Problem Set #2:

Due in class on 1/31.

1. Exercise 8.2
2. Exercise 8.4
3. Exercise 12.2
4. If \( \{ \mathcal{F}_a \} \) is a POVM, as defined in class, as the restriction of a projective measurement on a larger subspace, show that:
   (i) \( \mathcal{F}_a^+ = \mathcal{F}_a \)
   (ii) \( \mathcal{F}_a \) is positive
   (iii) \( \mathcal{F}_a \leq \mathcal{I} \)
5. Let \( |\alpha\rangle_{ra} \) be a purification of \( \mathcal{F}_a \). We define the entropy exchange at \( E \) upon input of \( \mathcal{P} \), \( \mathcal{S}(\mathcal{P}, \mathcal{E}) \), as:
   \[
   \mathcal{S}(\mathcal{P}, \mathcal{E}) = \mathcal{S}(I \otimes \mathcal{E} / |\alpha\rangle_{ra}\langle\alpha|)
   \]
   Show that:
   (i) \( \mathcal{S}(\mathcal{P}, \mathcal{E}) \) does not depend on the purification
   (ii) \( \mathcal{S}(\mathcal{P}, \mathcal{E}) \) is the entropy introduced to an initially pure environment
6. Prove the following entropy inequalities:
   (i) \( \mathcal{S}(A \otimes B | C) \leq \mathcal{S}(A | B) \)
   (ii) \( \mathcal{S}(A | B | C) \leq \mathcal{S}(A | C) + \mathcal{S}(B | C) \)
   (iii) \( \mathcal{S}(A | B | C | D) \leq \mathcal{S}(A | C) + \mathcal{S}(B | D) \)

Exercise 11.14

Exercise 11.19