# An Exploration of Non-Isomorphic 3D Rotation in Surround Screen Virtual Environments

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## ABSTRACT

Non-isomorphic rotational mappings have been shown to be an effective technique for rotation of virtual objects in 3D desktop environments. In this paper, we present an experimental study that explores the performance characteristics of isomorphic and nonisomorphic rotation techniques in a surround screen virtual environment. Our experiment compares isomorphic rotation with nonisomorphic rotation techniques utilizing three separate amplification factors, two different thresholds for task completion, and two different angular ranges for virtual object rotation. Our results show that a non-isomorphic mapping with an amplification factor of three is both optimal in terms of completion time and accuracy and is most preferred by our test subjects. In addition, our results suggest that, in a surround screen virtual environment, rotation tasks using both isomorphic and non-isomorphic rotational mappings can be completed faster and more accurately compared to previous studies exploring rotation in 3D user interfaces.

**Keywords:** non-isomorphic rotation, 3D interaction, evaluation. surround screen virtual environments

**Index Terms:** I.3.6 [Computer Graphics]: Methodology and Techniques—Interaction techniques; H.5.2 [Information Interfaces and Presentation]: User Interfaces—Evaluation/methodology

## **1** INTRODUCTION

The ability to effectively rotate objects in 3D space is an important part of many 3D user interfaces. In fact, rotating 3D objects is part of one of the fundamental 3D interaction tasks (i.e., selection and manipulation) used in 3D applications [1]. Given the importance of 3D rotation tasks in 3D user interfaces, it is worthwhile to design, evaluate, and understand how 3D rotation techniques perform under different conditions so guidelines can be established. These guidelines can then assist 3D user interface designers in choosing appropriate 3D rotation techniques that maximize speed and efficiency while minimizing rotational error.

One approach to rotating objects in 3D space is to use nonisomorphic mappings [1]. Non-isomorphic mappings let users interact with virtual world objects at an amplified scale, in contrast to isomorphic mappings (i.e., one-to-one mappings) that maintain a direct correspondence with the physical and virtual worlds. For example, with a non-isomorphic mapping, a user rotating a tracked input device 20 degrees about the *z*-axis in the physical world would rotate the corresponding virtual object 40 degrees (with the appropriate amplification factor). In the isomorphic case, the virtual object would be rotated only 20 degrees. Thus, although isomorphic mappings are the most natural in terms of interaction in the physical world, they have significant shortcomings due to limited ranges of input devices and anatomical constraints of users. These shortcomings are especially evident for rotation tasks, since tracking a full 360 degrees of rotation can be difficult with vision-based tracking systems. In addition, human joints have limited rotation. Clutching (i.e., releasing a virtual object, re-adjusting the hand, and continuing the rotation) is often used to compensate for these limitations. However, clutching can be cumbersome and tiring when dealing with large rotations.

To explore the effectiveness of non-isomorphic rotation techniques, Poupyrev et al. [14] conducted an experiment to investigate the performance characteristics of a non-isomorphic rotation technique compared with conventional isomorphic rotation for a 3D object rotation task. The results of their experiment showed that the non-isomorphic rotation technique performed 13% faster than the isomorphic technique without a statistically significant loss in accuracy. In addition, subjects significantly preferred the nonisomorphic rotation technique over standard isomorphic rotation.

Although Poupyrev et al.'s experiment showed the value of nonisomorphic rotation, it left questions that have yet to be answered. Therefore, the focus of this paper is to extend and augment the knowledge gained from their experiment in two fundamental ways and broaden what we know about non-isomorphic rotation in 3D interfaces. First, with Poupyrev et al.'s experiment, only one nonisomorphic mapping (an amplification factor of 1.8) and only one threshold (rotation tasks were considered complete when the error fell below 18 degrees) were used in their experimental design. In this paper, we extend their experimental setup to include three nonisomorphic mappings, in addition to standard isomorphic rotation, and include a second, smaller threshold to see how these techniques affect performance when more precise rotations are required. Second, Poupyrev et al.'s study was conducted using a desktop environment with a Polhemus SpaceBall, a solid sphere embedded with a 6DOF tracker. In this paper, we perform our experiment in a surround screen virtual environment (SSVE) that has head tracking and a stereoscopic display. This environment lets us determine if the results found in Poupyrev et al.'s study transfer to a different type of virtual environment and hardware configuration.

In the next section, we discuss work related to non-isomorphic mappings and non-isomorphic rotation. Section 3 briefly reviews the mathematics used to implement non-isomorphic rotation in our study. Section 4 describes our experiment in detail along with statistical results. Section 5 presents a discussion of our experimental findings and ties them to prior work. Finally, Section 6 concludes the paper.

## 2 RELATED WORK

Non-isomorphic mappings can be applied to both the translation and rotation of virtual objects in 3D user interfaces. For translation, there have been several non-isomorphic techniques for both translation of virtual objects and navigation through virtual environments [2, 11, 12, 13, 17]. Other non-isomorphic mapping techniques for translation can be found in [1].

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In contrast with non-isomorphic translation, non-isomorphic 3D rotation techniques have received less attention. Early studies by Chen et al. [3], and Hinckley et al. [6], explored user performance with different 3D rotation techniques. However, their work did not focus on non-isomorphic mappings using 3D input devices. Chen et al. focused on the effectiveness of 3D rotation with 2D controllers while Hinckley et al. compared 3D rotation using 6DOF tracking devices with two standard 2D rotation techniques: ARCBALL and the Virtual Sphere. Ware and Rose also conducted studies with 3D rotation [16]. Their work was focused on understanding the differences between rotating virtual objects and real objects.

Poupyrev et al.'s [14] work introduced a mathematical framework and design guidelines for developing non-isomorphic 3D rotation techniques and was the first to conduct an experiment exploring their effectiveness. This work spawned further research into the development and evaluation of non-isomorphic rotational mappings. For example, LaViola et al. [10] and Jay and Hubbold [8] both developed non-isomorphic rotation techniques for amplifying head rotations in virtual environments to counteract field of view problems. LaViola et al. developed a technique that gave users a full 360 degree field of regard in a surround screen virtual environment that had only three walls. However, they did no evaluation to determine the effectiveness of their technique. Jay and Hubbold developed a similar technique that targeted field of view problems in head mounted displays, Their experimental results showed significant performance improvements for a visual search task but that users cannot interact normally without the corresponding body movements amplified to the same degree as the head movements. In both these cases, the work focused on navigation rather than rotation of virtual objects.

More recently, Froehlich et al. [5] used a non-isomorphic rotational mappings as part of the design of desktop-based input devices, the GlobeFish and GlobeMouse, for creating larger rotations and increase the sensitivity of smaller rotations. They pilot tested several scaling factors from one to five and found three to be most appropriate for their devices but did not report any performance results for the other factors. Dominjon et al. [4] compared nonisomorphic rotation with a hybrid haptic-based approach for performing rotations. They used a scaling factor of four and found that their approach had better performance than the non-isomorphic approach. However, as with Froehlich et al., Dominjon et al. did not experiment with non-isomorphic rotation techniques in a SSVE. In fact, to the best of our knowledge, this paper presents the first study on non-isomorphic rotation in a tracked, stereoscopic SSVE.

## **3** Non-Isomorphic Rotation

There are many different angular representations for representing 3D rotations in virtual environments. One of the more powerful representations is with quaternions because they provide a compact representation, avoid problems with gimbal lock, and are relatively straightforward to use [15]. Given these advantages and that Poupyrev et al. [14] developed a framework for designing both isomorphic and non-isomorphic mappings using quaternions, we chose to use them in our work. The details of using quaternions for rotations are beyond the scope of this paper. Thus, we only present the necessary details for implementing the isomorphic and non-isomorphic rotation techniques used in our experiment. Further detail on quaternions can be found in [9].

A quaternion is a four dimensional vector represented as a pair  $(\vec{v}, w)$  where *w* is a real number and  $\vec{v}$  is a 3D vector. For a quaternion to be a valid rotation it must be of unit length. A unit quaternion can represent a single rotation about a unit axis  $\vec{u}$  and angle  $\theta$  in the following forms:

$$q = \left(\sin\frac{\theta}{2}\vec{u}, \cos\frac{\theta}{2}\right) = e^{\frac{\theta}{2}\vec{u}}.$$
 (1)

A non-isomorphic rotation involves amplifying the rotation while maintaining the direction of rotation. To perform this operation, we can apply a coefficient *k* to  $\theta$  and define a quaternion

$$q_d = \left(\sin\frac{k\theta}{2}\vec{u}, \cos\frac{k\theta}{2}\right) = e^{\frac{k\theta}{2}\vec{u}} = q_c^k \tag{2}$$

where  $q_c^k$  is the amplified rotation from an input device and  $q_d$  is the orientation applied to a virtual object. This equation assumes an unspecified initial orientation defined by the identity quaternion (0,0,0,1) and is referred to as an absolute rotational mapping [14]. However in our experiments, we chose to use a relative rotational mapping given the output of our tracking device. To perform relative rotation, an explicit reference orientation  $q_0$  is required that connects to  $q_c$  and is computed as

$$q_d = (q_c q_0^{-1})^k q_0. aga{3}$$

Given this equation, we can compute a relative non-isomorphic rotation at each step i of the event loop by calculating the relative orientation of the input device from its orientation at step i - 1, amplifying it, and then combining it with the orientation of the virtual object at step i - 1. The resulting equation is then

$$q_{d_i} = (q_{c_i} q_{c_{i-1}}^{-1})^k q_{d_{i-1}}.$$
(4)

Note that if k = 1, a relative isomorphic rotation is performed.

# 4 EXPERIMENTAL STUDY

We conducted an experimental study to further explore nonisomorphic rotation of virtual objects. Our study had two main goals. First, we wanted to expand Poupyrev et al.'s [14] experimental design. There has been no previous work on non-isomorphic rotation in 3D user interfaces (see Section 2) that has performed a systematic evaluation of the effect different rotation amplifications have on speed and accuracy. Poupyrev et al.'s study chose to compare conventional isomorphic rotation with a non-isomorphic rotation technique using an amplification factor of 1.8. To further understand the utility of non-isomorphic rotation, we tested nonisomorphic rotation with amplification factors of two, three, and four along with the isomorphic rotation technique to see if there are any benefits to higher amplification factors. Additionally, we wanted to determine the benefits of non-isomorphic rotation techniques when users need to rotate virtual objects very accurately. In Poupyrev et al.'s study, a threshold of 18 degrees was chosen to determine when a virtual object rotation was complete. In our experiment, we added a second threshold of six degrees so we could measure the performance benefits of non-isomorphic rotation when more accurate rotations are required.

The second goal of our study is to understand the benefits of non-isomorphic rotation in a surround screen virtual environment (SSVE). Poupyrev et al.'s study was conducted in a desktop environment with no head tracking and stereoscopic vision. Therefore, we wanted to determine if Poupyrev et al.'s results transfer to such an environment. Since there have been no previous efforts to compare non-isomorphic rotation with conventional isomorphic rotation in a SSVE, we felt it was important to determine if 3D user interface guidelines regarding non-isomorphic rotation from Poupyrev et al.'s work should be updated.

# 4.1 Subjects and Apparatus

Sixteen subjects (13 male, 3 female) were recruited from the undergraduate population at Brown University with ages ranging from 18 to 23. Of the 16 subjects, 13 were right handed while two were left handed and one subject was ambidextrous. All of the subjects had little or no experience with 6DOF input devices. Since handeye coordination is related to the participants' ability to perform the



Figure 1: The Wanda (left) and the 6DOF tracker embedded sphere (right) used in our experiment.



Figure 2: A subject rotating the house model to its target orientation.

experiment, we also asked participants if they played video games. 11 out of the 13 males and all three females answered yes to this question. The experiment took 25 to 35 minutes per subject and all subjects were paid \$10 dollars for their time.

The experiments were conducted in Brown University's surround screen virtual environment (three walls and a floor) at a resolution of 1024x768 per wall. The refresh rate was 120Hz (60Hz per eye). A 6DOF Polhemus FASTRAK magnetic sensor was placed inside a rubber ball and used as the input device for rotating the virtual objects. A Wanda was used as a triggering device in the non dominant hand. Figure 1 shows the input devices used in the experiment.

## 4.2 Experimental Task

The experimental task design followed the design of orientation matching experiments by Poupyrev et al. [14]. Participants were instructed to rotate a solid shaded 3D model of a house from a randomly generated orientation into a target orientation (see Figure 2). They were told that while they should not rush, they should aim to minimize their time and maximize their accuracy. The target orientation was such that the house lay flat on a checkerboard plane, and its front (indicated by a door) faced the opening of the SSVE. As with Poupyrev et al., the house was designed to provide maximum cues to understanding it's orientation from any angle, with asymmetric placement of windows, its chimney, and the coloring of its walls. In addition, text on the screen displayed a description of the amplification coefficient, describing the amplification factor as none, small, moderate, or large which equates to one to one isomorphic mapping and non-isomorphic mappings with amplification (i.e., scale) factors of two, three, and four, respectively.

Users could rotate the house when the button on the Wanda was depressed. The user would start or stop the rotation by pressing or releasing the button on the Wanda. The user could iteratively rotate the house by holding the button, rotating the ball device, releasing the button, repositioning the ball device, holding the button, etc. as many times as necessary. Each time the user released the button, the orientation error (defined as the angular distance between the current and goal orientations) was calculated. When the error was below the threshold, the house would immediately disappear and reappear in a new random orientation, indicating that the trial had been accomplished.<sup>1</sup> Participants were told that the time measured

for each trial began when they first pressed the button, and ended when they released the button and the error of orientation was under the threshold.

# 4.3 Experiment Design and Procedure

We used a 4 x 2 x 2 balanced, within subjects factorial design where the independent variables were coefficient of amplification (i.e., scaling factor), amplitude of rotation (i.e., angular range), defined as the angular distance between the starting and target orientations, and the orientation error threshold. The coefficient of amplification varied as an integer between one and four, the amplitude was always random but constrained to be between 20 and 60 degrees (small) or between 70 and 180 degrees (large), and the orientation error threshold was either six or 18 degrees.

The dependent variables were completion time and orientation error. Completion time is the time from the user first pressing the Wanda button until releasing the button while the orientation error is below the error threshold. Orientation error is the angular distance between the orientation of the house upon completing a trial and the house's target orientation.

The experiments began with a pre-questionnaire, followed by an explanation of the SSVE, the devices involved, the experimental task and procedure, and the techniques involved in accomplishing the task. There was then a training session where the subject was given one trial under each of the 16 conditions to be tested (each possible combination of four amplification coefficients (i.e., scaling factors), two amplitudes of rotation, and two error thresholds). This allowed the user to get used to the techniques, devices, and conditions in the experiment. After the training session, subjects were asked whether they felt comfortable with the isomorphic and non-isomorphic rotation techniques. In each case, the subject said yes and the experiment was started. The subject was then given 16 sets of 10 trials, each set represented one of the test conditions, and each of the 10 trials within a given set had the same amplification coefficient, amplitude of rotation, and orientation error threshold. To control for order effects, the ordering of the 16 sets was randomized for each of the 16 subjects.

In the post-questionnaire, subjects were asked which of the four amplification coefficients they preferred and if they had any further comments on the experiment.

#### 4.4 Results

A repeated measures three-way analysis of variance (ANOVA) was performed for each of the dependent variables with scaling factor (S), threshold (T), and angular range (A) as the independent

<sup>&</sup>lt;sup>1</sup>Note that in Poupyrev et al's study, a three second delay was used in between trials.

variables. Table 1 summarizes the main effects of the independent variables as well as their interaction for both time and error. Both threshold and angle significantly affected completion time while both scaling factor and threshold significantly affected error. For completion time, there also was a significant interaction effect between threshold and angle. These results make intuitive sense given the nature of the independent variables. Given that we tested two angular ranges, between 20 and 60 degrees (small amplitude) and between 70 and 180 degrees (large amplitude), the larger amplitude requires more rotation to place the house model in its target orientation thus requiring more time to complete the task. For the threshold condition, subjects often had to perform more than one clutching step to obtain a correct target orientation during trials with the 6 degree threshold requirement. Thus, completion times took longer. For error, the nature of the threshold condition created a significant effect because subjects had to be more accurate with the 6 degree threshold than the 18 degree threshold.

Effect	Time	Error
S	$F_{3,13} = 3.26$ p = 0.056	$F_{3,13} = 4.8$ p < 0.05
Т	$F_{1,15} = 13.66$ p < 0.05	$F_{1,15} = 22.96$ p < 0.05
А	$F_{1,15} = 55.46$ p < 0.05	$F_{1,15} = 0.001$ p = 0.979
$\mathbf{S}\times\mathbf{T}$	$F_{3,13} = 0.29 \\ p = 0.832$	$F_{3,13} = 1.575$ p = 0.243
$\mathbf{S}  imes \mathbf{A}$	$F_{3,13} = 0.78 \\ p = 0.523$	$F_{3,13} = 0.562 \\ p = 0.649$
$\mathbf{T} \times \mathbf{A}$	$F_{1,15} = 5.03$ p < 0.05	$F_{1,15} = 0.573$ p = 0.46
$S \times T \times A$	$F_{3,13} = 0.73 \\ p = 0.552$	$F_{3,13} = 0.97$ p = 0.436

Table 1: The main and interaction effects for scale factor (S), threshold (T), and angle (A) for both time and error.

We performed a post-hoc analysis on scaling factor for both completion time and error to gain a better understanding of the relationship between scaling factor and user performance.<sup>2</sup> For both completion time and error, we performed pairwise comparisons using Holm's sequential Bonferroni adjustment [7] with three comparisons at  $\alpha = 0.5$  for isomorphic rotation (S1) and each of the scaling factors S2, S3, and S4. For error, there were no significant differences between S1 and S2 ( $t_{15} = -0.543$ , p = 0.595), and S1 and S3 ( $t_{15} = -1.72$ , p = 0.105) but errors were significantly higher for S4 than S1 ( $t_{15} = -3.61$ , p < 0.0167). For completion times, there was a significant difference between S1 and S2 ( $t_{15} = 2.71$ , p < 0.0167), and S1 and S3 ( $t_{15} = 2.54$ , p < 0.025), but not between S1 and S4 ( $t_{15} = 1.09$ , p = 0.292). These results show that subjects performed 11.5% faster with the S2 scaling factor and 15.0% faster with the S3 scaling factor with no statistically significant loss in accuracy.

Figure 3 shows the mean values for completion time across S1-S4 along with 95% confidence bands. The figure shows that the mean completion times decrease from S1 to S2 and S3 before increasing with S4. In addition, the standard deviations followed a similar trend, decreasing from S1 to S2 and S3, then increasing with S4. The figure also shows S3 having the lowest mean completion time. The results from the post-questionnaire (see Figure 4) show that subjects overwhelmingly preferred the S3 scaling factor. These two figures suggest there is a correlation between subject preferences and mean completion time for the rotation task. Since there was no significant difference between S1 and S3 for error (see





Figure 3: Mean completion times (in seconds) for each scaling factor with threshold and angle collapsed. There are significant differences between S1 and S2 and between S1 and S3.

Figure 5), the data suggests that a scaling factor of 3 is preferable amplification coefficient in a SSVE.



Subject Scaling Factor Preferences

Figure 4: Subject preferences for scaling factor.

As part of the post questionnaire, we also asked subjects to comment on their experience with the techniques. Five subjects reported that they needed a few extra trials before they felt their performance was at a high level. This comment suggests that some of the subjects may have needed more training time to get acclimated to the techniques. Given the fast completion times and low error rates for the different conditions, we feel that giving subjects any more training time would have not significantly improved performance. In addition, two subjects thought that having smaller amplifications would be better for tasks where only a small amount of rotation was needed.

<sup>&</sup>lt;sup>2</sup>These tests collapse the threshold and angle conditions.

Mean Error per Scaling Factor





## 5 DISCUSSION

Our experimental findings show some striking differences with other studies reported in the literature. Poupyrev et al. [14] reported an average of 6.8 degrees of error across both isomorphic and non-isomorphic rotation techniques while Hinckley et al. [6] reported 6.7 degrees of error. Both of these studies used a similar experimental design to our own with Poupyrev et al. emphasizing speed and Hinckley et al. emphasizing accuracy. Our results show an average of 3.9 degrees of error for all scaling factors. When we separate the trials with a threshold of six from those with the threshold of 18, we get average errors of 3.41 and 4.40, respectively. These are also below Ware and Rose's [16] result of 4.64 degrees of error for rotating ordinary physical objects. For completion time, Poupyrev et al. reported an average of 5.15 seconds for isomorphic rotation and approximately 4.75 seconds for nonisomorphic rotation, and Hinckley et al. reported an average of 17.8 seconds for isomorphic rotation. Note 17.8 seconds for Hinckley et al. is based on the subjects focusing on accuracy with little training on the rotation techniques. Our results show task completion times at an average of 2.2 seconds for isomorphic rotation and 1.96 seconds for the non-isomorphic techniques.

We believe that these differences in completion time and accuracy can be attributed to the different hardware configurations used in the experiments. The experiments discussed in the literature were all conducted using a desktop configuration, while our experiment was conducted in a SSVE. Hinckley et al.'s [6] observation that the accuracy of rotation might be less affected by the manipulation capabilities of the interface then by the difficulties subjects have in perceiving and adjusting the rotation error appears to be justified in our case. A SSVE with head tracking and stereoscopic viewing provides a much more natural representation of virtual objects (i.e., closer to physical realty) than a desktop configuration. We believe this conjecture also extends to completion times as well which would explain why the completion times for our tasks were much faster than those in the reported literature.

Another difference with our experimental results and Poupyrev et al's results is the scaling factor used for non-isomorphic rotation. Poupyrev et al. used a scaling factor of 1.8 based on empirical evidence. Our results show that this factor may not be the most efficient or preferred given that a scaling factor of three was preferred by subjects in our study and also provided the fastest task completion times. We believe that this preferred amplification factor is also related to the fact that we used a SSVE configuration.

Clearly, using a SSVE makes a difference in user performance for both isomorphic and non-isomorphic rotational mappings when rotating virtual objects. However, there are a number of factors that still need to be addressed to determine the precise reasoning behind this difference. Head tracking that allows for motion parallax and/or stereoscopic vision could be the distinguishing factor. Also, the size of the size of the display, refresh rate, age of subjects (effects of being within a different generation), proficiencies with video games (hand-eye coordination) as well as tracking lag could all play a role in determining the differences between our results and results from prior work. The common thread with these factors is that they contribute to how virtual objects are perceived in a VE as well as how subjects' cognitive abilities are tailored toward certain 3D tasks. Thus, there are several experimental studies that should be conducted in future work to further determine how these factors contribute to user performance with both isomorphic and non-isomorphic rotation in 3D user interfaces.

## 6 CONCLUSION

We have presented an experiment which explores non-isomorphic rotation in surround screen virtual environments (SSVE). Our study compared conventional isomorphic rotation with non-isomorphic rotation using three separate amplification factors under two different thresholds for accuracy and two different angular ranges. Our results have shown that rotation tasks can be completed 15.0% percent faster with an amplification factor of three than with isomorphic rotation without any statistically significant loss in accuracy. In addition, we found that test subjects greatly preferred nonisomorphic rotation with this amplification factor. Our results also suggest that both isomorphic and non-isomorphic rotation can be performed faster and more accurately in a SSVE, where perception of virtual objects is more closely matched with physical reality. More experiments are needed to determine the exact factors that contribute to this enhanced performance. However, we believe that this paper extends the knowledge regarding performance of non-isomorphic rotation and presents a good foundation for further analysis.

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