Solutions

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EE 3657 Test 1 - Feb 22, 2010

1. Find the inverse Laplace transform of $F(s) = \dfrac{5e^{-s}}{s+1}$. (12)

$$f(t) = 5e^{-(t-1)}u(t-1)$$

u is the unit step. (12)

2. Find all fixed points for the following dynamical system and linearize the system about those points: (48)

$$\begin{aligned} \dot{x} &= -x + x^3 + y(1+y) \\ \dot{y} &= -x(1+x) \end{aligned}$$

(12)+(12)+(12)+(12)

Fixed Points: $(0,0), (0,-1), (-1,0), (-1,-1)$

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{pmatrix} = \begin{pmatrix} -1+3x^2 & 1+2y \\ -1-2x & 0 \end{pmatrix}$$

$$J_1 = \begin{pmatrix} -1+3x^2 & 1+2y \\ -1-2x & 0 \end{pmatrix}\Bigg|_{0,0} = \begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x-0 \\ y-0 \end{pmatrix}$$

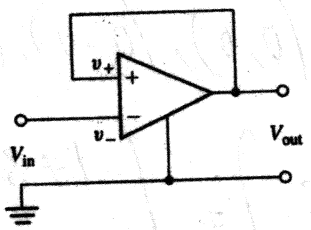
Similarly $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x-0 \\ y+1 \end{pmatrix}$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x+1 \\ y-0 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x+1 \\ y+1 \end{pmatrix}$$

3. A realistic model of an op-amp is given by the equation below. Find the transfer function from input to output and using that transfer function, determine if the circuit configuration shown below is stable or unstable. (40)

$$V_{out} = \frac{10^7}{s+1}[V_+ - V_-] \qquad i_+ = i_- = 0$$



(10)

$V_- = V_{in}$, $V_+ = V_{out}$ $\Rightarrow$ $sV_{out} + V_{out} = 10^7 V_{out} - V_{in}$

$\Rightarrow$ $\frac{V_{out}}{V_{in}} = \frac{-1}{s + (1 - 10^7)}$ (20)

(10)

The root is at $s = 10^7 - 1$ and is in the right half plane. System is unstable.