be distributed among islands such that to obtain a balanced parallel restoration with an optimised overall restoration time.
• After island formation, it would be preferable to have all buses, particularly boundary buses, observable to facilitate synchronisation task when needed [8, 11].
• After dis-connection of the boundary lines, the voltage stability limits of other lines should not be violated [12]. Also, the power flows through the transmission lines should not exceed their thermal limits.

In the present work, the above requirements are addressed through defining an appropriate objective function and constraints.

3 Mathematical formulations of network partitioning problem

The objective function of partitioning problem is to minimise the outage duration of generators and loads as described in (1)

$$\min \sum_{s \in S} \left( \sum_{t \in T} t_{i}^{s} + \sum_{l \in L} a_{l}^{s} \right)$$

where integer variable $t_{i}^{s}$ is on time of generator $i$ in island $s$. Integer variable $a_{l}^{s}$ shows the energisation time of load $i$ in island $s$ with the priority of $a_{i}$. Sets of generators, loads, and islands are $I$, $L$, and $S$, respectively.

3.1 Restoration time and islanding constraints

Integer variables $t_{i}^{s}$ and $a_{l}^{s}$ are defined in (2) and (3), where $u_{i}^{s}$ is a binary variable with 1 showing that generator $i$ is on at restoration time $t$, and 0 otherwise. Binary variable $u_{l}^{s}$ equals 1 only when the load bus $i$ is energised at restoration time $t$ and the load belongs to island $s$. $T$ denotes the set of restoration times

$$t_{i}^{s} \geq \sum_{t \in T} u_{i}^{s} \quad \forall i \in I, s \in S$$

$$a_{l}^{s} \geq \sum_{t \in T} u_{l}^{s} \quad \forall i \in L, s \in S$$

Constraints (4) and (5) assign each generator and load to only one island. Once assigned to a specific island, it will belong to that island for the entire restoration time

$$\sum_{t \in T} u_{i}^{s} \leq 1 \quad \forall i \in I, t \in T$$

$$u_{l}^{s} \leq u_{l}^{s} \quad \forall i \in I, t \in T, s \in S$$
The set of transmission lines is denoted by $K$.

\[
\begin{align*}
\sum_{i \in S} q^{l, s}_{\text{load}} & \leq 1 \quad \forall l \in L, t \in T \\
q^{l, s}_{\text{load}} & \geq 0 \quad \forall l \in L, t \in T, s \in S
\end{align*}
\] (5)

### 3.2 Bus and line energisation constraints

These constraints are defined to determine the binary variables $a_{lbus}$ and $a_{line}$ at each restoration time and in each island. Constraint (6) denotes that NSBUSs can become online at least one restoration time after the energisation of their corresponding generation buses, $b_i$. For the sake of simplicity, we neglect the start-up duration of generators, which can be simply added to the problem formulation. In (7), loads can be restored one restoration time after the energisation of their respective load buses $b_i$. Constraint (8) shows that transmission line $mn$ remains de-energised if the buses at both ends are de-energised. Constraint (9) denotes that each transmission line $mn$ can be energised one restoration time after the energisation of its connecting bus $m$ or $n$ [24].

\[
\begin{align*}
q^{\text{on}}_{\text{gen}} & \leq a_{lbus} \quad \forall l \in \text{NSBUS}, b_i \in B, t \in T, s \in S \\
a_{lbus} & \leq a_{line} \quad \forall l \in L, b_i \in B, t \in T, s \in S \\
q^{\text{on}}_{\text{line}} & \leq a_{lbus} + a_{line} \quad \forall mn \in K, (m, n) \in B, t \in T, s \in S \\
a_{lbus} & \leq a_{line} \quad \forall mn \in K, (m, n) \in B, t \in T, s \in S
\end{align*}
\] (6-9)

Constraints (2)-(9) are checked for each restoration time and dealt with the sequential recovery process in each island, whereas the following constraints are only checked at $t = T$, after each island formation. Thus, for notation brevity, $t$ is dropped from these constraints.

### 3.3 Load and generation balance constraints

Total generation capability in each island should be greater than the total restorable load

\[
\begin{align*}
\sum_{i \in I} p^{\text{gen}}_{\text{on}, i} b_{\text{bus}, i} & \geq \sum_{i \in L} p^{\text{load}}_{\text{on}, i} b_{\text{bus}, i} \quad \forall (b_i, s) \in B, s \in S
\end{align*}
\] (10)

where $p_{\text{gen}}^{\text{on}, i}$ denotes maximum generation capability of unit $i$ located at generation bus $b_i$ and $p_{\text{load}}^{\text{on}, s}$ shows the maximum restorable load.

### 3.4 Power balance and load flow constraints

In constraints (11) and (12), active and reactive power balances are enforced in each island, where $p_{lbus}^{K}$ and $q_{lbus}^{K}$ are scheduled active and reactive powers of each generator in each island. Variables $p_{\text{flow}}^{K}$ and $q_{\text{flow}}^{K}$ are active and reactive power flows of transmission line $k$ connecting buses $m$ and $n$, which is expressed in (19) and (20), respectively.

\[
\begin{align*}
\sum_{i \in I} p^{l}_{\text{gen}} - \sum_{k \in L} p_{\text{flow}}^{l} & = \sum_{k \in K} p_{\text{flow}}^{l, s} \quad \forall s \in S, n \neq m \\
\sum_{i \in I} q^{l}_{\text{gen}} - \sum_{k \in L} q_{\text{flow}}^{l} & = \sum_{k \in K} q_{\text{flow}}^{l, s} \quad \forall s \in S, n \neq m
\end{align*}
\] (11-12)

In constraint (13), the BSUs are considered as the slack generators in each island. Maximum and minimum voltage limits are enforced in (14). The upper and lower limits of active and reactive power of generators and loads are enforced in (15)-(18).

\[
\begin{align*}
\theta_{s}^b & = 0 \quad \forall b_i \in B, i \in I_{\text{BSU}} \\
V_{\text{min}} & \leq V_{mn} \leq V_{\text{max}} \quad \forall m \in B \\
0 & \leq P_{\text{gen}}^{s} \leq P_{\text{gen}}^{\text{max}} \quad \forall i, s \in S \\
0 & \leq q_{\text{gen}}^{s} \leq q_{\text{gen}}^{\text{max}} \quad \forall i, b_i \in B, s \in S \\
0 & \leq P_{\text{load}}^{s} \leq P_{\text{load}}^{\text{max}} \quad \forall l \in L, b_i \in B, s \in S \\
0 & \leq q_{\text{load}}^{s} \leq q_{\text{load}}^{\text{max}} \quad \forall l \in L, b_i \in B, s \in S
\end{align*}
\] (13-18)

Linearised model of AC power flow equations are presented in (19) and (20), where $d_{\phi mn}$ is the cosine function approximation by a set of linear functions (more details can be found in [25]). In constraints (19) and (20), $V^s$ denotes voltage of bus $n$ and $\theta^s$ is angle difference between buses $m$ and $n$. Parameters $g_{mn}$, $b_{mn}$, and $b_{mn}^o$ are conductance, susceptance, and shunt susceptance of transmission line between bus $m$ and $n$.

\[
P_{\text{flow}}^{\text{mn}} = (2V^m - 1)g_{mn} - (V^m + V^n + d_{\phi mn} - 2)d_{\phi mn} \quad \forall (n, m) \in B, mn \in K, s \neq m
\] (19)

(see (20))

### 3.5 Thermal limit constraint

The maximum active power flows through the line in each island should be restricted to the line thermal capacity

\[
P_{\text{flow}}^{\text{mn}} \leq P_{\text{flow}}^{\text{mn}, s} \quad \forall mn \in K, s \in S
\] (21)

### 3.6 Observability constraints

Considering the pre-specified PMU locations, to ensure the observability of each island, the linear observability constraints are developed and added to the partitioning problem. The impact on the partitioning objective function will be discussed in this paper. Four different PMU placement methods are considered including the normal operating conditions and single branch outage condition, each of which is studied with/without considering zero-injection bus (ZIB) effect.

We define an observability constraint for each resulting island $s$ as

\[
\alpha_{\text{obs}} \leq \sum_{i \in I} a_{\text{bus}}^{l, i} + \sum_{i \in I} b_i a_{\text{bus}}^{l, i} + \sum_{i \in I} c_i a_{\text{bus}}^{l, i} \quad \forall s \in S
\] (22)

where parameter $\alpha_{\text{obs}}$ shows the degree of observability and varies between 0 and 1, which can be set by system operators. Parameter $a_{\text{bus}}$, ranging from 2 and 10, reflects the importance of the specific bus. For load buses, it equals to the load priority, $a_{\text{bus}}$ multiplied by 10; and for generator buses, it takes the value of 10; otherwise, it takes the lowest priority of 2. Binary variable $a_{\text{bus}}^{l, i}$ is 1 if bus $b_i$ is observable, and 0 otherwise. In PMU placement schemes 1 and 3 (can be referred in Table 1), $a_{\text{bus}}^{l, i}$ can be derived from the inequality constraint (23).
where binary parameter PMU\(^n\) indicates the PMU status with 1 showing that bus \(n\) has an installed PMU, and 0 otherwise. Constraint (23) implies that a specific bus \(n\) will remain observable after partitioning either through its own PMU or from the neighbouring buses having the installed PMUs. This data will be available for a given network which can be derived from the PMU placement schemes.

To incorporate ZIB effect, constraint (23) needs to be modified as (24). Now, a certain bus can be observable either through the neighbouring buses or the ZIBs incident to that bus. Binary decision variable \(h\)\(^nm\) reflects the effect of ZIBs incident to bus \(n\). Also, constraint (25) applies only to the ZIBs and enforces that only one bus incident to ZIB or the ZIB itself can be observable when the other buses are already observable. Parameter \(\alpha\)\(^nm\) is 1 if bus \(m\) is a ZIB, and 0 otherwise.

\[
w_{bus}^s \leq \sum_{nm \in K \atop m \neq n} u_{line}^{nm}\text{PMU}^n + \sum_{nm \in K \atop m \neq n} u_{line}^{nm} h_{nm}^m + \text{PMU}^n + h_{nm}^m \quad \forall n \in B, s \in S
\]  

(24)

\[
\sum_{nm \in K} u_{line}^{nm} h_{nm}^m + h_{nm}^m \leq \alpha_{nm} \quad \forall m \in B, s \in S
\]  

(25)

Note that constraints (24) and (25) contain the non-linear term \(u_{line}^{nm} h_{nm}^m\), which is the product of two binary variables. It can be linearised using (26), where \(p_{line}^{nm}\) is an auxiliary binary variable equals to the product of two binary variables \(u_{line}^{nm}\) and \(h_{nm}^m\).

\[
p_{line}^{nm} \leq h_{nm}^m
\]  

(26)

3.7 Load pickup constraints

Objective function (1) ensures a quick energisation of generation buses as well as loads with high priorities. This helps to expedite the recovery process, but it would not be a sufficient condition for the whole recovery period. For example, after load energisation, the load pickup capability should be distributed equally among different islands to speed up the overall recovery process. Therefore, an appropriate constraint should be incorporated into the partitioning problem to ensure a balanced recovery.

We define a new constraint to measure the load pickup capability of each island with respect to the total load pickup capability of the system in (27)

\[
\alpha_{p}^s \leq \frac{(\Delta P_{load}^s)(\sum_{s \in S} P_{load}^s)}{(\sum_{s \in S} \Delta P_{load}^s)} \quad \forall s \in S
\]  

(27)

where \(\alpha_{p}^s = 1\) indicates that the generation units are equally distributed. It would be beneficial to have \(\alpha_{p}^s = 1\) for all islands; however, it seems impossible in practise. Our aim is to distribute the NBUs, so that the load pickup capability is distributed with respect to the maximum restorable load in each island. In this way, islands with higher restorable load level acquire more load pickup capability. The parameter \(\alpha_{p}^s\) can be set by system operators.

To specify the value of \(\Delta P_{load}^s\) we need to compute the maximum frequency drop (nadir) after a load pickup step in each island. In this way, compared with the slow response and low inertia generation units, generators with higher inertia constant and faster active power ramping are capable of restoring larger blocks of loads. The general formulation of frequency dynamic in each island can be extracted from the swing equation (26) by neglecting the damping coefficient of load as (28)

\[
\frac{d\Delta f_s(t)}{dt} = \frac{f_0}{2H_{eq}^sB}(\Delta P_{in}^s - \Delta P_{load}^s) \quad \forall s \in S
\]  

(28)

where \(\Delta f_s\) (Hz) is the frequency deviation at the centre of inertia in island \(s\), \(H_{eq}^s\) denotes the total inertia in island \(s\), \(\Delta P_{in}^s\) (MW) and \(\Delta P_{load}^s\) (MW) represent changes in mechanical and electrical power following a load pickup value, and \(B\) is the base power. A load pickup will cause the frequency to decline at very first instances after a load pickup, due to the mismatch between mechanical and electrical powers. The minimum permissible frequency (namely frequency nadir) in each island should not go below a certain limit.

Otherwise, it will result in unfavourable under-frequency load shedding activation. Thus, the maximum load pickup step must be specified to assure that frequency nadir will not violate that limit. The frequency nadir in each island is a function of total inertia, governors’ ramping rates, and amount of load being restored \(\Delta f_s \approx g(H_{eq}^s, R_{eq}^s, \Delta P_{load}^s)\). As in load pickup stage, our emphasis is on the maximum drop of frequency (nadir) and the detailed dynamic model of generator has a very complex and non-linear nature. Therefore, a simplified model proposed in [27] is adopted in this work. Note that, we assume that the governors’ dead band is equal to zero. The frequency nadir time can be calculated from (29)

\[
\frac{d\Delta f_s(t_{load})}{dt} = 0 \Rightarrow t_{load} = \frac{\Delta P_{load}^s}{R_{eq}^s} \quad \forall s \in S
\]  

(29)

where \(R_{eq}^s\) (MW/s) is the sum of the ramping rate of all generators in island \(s\). After finding the frequency nadir time, (30) shows the relationship between frequency nadir, inertia, governors’ ramping rates, and the size of the contingency, which is equal to the total load pickup. With \(\Delta f(0) = 0\), (30) can be rearranged for \(\Delta P_{load}^s\) which yields (31)

\[
\Delta f(t_{load}) - \Delta f(0) = \frac{f_0}{2H_{eq}^sB} \frac{(\Delta P_{load}^s)^2}{2R_{eq}^s} \quad \forall s \in S
\]  

(30)

\[
\Delta P_{load}^s = D\sqrt{(4S_b\Delta f)/(f_0)} \quad \forall s \in S
\]  

(31)

where \(D = \sqrt{(4S_b\Delta f)/(f_0)}\) will become a constant value after choosing the desired value of frequency nadir \(\Delta f_s\) (Hz) in each island. Equation (31) implies that the value of \(R_{eq}^s\) will reflect the load pickup capability of island \(s\) which is related to the characteristics of generation units in that island.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Locations of PMUs in 39-bus under different PMU placement schemes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal case</td>
<td>PMU location (bus number)</td>
</tr>
<tr>
<td>scheme 1 – ignore ZIB</td>
<td>2, 6, 9, 10, 13, 14, 17, 19, 22, 23, 29, 34, 37</td>
</tr>
<tr>
<td>scheme 2 – include ZIB</td>
<td>3, 8, 13, 16, 20, 23, 25, 29</td>
</tr>
<tr>
<td>one line outage</td>
<td>PMU location (bus number)</td>
</tr>
<tr>
<td>scheme 3 – ignore ZIB</td>
<td>1, 3, 5, 7, 9, 11, 13, 15, 17, 20, 21, 24, 26, 28, 30–38</td>
</tr>
<tr>
<td>scheme 4 – include ZIB</td>
<td>3, 8, 16, 24, 26, 28, 30–38</td>
</tr>
</tbody>
</table>

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3.8 Voltage stability constraint

It is imperative to measure the voltage stability margin after splitting the bulk network into several islands. Thus, we propose a linear voltage stability constraint to be integrated into the partitioning problem to maintain certain level of stability margin after partitioning. There are different methods used in the previous works on the subject of voltage stability assessment and reported various criteria as the voltage stability indicators. Among those works, line stability margins have been introduced in [28, 29] to determine the weakest lines in the system. Here, by taking the advantage of linearised AC power flow and the receiving end reactive power equation of transmission line, a linear voltage stability constraint is proposed.

Assume \( \pi \) model of transmission lines connected two buses \( m \) and \( n \), as shown in Fig. 2. Sending and receiving buses voltages and angles are depicted by \( V^m \) and \( V^n \), \( Q^m,s \) shows the line reactive power before the charging capacitor at the receiving end. Then, the current flowing between buses \( m \) and \( n \) is written as

\[
\begin{align*}
V^m - V^n &= \frac{R^m}{\sqrt{v^m} + jX^m} \left( P^m,s + jQ^m,s \right) \\
&= \left( \frac{R^m P^m,s + jX^m Q^m,s}{\sqrt{v^m}} \right)
\end{align*}
\]

where \( R^m \) is the line resistance and \( X^m \) is the line reactance. Separating the real and imaginary parts of (32) gives

\[
\begin{align*}
V^m V^n \cos(\theta^m - \theta^n) - (V^m)^2 &= R^m P^m,s + X^m Q^m,s \\
-V^m V^n \sin(\theta^m - \theta^n) &= X^m P^m,s - R^m Q^m,s
\end{align*}
\]

Rearranging the imaginary part for \( P^m,s \) gives

\[
P^m,s = \frac{R^m Q^m,s - V^m V^n \sin(\theta^m - \theta^n)}{X^m}
\]

Substituting \( P^m,s \) from (34) into real part of (33) gives an equation of order two of \( V^n \)

\[
(V^n)^2 - V^m V^n \cos(\theta^m - \theta^n) + \frac{(R^m)^2}{X^m} Q^m,s = 0
\]

The condition for \( V^n \) to have at least one solution is

\[
\frac{4(Z^m)^2 Q^m,s X^m}{(V^n R^m \sin(\theta^m - \theta^n) + V^m X^m \cos(\theta^m - \theta^n))} \leq 1
\]

The left-hand side of the inequality (36) can be defined as the voltage stability index with the value between 0 and 1. The following approximations have been applied to derive a linear equation for voltage stability constraint: in transmission line usually \( X^m \gg R^m \) and \( \sin(\theta^m - \theta^n) \approx \theta^m - \theta^n \) is a very small value; therefore (36) can be approximated as:

\[
\alpha_{\text{vst}} = \frac{4(Z^m)^2 Q^m,s X^m}{(V^n R^m \sin(\theta^m - \theta^n) + V^m X^m \cos(\theta^m - \theta^n))} \leq 1
\]

To linearise (37), the following equation and approximation can be applied:

\[
(V^n)^2 (\cos(\theta^m - \theta^n))^2 = (V^n)^2 \frac{1 + \cos 2(\theta^m - \theta^n)}{2}
\]

\[
\frac{1}{4} \cos 2(\theta^m - \theta^n) - \frac{3}{7}
\]

Applying the above approximation, the voltage stability constraint can be written in (40). Note that, the voltage stability parameter \( \alpha_{\text{vst}} \) indicates the closeness of transmission lines to its collapse point after partitioning. It can be set to the desired value (non-zero) or the value of this parameter should be kept <1 to maintain secure operation of power grid. The closer to 1 indicates that the particular line is closer to its instability point

\[
V^n \geq \frac{2(Z^m)^2 Q^m,s X^m}{\alpha_{\text{vst}}} - \frac{\cos 2(\theta^m - \theta^n)}{4} + \frac{3}{4}
\]

As stated in the linearised AC load flow constraint, the cosine function can be approximated by a set of linear functions, and \( Q^m,s \) can be obtained from the load flow equations. Constraint (40) sets the lower value of sending end voltage whose upper level is enforced in (14).

4.3 Partitioning optimisation problem

The proposed network partitioning optimisation problem formulation with/without considering the ZIB effect can be summarised as

\[
\text{minimise} \sum_{s \in S} \left( \sum_{i \in I} \bar{\alpha}_{i,s}^l + \sum_{t \in L} \alpha_{s,t}^l \right)
\]

subject to

\[
(2) - (22), (27), (31), (40), \begin{cases} (23) & \text{without ZIB} \\ (24) - (26) & \text{with ZIB} \end{cases} \text{for } \forall i \in I, (b, h) \in B, l \in L, mn \in K, (m, n) \in B, \alpha_{s,t}^l \in T, s \in S
\]

4. Numerical results

In this section, we present the experimental results to demonstrate the effectiveness of the proposed partitioning method. We have tested the proposed algorithm on two IEEE standard cases: 39-bus and 118-bus systems. The data of each test case were taken from [30], assuming that voltage limits at all buses vary between 0.95 and 1.05 pu. The minimum frequency decline after a load pickup step is restricted to 59.6 Hz. Our model was implemented using C++ with IBM ILOG CPLEX 12.6 on a personal computer with Intel Core™ i5 central processing unit at 3.30 GHz and 8 GB random access memory.

4.1 IEEE 39-bus system simulation results and analysis

IEEE 39-bus system contains ten generators with nominal generating capacity of 6150 MW and maximum restorable load of 5950 MW. Generators G10 and G3 are the BSUs and the other generators serve restoration as the NBSUs. We apply the proposed partitioning method to split the system into two islands. Table 1 indicates the optimal locations of PMUs for different schemes. In scheme 3, where a single line outage contingency has been
can observe that the maximum values of $d_{obs}$ using scheme 1 can reach to 0.94 and 1.0 for islands 1 and 2, respectively. However, this will bring higher objective function value, which causes more recovery time of the whole system. It is important to note that two infeasible regions can be observed in Fig. 3. The infeasible region on the left-hand side is caused by the shortage of reactive power sources, which results in a non-convergent load flow, whereas the infeasible solution on the right-hand side implies that adopting PMU placement scheme 1 cannot render both islands fully observable.

4.1.2 Sensitivity to load pickup capability: Fig. 4 shows the sensitivity of four partitioning solutions from the solution pool with respect to parameter $d_p$. As stated in Section 3.7, when $d_p$ in each island becomes close to 1, various islands will be recovered uniformly. In solution 1, the load pickup capabilities of islands 1 and 2 are 0.80 and 1.12, respectively. In the second solution, these values become closer to each other, showing an improvement over solution 1 with the cost of increasing the objective function value from 179.3 to 180.3. In solution 3, the best load pickup capabilities are obtained for both islands and close to value 1. However, the objective function increases to 203.4. In solution 4, the values of load pickup capabilities are diverged and the objective function decreases to 180.3. One should note that though in solution 3 the higher objective function was obtained, it will provide a uniform load restoration in two islands which reduces the total load restoration time.

4.1.3 Voltage stability analysis: The linear voltage stability constraint presented in Section 3.8 is evaluated here. We set the maximum value of $d_{stability}$ to 0.80 for all lines. When this parameter becomes closer to unity, the respective line is being operated closer to its instability point, and a sudden voltage drop may occur by the reactive power variation. Fig. 5 shows the values of $d_{stability}$ in the candidate lines based on their lower stability margins. The result shows that after applying the voltage stability constraint, the partitioning solutions will maintain the desired stability margin for all lines.

4.1.4 Optimal partitioning solution: Fig. 6 indicates the optimal partitioning solution of IEEE 39-bus system fulfilling all the required constraints, specifically, parameters $d_p = 0.9$, $d_{obs} = 0.92$, and $d_{stability} = 0.8$ ($\forall s \in S, \forall v \in K$). The red lines signify the boundary lines, the locations of BSU1 and BSU2 are also indicated. Assuming that the BSUs will be on at $t = 1$ pu, the on time of generation units are listed in Table 3.

4.2 IEEE 118-bus system simulation results and analysis

We implemented the developed partitioning algorithm on larger test network, IEEE 118-bus system. It contains 54 generators including three BSUs placed at buses 31, 49, and 82. Thus, the target is to split the system into three islands. The optimal PMU placement solution is indicated for different schemes in Table 4.

Table 2  Partitioning solutions for IEEE 39-bus system with different $d_{obs}$

<table>
<thead>
<tr>
<th>Case</th>
<th>Objective function</th>
<th>Scheme</th>
<th>$d_{obs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Infeasible</td>
<td>scheme 1</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>scheme 2</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>scheme 3</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>scheme 4</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>180.3</td>
<td>scheme 1</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>scheme 2</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>scheme 3</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>scheme 4</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Similar to former test case, scheme 3 brings the largest number of PMUs and scheme 2 requires the smallest number of PMUs.

4.2.1 Sensitivity to observability: Table 5 reports the values of $\bar{\alpha}_{obs}$ for different schemes with respect to different values of $\alpha_p$. Note that the observability of each island varies for different schemes as the value of $\alpha_p$ changes. One can observe that with PMU placement scheme 4, the observability of three islands is ensured under different $\alpha_p$ values, whereas for all other PMU placement schemes, the full observability cannot be achieved. Particularly, we observe the improvement in some cases while the deterioration in other cases. Note that the results have been reported for the best feasible solution in terms of the objective function. That is, when the value of $\bar{\alpha}_p$ is set to 0.9, according to the constraint (27), it forces the minimum value of load pickup capability to become $>0.90$. Thus, when the value of $\bar{\alpha}_p$ increases, the new values of $\bar{\alpha}_{obs}$ will also be valid for the case of $\alpha_p = 0.9$, with the cost of higher objective function or longer restoration time.

4.2.2 Sensitivity to load pickup capability considering different PMU placement schemes: Fig. 7 shows the sensitivity of objective function with respect to the different load pickup capabilities under various PMU placement schemes. It is important to note that the degree of the observability in each scheme can be obtained from Table 5. For instance, when $\bar{\alpha}_p = 0.9$, adopting scheme 2 causing the highest objective function (i.e. the highest restoration time) and the lowest degree of observability according to Table 5. The other schemes give almost the similar objective function; however, schemes 1 and 4 outperform scheme 3 from the observability viewpoint. From Fig. 7, some observations are summarised as follows: (i) by improving the load pickup capability, the objective function increases. In other words, the energisation time of the loads and on time of generators will increase. On the other hand, having better load pickup capability reduces the time of load restoration in each island. Additionally, it brings a uniform restoration of three islands. (ii) Among different schemes, scheme 4 is the best choice in terms of the objective function for various load pickup capabilities. (iii) Schemes 1–3 present the same objective functions for $\bar{\alpha}_p = 0.99$; however, scheme 3 outperforms schemes 1 and 2 from the observability viewpoint according to Table 5.
4.2.3 Voltage stability analysis: Similar to the IEEE 39-bus system, we set the $\alpha_{\text{vmn}} = 0.8$ and the voltage stability margins are evaluated for the candidate lines, as shown in Fig. 8. From Fig. 8, one can observe that transmission lines (54–56) and (80–79) are closer to the voltage stability limit with the values of 0.77 and 0.75, respectively. However, constraint (40) restricted the value to under the limit of 0.8.

4.2.4 Optimal partitioning solution: Fig. 9 shows the partitioning solution for IEEE 118-bus system fulfilling all the presented constraints, specifically, parameters $\alpha_p = 0.9$, $\alpha_{\text{vmn}} = 0.80$, and $\alpha_{\text{obs}} = 0.95$ ($\forall s \in S$, $mn \in K$). Also, the locations of BSUs and nine boundary lines are highlighted with red colour. Note that, when the values of aforementioned parameters change, the optimal solution will be changed as well, and a new partitioning solution can be generated. Thus, our model can provide more flexibility to explore a broad range of solutions to satisfy various requirements.

4.3 Comparison to the prior works
To further assess the effectiveness of the proposed algorithm, we compare the partitioning solutions obtained from this work with the other references. In particular, IEEE 39-bus and IEEE 118-bus test cases are studied, where IEEE 39-bus is split into four islands [11] and three islands [7], and IEEE 118-bus is split into two islands [12]. Ultimately, objective function and different indices introduced in this paper are utilised to indicate the advantage of the proposed method. It should be noted that to perform a comparison between different approaches, we have extracted the partitioning solutions presented in [7, 11, 12], and calculated the objective function (41) based on the proposed solutions.

4.3.1 IEEE 39-bus four islands case: This problem has been solved in [11] assuming that BSUs are located at buses 30, 33, 36, and 37. However, the aforementioned work only presents a single solution without discussing the solution’s quality. Also, it proposed to measure the weighted observability percentage of resulting islands to ensure the observability of all islands. The results of partitioning problem are compared with the method proposed in this work, as shown in Table 6. It can be seen that the full observability has been achieved in both methods. The second column shows the objective function of the proposed method [as expressed in (41)] which is smaller than the one calculated based on the partitioning solution of [11]. As stated before, the objective function of the partitioning problem reflects the restoration time of total system. In other words, the overall restoration time will be reduced using the proposed method. Furthermore, to explore a broad range of solutions and show the flexibility of the proposed model, Fig. 10a indicates various partitioning solutions with respect to the desired load pickup capability index. As shown in Fig. 10a, in order to achieve more equal load pickup capability, resulting in the uniform restoration of different islands, the
objective function of the proposed model will increase. It should be noted that the proposed method generates partitioning solutions for \(0.6 \leq \alpha_p^* \leq 0.9\), whereas adopting the partitioning solution in [11], the maximum achievable \(\alpha_p^*\) is 0.7. As \(\alpha_p^*\) increases, it causes the solution infeasible, shown as the shaded region in Fig. 10a. Furthermore, when \(\alpha_p^* = 0.9\), our method yields smaller value for the objective function compared with [11] with \(\alpha_p^* = 0.7\).

### 4.3.2 IEEE 39-bus three islands case

This case has been studied in [7] in which BSUs are located at buses 30, 33, and 34. In Table 6a, objective function of the proposed method is compared with [7] under two cases. In case 1, the objective function becomes 159.3 which is smaller than the one obtained using partitioning solution proposed in [7]. In addition, the partitioning solution in [7] does not yield fully observable islands, whereas the proposed approach can give fully observable islands in case 2 with the cost of larger objective function, meaning the longer restoration time. Again, this shows that the proposed model can generate multiple solutions for any given conditions, which enables system operators to select the best one to satisfy the requirements. Fig. 10b depicts the sensitivity of the object function with respect to load pickup capability index. One also can observe the quality of solution, from load pickup capability standpoint, cannot be improved more than \(\alpha_p^* = 0.7\) in [7]. The shaded area shows the infeasible region where the solution proposed in [7] cannot be converged, whereas by adopting the proposed method, the quality of partitioning solution can be enhanced to \(\alpha_p^* = 0.95\) with smaller objective function.

### 4.3.3 IEEE 118-bus two islands case

This case has been studied in [12] in which BSUs are located at buses 31 and 87. Table 7 compares the objective function of the proposed model with [12]. One can observe that the proposed model gives the lower objective function with/without the fully observable islands. Two cases have been shown using the proposed model; case 1 has the lowest objective function (1266.6) without giving fully observable islands, whereas case 2 gives the higher objective function (1267.4) with the fully observable islands. One should note that in both cases, the objective functions have been improved with respect to [12]. That is, adopting the solution presented in [12] causes the longer restoration time without achieving fully observable islands. Furthermore, our study shows that the maximum value of \(\alpha_p^*\) to have a feasible solution is 0.75 in [12], whereas in our method, it can be increased to 0.95 without affecting the objective function. This obviously indicates the quality of solution obtained from the proposed model. The optimal partitioning solution of IEEE 118-bus system for \(\alpha_p^* \geq 0.95\) is shown in Fig. 11. The BSUs and boundary lines are highlighted red.

### Table 6a

Comparison between the proposed partitioning approach and [7, 11]

<table>
<thead>
<tr>
<th>Approach</th>
<th>Objective function</th>
<th>(\alpha_{\text{obs}})</th>
<th>Island 1</th>
<th>Island 2</th>
<th>Island 3</th>
<th>Island 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>[11]</td>
<td>161.3</td>
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<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>proposed method</td>
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<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

### Table 6b

<table>
<thead>
<tr>
<th>Approach</th>
<th>Objective function</th>
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<th>Island 2</th>
<th>Island 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>[7]</td>
<td>162.9</td>
<td>0.89</td>
<td>1.0</td>
<td>1.0</td>
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</tr>
<tr>
<td>proposed method</td>
<td>159.3</td>
<td>0.76</td>
<td>0.91</td>
<td>0.92</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>164.7</td>
<td></td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

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Fig. 10  Partitioning solutions with respect to the desired load pickup capability index

(a) Sensitivity of objective function with respect to the load pickup capability index in the proposed method and [11], (b) Sensitivity of objective function with respect to the load pickup capability index in the proposed method and [7].
5 Conclusions
As a major step toward the self-healing power grid, this paper investigated the network partitioning problem and developed a novel partitioning approach. We proposed to integrate the restoration actions into the partitioning problem in addition to other practical constraints. Particularly, we incorporated three new constraints to ensure the quality of the solutions from different aspects. The proposed approach was tested on both small and large sizes of power grids, and simulation results proved its applicability and effectiveness. Also, the sensitivity of the objective function to different parameters has been presented. The results confirm that our method outperforms previous works by providing a shorter restoration time. The proposed approach can swiftly generate a solution pool, and the feasibility of each solution in the pool is evaluated after applying the linear load flow equations. It shows that our approach is very simple for real-world implementation without computational difficulties. Also, it brings more flexibility by enabling system operators to include or exclude any of the aforementioned constraints as well as other required constraints.

6 References
[29] ‘Illinois Center for a Smarter Electric Grid’. Available at http://icseg.iti.illinois.edu