COT 6938: Approximation Algorithms

Assignment 3

Out: 10/22/2008 Due: 11/12/2008 (before class)

1. Find an approximation algorithm having approximation gurantee 2 for the following clustering problem:

   - Input: Points $x_1, \ldots, x_n \in \mathbb{R}^n$ and an integer k
   - Output: A partition of the points into k clusters $C_1, \ldots, C_k$
   - Goal: Minimize the diameter of the clusters

   $$\max_{j=1,\ldots,k} \max_{x_a, x_b \in C_j} d(x_a, x_b)\,,$$

   where $d(x_a, x_b)$ denotes the Euclidean distance between the points x_a and x_b.

2. An obvious algorithm for the knapsack problem is to sort the objects by decreasing ratio of profit to size, and then greedily pick objects in this order. Show that this greedy algorithm can be made to perform arbitrarily badly.

3. Consider the following improvement of the algorithm in Problem 2. Let the sorted order of objects be $a_1, a_2, \ldots, a_n$. Find the lowest k such that the size of the first k objects exceeds B. Now, pick the more profitable of $\{a_1, \ldots, a_{k-1}\}$ and $\{a_k\}$. Show that this algorithm achieves an approximation factor of 2.

4. Find an FPTAS for the following subset-sum ration problem. Given n positive integers, $a_1 < \ldots < a_n$, find two disjoint nonempty subsets $S_1, S_2 \subseteq \{1, \ldots, n\}$ with $\sum_{i \in S_1} a_i \geq \sum_{j \in S_2} a_j$, such that the ratio

   $$\frac{\sum_{i \in S_1} a_i}{\sum_{j \in S_2} a_j}$$

   is minimized.

   Hint: First, obtain a pseudo-polynomial time algorithm for this problem. Then, scale and round appropriately.

5. Give an example on which First-Fit does as bad as $5/3 \cdot OPT$.

6. Prove the bounds on R and P stated in Lemma 9.4.

7. Prove the following statement made in Lemma 9.5, "A packing for instance J' yields a packing for all but the largest Q items of instance J."

   Hint: Throw away the largest Q items of J and the smallest items of J', and establish a domination.

8. Consider the following improvement of Algorithm 10.2 (Minimum makespan scheduling). Sort the jobs by decreasing processing times before scheduling them. Show that this leads to a 4/3 approximation algorithm. Provide a tight example for this algorithm.

9. Give a FPTAS for the variant of the minimum makespan scheduling problem in which the number of machines m is fixed.