EE 3657 Makeup Test (Optional) - April 18, 2011

1. Find the inverse Laplace transform of $F(s) = \frac{5e^{-s}}{s+1}$. (10)

$\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t}$ $\cancel{f(t) = 5e^{-(t-1)}u(t-1)}$ 10

$f(t) = \mathcal{L}^{-1}\left\{\frac{5e^{-s}}{s+1}\right\} = 5e^{-(t-1)}u(t-1)$

2. Find all fixed points for the following dynamical system and linearize the system about those points: (30)

$$
\begin{aligned}
\dot{x} &= -x + x^3 + y(1+y) \\
\dot{y} &= -x(1+x)
\end{aligned}
$$

Fixed points are $(0,0), (0,-1), (-1,0), (-1,-1)$ 8

$J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} = \begin{bmatrix} -1+3x^2 & 1+2y \\ -1-2x & 0 \end{bmatrix}$ 10

$(0,0)$: $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ 3

$(0,-1)$: $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{bmatrix} -1 & -1 \\ -1 & 0 \end{bmatrix}\begin{pmatrix} x \\ y+1 \end{pmatrix}$ 3

$(-1,0)$: $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} x+1 \\ y-0 \end{pmatrix}$ 3

$(-1,-1)$: $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} x+1 \\ y+1 \end{pmatrix}$ 3

3. For a second order underdamped system whose characteristic equation is given by $Js^2 + Bs + K = 0$, the gain K has been fixed, all other parameters can be varied. How can you increase damping ratio without affecting settling time. (25)

(5) $\zeta = \frac{B}{2\sqrt{KJ}}$ (5) $t_s \propto \frac{2J}{B}$, to keep t_s unchanged,

$\frac{J}{B}$ = constant, increase J and B by factor n

where $n > 1$, we get $\zeta = \frac{nB}{2\sqrt{nKJ}} = \frac{\sqrt{n}\,B}{2\sqrt{KJ}}$

(15)

Thus ζ increases but t_s doesn't change.

4. A closed-loop system has a feedforward transfer function that consists of a proportional controller K in series with the plant $G(s) = \frac{1}{s^2}$. The feedback transfer function is given by $H(s) = \frac{1}{1+s}$. Find a range of K (if it exists) for which this system is stable in the closed-loop. (15)

Ch. poly. for closed-loop system is $s^3 + s^2 + k$, since 's' term is missing, system is unstable for all k values.

(15)

5. When are poles closest to the imaginary axis not dominant? (10)

When they are close to zeros. (10)

6. Unity feedback systems of Type n exhibit a zero steady-state error to a ramp input. What value(s) of n will make the preceding statement true? (10)

$n \geq 2$ (10)