1. Find the inverse Laplace transform of $F(s) = \dfrac{5e^{-2s}}{(s+1)^2}$. (20)

$\mathcal{L}^{-1}\left\{\left(\frac{1}{s+1}\right)^2\right\} = te^{-t}u(t)$ (10)

$\mathcal{L}^{-1}\left\{\frac{5e^{-2s}}{(s+1)^2}\right\} = 5(t-2)e^{-(t-2)}u(t-2)$ (10)

2. Find all fixed points for the following dynamical system and linearize the system about those points: (30)

$$\begin{aligned}\dot{x} &= -x + x^2 \\ \dot{y} &= -xy + 2y\end{aligned}$$

At fixed points: $-x + x^2 = 0 \Rightarrow x = 0, x = 1$

$-xy + 2y = 0 \Rightarrow y(2 - x) = 0 \Rightarrow y = 0, x = 2$ (10)

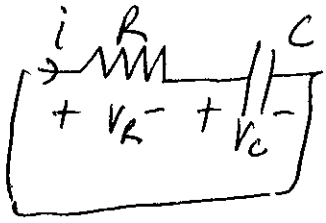
So, fixed points are: $(0,0)$ and $(1,0)$

$$J = \begin{pmatrix} -1 + 2x & 0 \\ -y & -x + 2 \end{pmatrix}$$ (8)

@ $(0,0)$ $\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix}_{lin} = \begin{pmatrix}-1 & 0\\ 0 & 2\end{pmatrix}\begin{pmatrix}x-0\\ y-0\end{pmatrix} = \begin{pmatrix}-x\\ 2y\end{pmatrix}$ (6)

@ $(1,0)$ $\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix}_{lin} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x-1\\ y-0\end{pmatrix} = \begin{pmatrix}x-1\\ y\end{pmatrix}$ (6)

where V_C denotes the voltage across the capacitor. Find an expression for $V_C(t)$ for $t \geq 0$. (40)



(10) $i_R = \frac{V_R}{R} = i_C = C\frac{dV_C}{dt}$

$V_R = -V_C$, $R = 1, C = 1$

(10) $V_C + \frac{dV_C}{dt} = 0 \Rightarrow$ (10) $V_C(s) + sV_C(s) - V_C(0) = 0$

$\Rightarrow V_C(s) = \frac{V_C(0)}{s+1} \Rightarrow V_C(t) = V_C(0)e^{-t}\ \forall t \geq 0$

(10) $= 10e^{-t}\ \forall t \geq 0$

4. What are the allowable pole locations for which the final value theorem is applicable? (10)

Pole locations are allowed in the strict LHP plus a single pole at the origin. No other pole locations are allowed.

(10)