EE 3657 Mid-term I - Feb 27, 2007 **Duration: 1 hour 15 minutes**

1. Linearize the nonlinear equations

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} xy \\ x^2 + 8xy + 3y^2 \end{bmatrix}$$

   in the region defined by $2 \leq x \leq 4, 10 \leq y \leq 12$. (50)

2. In response to a voltage source $v(t)$ applied to a series combination of a capacitor C $= 1$ F, an inductor L $= 1$ H, and a resistor R $= 2\ \Omega$, a current $i(t)$ flows in the circuit. Find an expression for the transfer function $\dfrac{I(s)}{V(s)}$. In detail, show whether the response of the system to an impulsive input voltage would be underdamped, overdamped, or critically damped. You can work in the time domain or in the Laplace domain. (40)

3. Why is it impossible to obtain oscillations in a first-order system? Use an example to illustrate your point if you like. (10)

Solutions - Test 1 - 3657

1.
$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} xy \\ x^2+8xy+3y^2 \end{bmatrix}$$

Choose $(x^*, y^*) = (3, 11)$

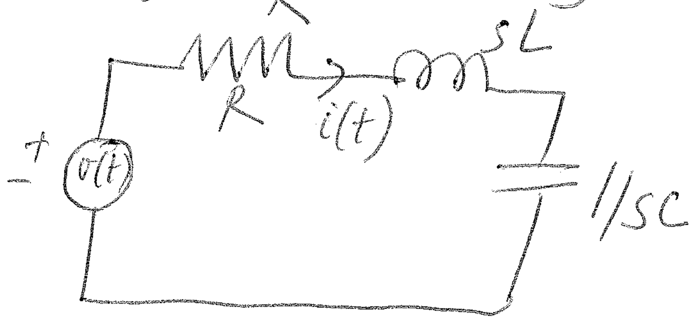
$$(z_1^*, z_2^*) = (3\times 11,\ 3^2 + 8\times 3\times 11 + 3\times 11^2) = (33, 636)$$ 10

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}_{lin} = \begin{bmatrix} z_1^* \\ z_2^* \end{bmatrix} + \begin{bmatrix} \frac{\partial(xy)}{\partial x} & \frac{\partial(xy)}{\partial y} \\ \frac{\partial(x^2+8xy+3y^2)}{\partial x} & \frac{\partial(x^2+8xy+3y^2)}{\partial y} \end{bmatrix}_{(x^*, y^*)} \begin{bmatrix} x-x^* \\ y-y^* \end{bmatrix}$$ 20

$$= \begin{bmatrix} 33 \\ 636 \end{bmatrix} + \begin{bmatrix} y & x \\ 2x+8y & 8x+6y \end{bmatrix}_{(3,11)} \begin{bmatrix} x-3 \\ y-11 \end{bmatrix}$$ 20

$$= \begin{bmatrix} 11x + 3y - 33 \\ 94x + 90y - 636 \end{bmatrix}$$

2.



By KVL, $v(t) = Ri(t) + L\frac{di}{dt} + \frac{1}{C}\int_0^t i(\tau)\,d\tau$

(2)

Take laplace transform (with zero initial conditions)

$$V(s) = \left(R + sL + \frac{1}{sC}\right) I(s)$$

$$\Rightarrow \frac{I(s)}{V(s)} = \frac{sC}{s^2LC + sRC + 1}$$

(25)

$R = 2, L = 1, C = 1 \Rightarrow s^2LC + sRC + 1$

$$= s^2 + 2s + 1$$

$$= (s+1)^2$$

(15)

Roots of ch polynomial are real and identical, system is critically damped. (10)

3. In a first order system, there are only real roots and we know that the response can only be monotonic. Only roots with imaginary parts show oscillations in the response such as underdamped second order systems.

(10)