Homework 4

1- Given \( F_1 = \sum m(0,2,5,7,9) \) and \( F_1 = \sum m(2,3,4,7,8) \) find the minterm expression for \( F_1 + F_2 \). State a general rule for finding the expression for \( F_1 + F_2 \) given the minterm expansions for \( F_1 \) and \( F_2 \). Prove your answer by using the general form of the minterm expansion.

2- Given: \( F(a,b,c,d) = (a + b + c' + d')(a' + b' + c')(a + b + d)(a' + c) \)

   (a) Express \( F \) as a minterm expansion. (use m-notation)
   (b) Express \( F \) as a maxterm expansion. (use M-notation)
   (c) Express \( F' \) as a minterm expansion. (use m-notation)
   (d) Express \( F' \) as a maxterm expansion. (use M-notation)

3- Find the minterm expansion of \( f(a,b,c,d) = a'(b' + d) + acd' \) and then design the result.

4- Design a combinational logic circuit which has one output \( Z \) and a 4-bit input \( ABCD \) representing a binary number. \( Z \) should be 1 if the input is at least 5, but is no greater than 11. Use one OR gate (three input), and three AND gates.

5- A half adder is a circuit that adds two bits to give a sum and a carry. Give the truth table for a half adder, and design the circuit using only two gates. Then design a circuit which will find the 2’s complement of a 4-bit binary number. Use four half adders and any additional gates. (Hint: recall that one way to find the 2’s complement of a binary number is to complement all bits, and then add 1)

6- Find the minimum sum of products for each function using a Karnaugh map.
   (a) \( f_1(a, b, c) = m_0 + m_2 + m_5 + m_6 \)
   (b) \( f_2(d, e, f) = \Sigma m(0, 1, 2, 4) \)
   (c) \( f_3(x, y, z) = xz' + x'y' + x'y \)
   (d) \( f_4(r, s, t) = M_0 \cdot M_5 \)

7- (a) Plot the following function on a Karnaugh map. (Do not expand to minterm form before plotting.)
   (b) \( F(A, B, C, D) = BC' + B'CD + ABC + ABCD + B'D' \)
   (c) Find the minimum sum of products.
   (d) Find the minimum product of sums.

8- Find the minimum product of sums for the following.
   (a) \( \Pi M(0, 2, 4, 6, 7, 9, 14) \cdot \Pi D(10, 11) \)
   (b) \( \Sigma m(1, 3, 7, 8, 15) + \Sigma d(5, 12) \)
9- A logic circuit realizes the function \( F(a, b, c, d) = ab'd' + a'b' + a'cd + ac'd \). Assuming that \( a = c \) never occurs when \( b = d = 1 \), find a simplified expression for \( F \).

10- Given \( F = AB'D' + A'B + A'C + CD \).
   (a) Use a Karnaugh map to find the maxterm expression for \( F \) (express your answer in both decimal and algebraic notation).
   (b) Use a Karnaugh map to find the minimum sum-of-products form for \( F' \).
   (c) Find the minimum product of sums for \( F \).

11- Assuming that the inputs \( ABCD = 0101, ABCD = 1001, ABCD = 1011 \) never occur, find a simplified expression for
   \[ F = AB'C'D + A'B'D + A'CD + ABD + ABC \]