Design of Synchronous Generator with Round Rotor
Part 1 – General Considerations
Synchronous Generator System
Rotor Peripheral Speed

The maximum allowable peripheral speed of the rotor is a central consideration in machine design. With present-day steel alloys, rotor peripheral speeds of 50,000 ft/min (or about 250 m/s) represent the design limit.

1 ft/min = 0.0051 m/s
The resistivity of copper versus temperature can be calculated using the following formula:

$$\rho_{Cu}(T) = \rho_{Cu}(T_0) + \alpha_{Cu}(T - T_0)$$

where $T_0$ is a reference temperature and $\alpha_{Cu}$ is a constant given by

$$\alpha_{Cu} = 2.668 \times 10^{-9} \quad \Omega \cdot \text{in/°C}$$

or:

$$\alpha_{Cu} = 6.775 \times 10^{-11} \quad \Omega \cdot \text{m/°C}$$

At $T_0 = 20^\circ \text{C}$, we have

$$\rho_{Cu}(T_0) = 0.679 \times 10^{-6} \quad \Omega \cdot \text{in/°C}$$

or:

$$\rho_{Cu}(T_0) = 1.724 \times 10^{-8} \quad \Omega \cdot \text{m/°C}$$
Part 2 – Armature Design
Number of Armature Slots

For a m-phase synchronous machine, the number of armature slots (S) must be multiples of m. This will guarantee all the phases are balanced.

A. Integral S/P

S is multiples of mP.

Example: For a 2 pole, 3 phase machine, the number of slots can be 6,12,18,24,30,36… For a 4 pole, 3 phase machine, the number of slots can be 12, 24,36,48,60…

Integral S/P may cause extensive cogging or detent torque since all pole faces will line up with slot openings at the same time.

Cogging torque: torque from the interactions between rotor poles and stator teeth. Use slot skew or fractional S/P can reduce it.

B. Fractional S/P

S/P takes a fractional number.
Number of Turns per Coil

\[ V_{\phi, \text{rated}} = \sqrt{2 \pi f_e \hat{N}_a \Phi_{g, pk}} \approx 4.44 f_e \hat{N}_a \Phi_{g, pk} \]

where \( \Phi_{g, pk} = \frac{2B_{g, pk} D_l}{P} \)

\[ \hat{N}_a = k_w N_a / 1.1 \quad k_w = k_p k_d k_s \quad N_a = P q N_c / C \]

\( N_a \) is the number of series turns per phase of armature winding
\( C \) is the number of parallel circuits of armature winding

\[ N_c = \frac{1.1 V_{\phi, \text{rated}} C}{2 \sqrt{2 \pi f_e q k_w B_{g, pk} D_l}} \]

To consider leakage flux
Number of Conductor Positions per Slot on Stator

\[ C_s = 2 N_c a \]  for double layer winding.

In the above expression, \( C_s \) includes hollow conductor positions for cooling (about 25%) and additional 15% - 25% of both height and width tolerance of conductors (for insulation, slot liner, etc.) in factor \( a \). \( a \) can take about 1.6 – 2.

\[
C_s = \frac{1.1 a V_{\phi, \text{rated}} C}{\sqrt{2 \pi f_e q k_w B_{g, p_k} D l}}
\]
Maximum and Average Flux Density

Average flux per pole:

\[ \Phi_{g,av} = \frac{\int_0^\pi \Phi_{g,pk} \sin \theta_{ae} d\theta_{ae}}{\pi} = \frac{2}{\pi} \Phi_{g,pk} \]

Average flux density per pole:

\[ B_{g,av} = \frac{\Phi_{g,av}}{\pi Dl / P} = \frac{2\Phi_{g,pk} P}{\pi^2 Dl} \]

or:

\[ \Phi_{g,pk} = \frac{\pi^2 Dl B_{g,av}}{2 P} \]

Since \[ \Phi_{g,pk} = \frac{2B_{g,pk} Dl}{P} \]

Specific magnetic loading \[ B_{g,av} = \frac{4}{\pi^2} B_{g,pk} \approx 0.4 B_{g,pk} \]

Typically, take \( B_{g,av} \approx 0.6T \) or \( B_{g,pk} \approx 1.5T \) in design.
Machine Size (1)

Specific electric loading:

rms current per unit length of the armature circumference

\[ K_a = \frac{m(2N_a C)(I_{A,\text{rated}} / C)}{\pi D} = \frac{2mN_a I_{A,\text{rated}}}{\pi D} \]

\( N_a \) is the number of series turns per phase of armature winding
\( C \) is the number of parallel circuits of armature winding

\[ \Rightarrow I_{A,\text{rated}} = \frac{\pi D K_a}{2mN_a} \]
Machine Size (2)

Apparent power

\[ S_{\text{rated}} = m V_{\phi, \text{rated}} I_{A, \text{rated}} \]

\[ V_{\phi, \text{rated}} = \sqrt{2} \pi f_e \hat{N}_a \Phi_{g, \text{pk}} \]

\[ \Phi_{g, \text{pk}} = \frac{2 B_{g, \text{pk}} D l}{P} \]

\[ I_{A, \text{rated}} = \frac{\pi D K_a}{2 m N_a} \]

\[ \Rightarrow S_{\text{rated}} = m \sqrt{2} \pi f_e \hat{N}_a \frac{2 B_{g, \text{pk}} D l}{P} \frac{\pi D K_a}{2 m N_a} \]

\[ \Rightarrow S_{\text{rated}} = \sqrt{2} \pi^2 f_e B_{g, \text{pk}} k_w K_a \frac{D^2 l}{P} \]

\[ f_e = \frac{n_m P}{120} \]

\[ \Rightarrow \frac{S_{\text{rated}}}{(D^2 l) n_m} = \frac{\sqrt{2} \pi^2}{120} k_w K_a B_{g, \text{pk}} \equiv \frac{\pi^2}{60} \sigma_m \]

where \( \sigma_m = k_w K_a \frac{B_{g, \text{pk}}}{\sqrt{2}} \)

proportional to power density

Defined as: magnetic shear stress
Machine Size (3)

\[ D^2 l = \left( \frac{60\sqrt{2}}{\pi^2 k_w} \right) \frac{S_{\text{rated}}}{n_m K_a B_{g,pk}} \propto \text{Volume of Machine} \]

Discussions:
- The more advanced cooling technology (larger \( K_a \)), the smaller the volume.
- The larger the rated apparent power \( S_{\text{rated}} \), the larger the volume.
- The faster the machine speed \( n_m \), the smaller the volume.
- The larger the gap magnetic field \( B_{g,pk} \) (through using advanced materials with larger magnetic saturation, etc.), the smaller the volume.
Generator Size - Experience

\[ \frac{D^2 l}{S_{\text{rated}} P} = C_0, \quad C_0 = \frac{1}{2\pi^2 f_e \sigma_m} \alpha \frac{1}{K_a} \text{ depends on cooling} \]

\[ C_0 = 1400 \quad \text{in}^3/\text{MVA (air-cooled)} \]
\[ C_0 = 700 \quad \text{in}^3/\text{MVA (hydrogen-cooled)} \]
\[ C_0 = 375 \quad \text{in}^3/\text{MVA (liquid-cooled)} \]

\[ f_e = 60 \text{ Hz} \]

common steel
Length/Diameter Ratio (1)

The length/diameter ratio of a machine is defined as the ratio of the length and the stator bore diameter:

\[ r_{ld} = \frac{l}{D} \]

Discussions:

For fixed mechanical speed, the machine power rating depends on \( D^2l \).
- As the \( l/D \) increases, the rotor diameter decreases and thus the moment of inertia decreases. Also the rotor peripheral speed decreases.
- As the \( l/D \) increases, the machine length increases and the rotor is prone to exhibit critical frequencies at lower speeds that can result in shaft flexure to the point that the rotor strikes the stator bore.
- If the \( l/D \) is too large, it is difficult to cool.
- If the \( l/D \) is too small, the leakage inductance of end turns can severely affects machine performance.
Some people use aspect ratio, which is defined as the ratio of the length and the pole pitch:

\[ r_{asp} = \frac{l}{\tau_P} \quad \text{where} \quad \tau_P = \frac{\pi D}{P} \]

We can find the relationship between aspect ratio and length/diameter ratio:

\[ r_{asp} = \frac{l}{\tau_P} = \frac{l}{D} \frac{P}{\pi} = r_{lD} \frac{P}{\pi} \]
Stator Core Diameter

Stator Core diameter (outer diameter) \( D_0 \)

Two-pole: \( D_0 \approx 2.1D \)

Four-pole: \( D_0 \approx 1.7D \)
Stator Slot Design

\[ \tau_s = \frac{\pi D}{S} \]

\[ 0.4 \tau_s \leq b_s \leq 0.6 \tau_s \]

\[ 3b_s \leq d_s \leq 7b_s \]

\[ t_s = \tau_s - b_s \]

- Use 65 V/mil ground insulation
- Use 0.375-in slot wedge
- Use 0.125-in coil separator and top stick
Stator Conductor Size

\[ J_s = \frac{I_{A,\text{rated}}}{aA_a} \]

where \( A_a \) is stator (armature) conductor cross section area and can be determined from the above formula together with:

Air-cooled: \( J_s \leq 2500 \, \text{A}_{\text{rms}}/\text{in}^2 \)

Hydrogen-cooled: \( J_s \leq 4000 \, \text{A}_{\text{rms}}/\text{in}^2 \)

Water-cooled: \( J_s \leq 7000 \, \text{A}_{\text{rms}}/\text{in}^2 \)

\[ 1 \, \text{A/in}^2 = 0.00155 \, \text{A/mm}^2 \]

Round Wire Structure

This figure shows the wire structure, including the bare conductor, an insulation layer and an optional bonding layer.

- $d_{wb}$: bare conductor diameter
- $d_{wc}$: covered wire diameter
- $A_{wb}$: bare conductor cross section area
- $A_{wc}$: covered conductor cross section area
American Wire Gauge (AWG)

\[ d_{wb} = 8.251463 \times (0.8905257)^G \]

Or

\[ G = \frac{\log \left( \frac{d_{wb}}{8.251463} \right)}{\log (0.8905257)} \]

G: wire gauge (typically an integer)

\( d_{wb} \): bare wire diameter in mm.
- Increasing wire gauge by 1, increases copper loss by 26%.
- Decreasing wire gauge by 1, decreases copper loss to about 79% of the previous gauge.
The maximum allowable current density varies roughly between $1 \text{ A}_{\text{rms}}/\text{mm}^2$ to $10 \text{ A}_{\text{rms}}/\text{mm}^2$. In confined volumes, the lower limit $1 \text{ A}_{\text{rms}}/\text{mm}^2$ may be too high. Similarly, with active cooling, the upper limit $10 \text{ A}_{\text{rms}}/\text{mm}^2$ may be too conservative.
## Round Wire with Film Insulation (1)

<table>
<thead>
<tr>
<th>AWG</th>
<th>Bare Diameter (in)</th>
<th>Total Diameter (in)</th>
<th>Weight (lb/1000 ft)</th>
<th>Resistance (20°C) (ohms/1000 ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.2053</td>
<td>0.2098</td>
<td>127.20</td>
<td>0.2485</td>
</tr>
<tr>
<td>5</td>
<td>0.1828</td>
<td>0.1872</td>
<td>100.84</td>
<td>0.3134</td>
</tr>
<tr>
<td>6</td>
<td>0.1628</td>
<td>0.1671</td>
<td>80.00</td>
<td>0.3952</td>
</tr>
<tr>
<td>7</td>
<td>0.1450</td>
<td>0.1491</td>
<td>63.51</td>
<td>0.4981</td>
</tr>
<tr>
<td>8</td>
<td>0.1292</td>
<td>0.1332</td>
<td>50.39</td>
<td>0.6281</td>
</tr>
<tr>
<td>9</td>
<td>0.1150</td>
<td>0.1189</td>
<td>39.98</td>
<td>0.7925</td>
</tr>
<tr>
<td>10</td>
<td>0.1024</td>
<td>0.1061</td>
<td>31.74</td>
<td>0.9988</td>
</tr>
<tr>
<td>11</td>
<td>0.0912</td>
<td>0.0948</td>
<td>25.16</td>
<td>1.26</td>
</tr>
<tr>
<td>12</td>
<td>0.0812</td>
<td>0.0847</td>
<td>20.03</td>
<td>1.59</td>
</tr>
<tr>
<td>13</td>
<td>0.0724</td>
<td>0.0757</td>
<td>15.89</td>
<td>2.00</td>
</tr>
<tr>
<td>14</td>
<td>0.0644</td>
<td>0.0682</td>
<td>12.60</td>
<td>2.52</td>
</tr>
<tr>
<td>15</td>
<td>0.0574</td>
<td>0.0609</td>
<td>10.04</td>
<td>3.18</td>
</tr>
<tr>
<td>16</td>
<td>0.0511</td>
<td>0.0545</td>
<td>7.95</td>
<td>4.02</td>
</tr>
<tr>
<td>17</td>
<td>0.0455</td>
<td>0.0488</td>
<td>6.33</td>
<td>5.05</td>
</tr>
<tr>
<td>18</td>
<td>0.0405</td>
<td>0.0437</td>
<td>5.03</td>
<td>6.39</td>
</tr>
</tbody>
</table>
## Round Wire with Film Insulation (2)

<table>
<thead>
<tr>
<th>AWG</th>
<th>Bare Diameter (in)</th>
<th>Total Diameter (in)</th>
<th>Weight (lb/1000 ft)</th>
<th>Resistance (20°C) (ohms/1000 ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>0.0361</td>
<td>0.0391</td>
<td>3.99</td>
<td>8.05</td>
</tr>
<tr>
<td>20</td>
<td>0.0322</td>
<td>0.0351</td>
<td>3.18</td>
<td>10.1</td>
</tr>
<tr>
<td>21</td>
<td>0.0286</td>
<td>0.0314</td>
<td>2.53</td>
<td>12.8</td>
</tr>
<tr>
<td>22</td>
<td>0.0254</td>
<td>0.0281</td>
<td>2.00</td>
<td>16.2</td>
</tr>
<tr>
<td>23</td>
<td>0.0227</td>
<td>0.0253</td>
<td>1.60</td>
<td>20.3</td>
</tr>
<tr>
<td>24</td>
<td>0.0202</td>
<td>0.0227</td>
<td>1.26</td>
<td>25.7</td>
</tr>
<tr>
<td>25</td>
<td>0.0180</td>
<td>0.0203</td>
<td>1.00</td>
<td>32.4</td>
</tr>
<tr>
<td>26</td>
<td>0.0160</td>
<td>0.0182</td>
<td>0.794</td>
<td>41.0</td>
</tr>
<tr>
<td>27</td>
<td>0.0143</td>
<td>0.0164</td>
<td>0.634</td>
<td>51.4</td>
</tr>
<tr>
<td>28</td>
<td>0.0127</td>
<td>0.0147</td>
<td>0.502</td>
<td>65.3</td>
</tr>
<tr>
<td>29</td>
<td>0.0114</td>
<td>0.0133</td>
<td>0.405</td>
<td>81.2</td>
</tr>
<tr>
<td>30</td>
<td>0.0101</td>
<td>0.0119</td>
<td>0.318</td>
<td>104.0</td>
</tr>
<tr>
<td>31</td>
<td>0.0090</td>
<td>0.0108</td>
<td>0.253</td>
<td>131.0</td>
</tr>
<tr>
<td>32</td>
<td>0.0081</td>
<td>0.0098</td>
<td>0.205</td>
<td>162.0</td>
</tr>
<tr>
<td>33</td>
<td>0.0072</td>
<td>0.0088</td>
<td>0.162</td>
<td>206.0</td>
</tr>
</tbody>
</table>
# Square Wire with Film Insulation

<table>
<thead>
<tr>
<th>AWG</th>
<th>Bare Width (in)</th>
<th>Total Width (in)</th>
<th>Area (in²)</th>
<th>Weight (lb/1000 ft)</th>
<th>Resistance (20°C) (ohms/1000 ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3279</td>
<td>0.3329</td>
<td>0.10420</td>
<td>403.1</td>
<td>0.07815</td>
</tr>
<tr>
<td>1</td>
<td>0.2922</td>
<td>0.2972</td>
<td>0.08232</td>
<td>318.4</td>
<td>0.09895</td>
</tr>
<tr>
<td>2</td>
<td>0.2602</td>
<td>0.2652</td>
<td>0.06498</td>
<td>251.4</td>
<td>0.1253</td>
</tr>
<tr>
<td>3</td>
<td>0.2317</td>
<td>0.2367</td>
<td>0.05125</td>
<td>198.4</td>
<td>0.1589</td>
</tr>
<tr>
<td>4</td>
<td>0.2063</td>
<td>0.2113</td>
<td>0.04037</td>
<td>156.4</td>
<td>0.2018</td>
</tr>
<tr>
<td>5</td>
<td>0.1837</td>
<td>0.1887</td>
<td>0.03171</td>
<td>122.9</td>
<td>0.2568</td>
</tr>
<tr>
<td>6</td>
<td>0.1636</td>
<td>0.1686</td>
<td>0.02536</td>
<td>98.39</td>
<td>0.3211</td>
</tr>
<tr>
<td>7</td>
<td>0.1457</td>
<td>0.1507</td>
<td>0.01994</td>
<td>77.42</td>
<td>0.4085</td>
</tr>
<tr>
<td>8</td>
<td>0.1298</td>
<td>0.1348</td>
<td>0.01563</td>
<td>60.74</td>
<td>0.5212</td>
</tr>
<tr>
<td>9</td>
<td>0.1155</td>
<td>0.1205</td>
<td>0.01251</td>
<td>48.66</td>
<td>0.6514</td>
</tr>
<tr>
<td>10</td>
<td>0.1029</td>
<td>0.1079</td>
<td>0.00980</td>
<td>38.19</td>
<td>0.8310</td>
</tr>
<tr>
<td>11</td>
<td>0.0917</td>
<td>0.0967</td>
<td>0.00788</td>
<td>30.74</td>
<td>1.033</td>
</tr>
<tr>
<td>12</td>
<td>0.0818</td>
<td>0.0868</td>
<td>0.00619</td>
<td>24.16</td>
<td>1.316</td>
</tr>
<tr>
<td>13</td>
<td>0.0730</td>
<td>0.0780</td>
<td>0.00496</td>
<td>19.41</td>
<td>1.641</td>
</tr>
<tr>
<td>14</td>
<td>0.0651</td>
<td>0.0701</td>
<td>0.00389</td>
<td>15.24</td>
<td>2.095</td>
</tr>
</tbody>
</table>
Air Gap Size

From

\[ B_{a, pk} = \frac{4 \mu_0 \hat{N}_a}{\pi g_{eff}} \frac{1.5 \sqrt{2} I_{A, rated}}{P} \]

\[ g_{eff} = k_c g \]

An empirical formula for effective air gap size:

\[ g_{eff} = \frac{4 \mu_0 \hat{N}_a}{\pi B_{a, pk} P} 1.5 \sqrt{2} I_{A, rated} \]

\[ g = g_{eff} / k_c \]

The actual air gap size can be further tuned using an electromagnetic simulation software.
Part 3 – Round Rotor Design for Generator with Field Winding
For round rotor machine with field winding, take $P = 2$ or $4$. 
Phasor Diagram

\[ E_A = V_\phi + jX_s I_A \]

Pick up torque angle \( \delta \) \( (T_{\text{full load}} = T_{\text{max}} \sin \delta) \) and power factor \( pf = \cos \theta \).

From \( X_s I_A \cos \theta = E_A \sin \delta \) \( \Rightarrow E_A = K_B X_s I_A, \quad K_B = \cos \theta / \sin \delta \)

Example: If \( \delta = 30^\circ \), \( pf = 0.85 \) lagging, \( K_B = 1.7 \).

\[ E_A = K_B X_s I_A \Rightarrow B_{f,pk}^{\text{Steady}} \approx K_B B_{a,pk}^{\text{Steady}} \]

\[ V_\phi = E_A \cos \delta - X_s I_A \sin \theta = (K_B \cos \delta - \sin \theta) X_s I_A \]

\[ \Rightarrow \frac{B_{a,pk}}{B_{g,pk}} = \frac{X_s I_A}{V_\phi} = \frac{1}{K_B \cos \delta - \sin \theta} \]
Rotor Slot Selection (1)

total number of slots on rotor $N_r = 2n_rP$

$n_r$ is integer

slots on rotor per pole half : $n_r = \frac{N_r}{2P}$

rotor pole pitch : $\tau_r = \frac{\pi D_r}{P}$

pole width : $0.2\tau_r \leq W_f \leq 0.3\tau_r$

Angular slot pitch (in elec. radian):

$$\gamma_2 = \frac{\pi D_r - PW_f}{2n_rP} \cdot \frac{1}{(D_r/2) / 2} = \frac{\pi D_r - PW_f}{2D_r n_r}$$

Arc length between two adjacent slots:

$$t_\gamma = \frac{\gamma_2 D_r}{P}$$

rotor slot width : $0.4t_\gamma \leq b_f \leq 0.5t_\gamma$  
rotor tooth width : $t_f = t_\gamma - b_f$
The length of the ith field coil: \( L_{fi} = 2(l + W_f + i\pi \frac{\gamma_2 D_r}{P}) \)

Assume the number of conductors \((C_f)\) are the same in the each slot

\[ \text{Total length of the field winding:} \quad L_F = C_f P \sum_{i=1}^{n_r} 2(l + W_f + i\pi \frac{\gamma_2 D_r}{P}) \]

Note: use single layer concentric winding on rotor.
Number of Conductors in Rotor (2)

\[ L_F = C_f X_f \]

where \( X_f = P \sum_{i=1}^{n_r} 2(l + W_f + i \pi \frac{\gamma_2 D_r}{P}) = 2n_r P(l + W_f) + \pi \gamma_2 D_r n_r (n_r + 1) \)

The field winding is to be supplied by a voltage source with maximum value \( V_{f_{\text{max}}} \). If 30 percent is to be held in reserve for field forcing, then at rated conditions

\[ 0.7V_{f_{\text{max}}} = I_{F,\text{rated}} \frac{\rho L_F}{A_f} = J_f \rho C_f X_f \]

where \( A_f \) is the cross section area of the field conductor, \( J_f \) is the allowable current density, and \( \rho \) is the resistivity of copper at working temperature.

\[ C_f = \frac{0.7V_{f_{\text{max}}}}{J_f \rho X_f} \]

The calculated results will be rounded to an integer. \( J_f \) depends on rotor cooling. See next slide
Rotor Cooling

Field winding conductors are cooled by one of two methods—indirect cooling and direct cooling. For the case of indirect cooling, heat generated within the field conductor must flow across the ground insulation to reach the cooling medium, which may be, for example, a channel beneath the rotor slot carrying an axial flow of air. The direct cooling methods are schemes that allow the cooling medium to directly contact the field conductor by passing through holes or slots that pierce the wide side. The usual cooling medium is hydrogen gas. The field winding should not exceed an average temperature of 125°C. This desired temperature can be maintained with the following current densities:

1. Indirect cooling \[ J_f = 2000 \text{ A/in}^2 \]
2. Direct cooling \[ J_f = 3500 \text{ A/in}^2 \]

\[ 1 \text{ A/in}^2 = 0.00155 \text{ A/mm}^2 \]
Empirical design requires maximum magnetic field from field winding is about $K_B$ times maximum magnetic field from armature winding:

$$B_{f, pk}^{\text{Steady}} \approx K_B B_{a, pk}^{\text{Steady}}$$

$$B_{a, pk}^{\text{Steady}} = \frac{4}{\pi} \frac{\mu_0}{g_{\text{eff}}} \frac{\hat{N}_a}{P} 1.5\sqrt{2} I_{A, \text{rated}}$$

$$B_{f, pk}^{\text{Steady}} = \frac{4}{\pi} \frac{\mu_0}{g_{\text{eff}}} \left( \frac{k_{wf} N_f}{P} \right) I_{f, \text{rated}}$$

$$I_{f, \text{rated}} = \frac{K_B \times 1.5 \times \sqrt{2} \hat{N}_a I_{A, \text{rated}}}{k_{wf} N_f} \approx \frac{2.12 K_B \hat{N}_a I_{A, \text{rated}}}{k_{wf} N_f}$$

where $N_f$ is total number of series turns in field winding: $N_f = P C_f n_r$

$k_{wf}$ is rotor winding factor given is next page.

rotor slot cross section area $A_f$:

$$A_f = \frac{I_{f, \text{rated}}}{J_f}$$
Round Rotor Winding Factor

This is the case when \( s_r \) is even.

\[
    s_r = 2n_r
\]

\[
    k_{wf} = \frac{\sum_{\nu=1}^{n_r} N_\nu \cos[(2\nu - 1)\gamma_r / 2]}{\sum_{\nu=1}^{n_r} N_\nu}
\]

If \( N_1 = N_2 = \ldots N_{n_r} \)

\[
    k_{wf} = \frac{\sum_{\nu=1}^{n_r} \cos[(2\nu - 1)\gamma_r / 2]}{n_r}
\]
Comprehensive Design Example

Design a 3 phase turboalternator with the following specifications:

- 500 MVA  Y connected 24 kV (terminal voltage)  60 Hz
- 3600 rpm 2 pole  0.85 pf lagging
- Maximum allowable rotor peripheral speed 50,000 ft/min for 20% overspeed
- Directly cooled stator (water)
- Directly cooled rotor (hydrogen)

In the design, initially picked up
- 48 stator slots, 20 rotor slots
- 5/6 stator coil pitch
- $V_{fmax} = 600V$
- Not skewed

Details in sgDesign.m
Part 4 – Round Rotor Design for Surface Mount Permanent Magnet Generator
Magnetic Circuit Analysis

For a multi-pole surface mount rotor

\[ gH_g + H_m d_m = 0 \]

\[ \Rightarrow gB_g + \mu_0 H_m d_m = 0 \]

\[ B_m A_m = B_g A_g \]

\[ \Rightarrow \frac{d_m}{g} = -\frac{B_m}{\mu_0 H_m} \frac{A_m}{A_g} \]
Maximum Energy Point

\[ B = \frac{B_r}{H_c} (H + H_c) \quad \Rightarrow \quad BH = \frac{B_r}{H_c} (H + H_c)H \]

To get \((BH)_{\text{max}}\) 
\[ \frac{\partial (BH)}{\partial H} = 0 \quad \Rightarrow \quad B_m = \frac{B_r}{2}, \quad H_m = -\frac{H_c}{2} \]
Working Point for Permanent Magnetics (2)

\[ \mu_{rm} = \frac{B_r}{\mu_0 H_c} \]
\[ 1 \leq \mu_{rm} \leq 1.2 \]

Load Line:
\[ B_m = - \frac{d_m}{g} \frac{A_g}{A_m} \left( \mu_0 H_m \right) \]
\[ = - P_c \left( \mu_0 H_m \right) \]

Define:
\[ B_m = \alpha_m B_r \]
\[ H_m = -(1 - \alpha_m) H_c \]

Typically pick up:
\[ \alpha_m \approx 0.5 \sim 0.8 \]

\[ P_c = \frac{d_m}{g} \frac{A_g}{A_m} = \frac{A_g / g}{A_m / d_m} \approx \frac{R_m}{R_g} = \frac{\mathcal{P}_g}{\mathcal{P}_m} \]

\[ \Rightarrow P_c = - \frac{B_m}{\mu_0 H_m} = \frac{\alpha_m}{1 - \alpha_m} \mu_{rm} \]
Airgap Magnetic Field from PM Rotor

\[ B_{g, \text{rotor}} = \sum_{h=1,3,5,...} B_{Rh} \]

\[ B_{Rh} = B_{rh} \cos(h\theta_{de}) = B_{rh} \cos(h\frac{P}{2}\theta_d) \]

\[ \Rightarrow B_{rh} = \frac{2}{2\pi} \left[ \int_{-\rho_{PM}/2}^{\rho_{PM}/2} B_m \cos(h\theta_{ae})d\theta_{ae} + \int_{\pi-\rho_{PM}/2}^{\pi+\rho_{PM}/2} (-B_m) \cos(h\theta_{ae})d\theta_{ae} \right] \]

\[ = \frac{4}{\pi} \sin\left( h \frac{\rho_{PM}}{2} \right) B_m \]

\[ k_{ph} = \sin\left( h \frac{\rho_{PM}}{2} \right) \]

PM embrace: \[ \frac{\rho_{PM}}{\pi} \rightarrow \text{electrical angle} \]

\[ \theta_{de} = \frac{P}{2} \theta_d \]
Phasor Diagram

From $\cos \theta = \frac{sA}{E}$ and $\sin \theta = \frac{sB}{E}$, we have

$$\theta = \arctan\left(\frac{sB}{sA}\right)$$

The phasor diagram shows

$$E_A = V_\phi + jX_s I_A$$

Pick up torque angle $\delta$ ($T_{\text{full load}} = T_{\text{max}} \sin \delta$) and power factor $pf = \cos \theta$.

From $X_s I_A \cos \theta = E_A \sin \delta \Rightarrow E_A = K_B X_s I_A$, $K_B = \cos \theta / \sin \delta$

Example: If $\delta=30^\circ$, $pf=0.85$ lagging, $K_B = 1.7$.

$$E_A = K_B X_s I_A \Rightarrow B_{f,pk}^{\text{Steady}} \approx K_B B_{a,pk}^{\text{Steady}}$$

$$V_\phi = E_A \cos \delta - X_s I_A \sin \theta = (K_B \cos \delta - \sin \theta) X_s I_A$$

$$\Rightarrow \frac{B_{a,pk}}{B_{g,pk}} = \frac{X_s I_A}{V_\phi} = \frac{1}{K_B \cos \delta - \sin \theta}$$
Air Gap Size and PM Thickness

From:

\[ B_{a,pk} = \frac{4}{\pi} \mu_0 \frac{\hat{N}_a}{\hat{g}_{total}} \cdot 1.5\sqrt{2} I_{A,\text{rated}} \]

Initial total effective air gap size:

\[ \hat{g}_{total} = \frac{4}{\pi} \mu_0 \frac{\hat{N}_a}{B_{a,pk}} \cdot 1.5\sqrt{2} I_{A,\text{rated}} \]

From:

\[ \hat{g}_{total} = k_c g'_{total} \quad g'_{total} = g + \frac{d_m}{\mu_{rm}} \quad \frac{d_m}{g} \approx P_c \quad (A_g \approx A_m) \]

Carter’s coefficient

\[ \Rightarrow g = (1 - \alpha_m) g'_{total} \]

\[ d_m = \alpha_m g'_{total} \mu_{rm} \]
Effective Air Gap

\[ \hat{g}_{\text{total}} = k_c g'_{\text{total}} \]

where the Carter's coefficient

\[ k_c = \frac{\tau_s}{\tau_s - \frac{2b_{s0}}{\pi} \left\{ \text{atan} \left( \frac{b_{s0}}{2g'_{\text{total}}} \right) - \frac{g'_{\text{total}}}{b_{s0}} \ln \left[ 1 + \left( \frac{b_{s0}}{2g'_{\text{total}}} \right)^2 \right] \right\}} \]

approximately

\[ k_c \approx \frac{\tau_s}{\tau_s - \frac{b_{s0}^2}{5g'_{\text{total}} + b_{s0}}} \]