EE 450/550
Test # 1 - Oct 20, 2005 in class

1. An extended object of mass m falling downward under gravity (assume constant acceleration due to gravity g) is known to experience a resistive force of the air called drag. We assume that the magnitude of this force is proportional to its speed v. Write down the dynamics for the object velocity. Solve the dynamical equation to calculate a formula for $v(t)$ assuming that $v(0) = v_0$. Use symbol k for the proportionality constant. Hint: You May Use Laplace. (50)

2. Linearize the following system

$$\dot{x} = x - \cos x$$

about its fixed point(s). Use the following plot if necessary. Hint: First find fixed point $\bar{x}$ and then apply Taylor series expansion upto linear term about that fixed point. (40)

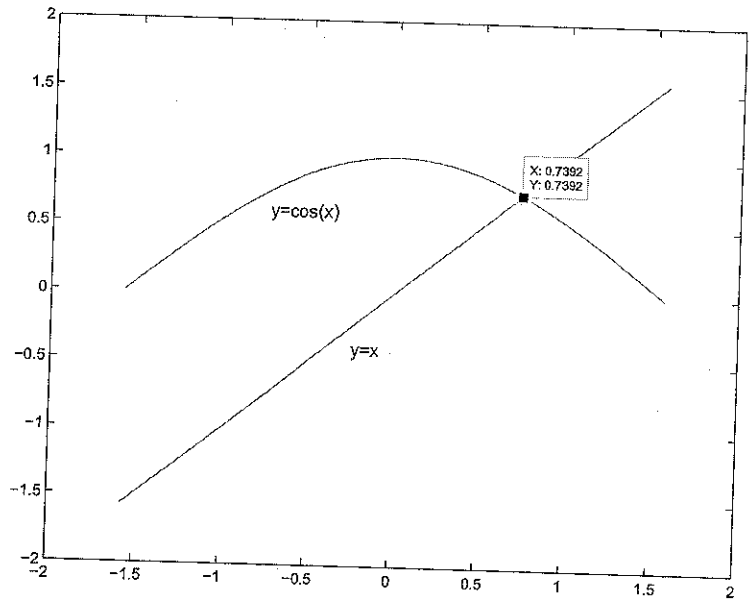
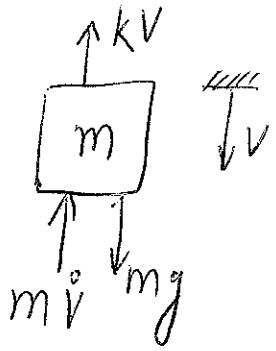


Figure 1:

3. Linear systems are characterized by the property of superposition. Explain what is meant by superposition? (10)

Solutions - Test 1

1.



$$m\dot{v} + kv = mg$$

$$msV(s) - mv(0) + kV(s) = \frac{mg}{s}$$

$$V(s)(ms + k) = mv_0 + \frac{mg}{s}$$

$$V(s) = \frac{mv_0}{ms+k} + \frac{mg}{s(ms+k)}$$

$$V(s) = \frac{v_0}{s + k/m} + \frac{g}{s(s + k/m)}$$

$$= \frac{v_0}{s+k/m} + \frac{mg}{k}\left[\frac{1}{s} - \frac{1}{s+k/m}\right]$$

$$v(t) = v_0 \exp(-k/m\,t) + \frac{mg}{k} - \frac{mg}{k}\exp(-k/m\,t)$$

$$= \left(v_0 - \frac{mg}{k}\right)\exp(-k/m\,t) + \frac{mg}{k}$$

2. From the graph, we can see that

$x = \cos x$ solves for $\bar{x} = 0.7392$

$$\dot{x} = x - \cos x$$

For fixed point $x - \cos x = 0$

$\Rightarrow \bar{x} = 0.7392$

Linearizing the right hand side gives

$$\dot{x} = f(\bar{x}) + \left.\frac{\partial f(x)}{\partial x}\right|_{\bar{x}} (x - \bar{x})$$

$$= 0 + \left.(1 + \sin(x))\right|_{\bar{x}} (x - 0.7392)$$

$$= (1 + \sin(0.7392))(x - 0.7392)$$

$$= 1.6737(x - 0.7392)$$

$$= 1.6737(x) - 1.2372$$

$$= 1.67x - 1.24$$

3. Given $y = f(x)$, then for any inputs x_1 and x_2, $y_1 = f(x_1)$, $y_2 = f(x_2)$, if given an input $x_3 = x_1 + x_2$, then $y_3 = f(x_1 + x_2) = f(x_1) + f(x_2)$, it means superposition applies.