

# COT 6602 Quantum Information Theory

## Problem set # 2.

• Due in class on 1/31.

① Exercise 8.2

② Exercise 8.4

③ Exercise 12.2

④ If  $\{F_a\}$  is a POVM, as defined in class ~~as~~ as the restriction of a projective measurement on a larger subspace, show that:

(i)  $F_a^\dagger = F_a$ ; (ii)  $F_a$  is positive (iii)  $\sum_a F_a = I$

⑤ Let  $|\psi\rangle_{RA}$  be a purification of  $\rho_a$ . We define the entropy exchange of  $E$  upon input of  $\rho$ ,  $S(\rho, E)$ , as;

$$S(\rho, E) \equiv S((I \otimes E) |\psi\rangle_{RA})$$

Show that: (i)  $S(\rho, E)$  does not depend on the purification

(ii)  $S(\rho, E)$  is the entropy introduced to an initially pure environment

⑥ Prove the following entropy inequalities:

(i)  $S(A|BC) \leq S(A|B)$

(ii)  $S(AB|C) \leq S(A|C) + S(B|C)$

(iii)  $S(AB|CD) \leq S(A|C) + S(B|D)$

⑦ Exercise 11.14

⑧ Exercise 11.19