

**COT 3100H Spring 2007**  
**Assignment #3**

Use **induction** to prove each of the following questions, and be sure to **mark clearly when and where the induction hypothesis is applied in each question**:

1. (15pts)

Prove that  $421 \mid (20^{n+2} + 21^{2n+1})$  for all integer  $n \geq 0$ .

2. (15pts)

Suppose a sequence  $a_0, a_1, \dots, a_n, \dots$  is defined by the following recurrence:

$$a_0 = 6, \quad a_1 = 13, \quad \text{and} \quad a_n = a_{n-1} + 6a_{n-2}, \quad \text{for } n \geq 2.$$

Prove that the sequence  $a_n$  satisfies the formula  $a_n = 5 \cdot (3)^n + (-2)^n$  for all integers  $n \geq 0$ .

3. (15pts)

Prove that for integer  $n \geq 0$ ,  $\sum_{j=0}^n C(r+j, j) = C(r+n+1, n)$  where  $r$  is an arbitrary positive constant (Hint: Recall the Pascal's Triangle identity  $C(r-1, k) + C(r-1, k-1) = C(r, k)$ .)

4. (15pts)

Prove that for integer  $n \geq 1$ ,  $\sum_{j=1}^n j5^j = \frac{5^{n+1}(4n-1) + 5}{16}$ .