EE 450/550 Final Exam - Dec 8, 2003

1. A unity feedback closed-loop system has a feedforward transfer function $G(s) = \frac{1}{s}$. Design a compensator such that the DCLPs are located at $s = -1 \pm j1$. (30)

2. Derive an expression for the resonant frequency for a second order underdamped system with damping ratio ζ and undamped frequency ω_n. Find the expression for the resonant peak. (25)

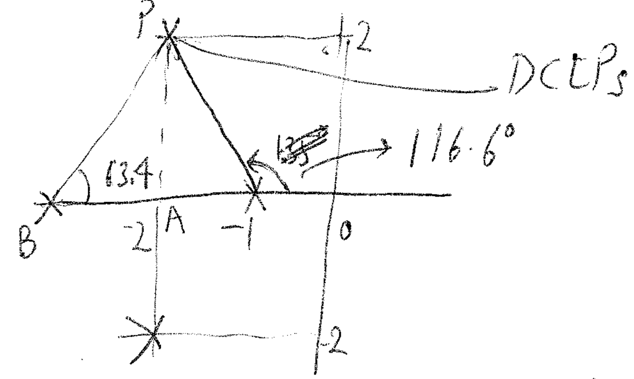
3. What is the critical value of open-loop gain (dB) for a system at the phase crossover frequency for closed-loop stability. (5)

4. The open loop transfer function of a system is $G(s) = \frac{K}{s(s+2)}$. The desired closed-loop poles are $s = -1 \pm j1$. Do you need a compensator. If so, what kind? Explain your choice. (10)

5. Draw rough Bode plots (both magnitude and phase) for a first order lead and lag network. (20)

6. A unity feedback closed-loop system has a feedforward transfer function $G(s) = \frac{1}{(s+1)^2}$. If the desired closed-loop poles are at $s = -2 \pm j2$. What is the angle excess/deficiency at that location? (10)

4.



angle excess : $\angle \frac{1}{s+1}\Big|_{-2\pm j2} = -116.6^\circ$

$AP = 2 \quad AB = ?$

$AP = AB \times \tan 63.4 \Rightarrow AB = \frac{AP}{\tan 63.4} = \frac{2}{2} = 1$

So pole is place at -3, compensator is $\frac{K_c}{s+3} = G_c$

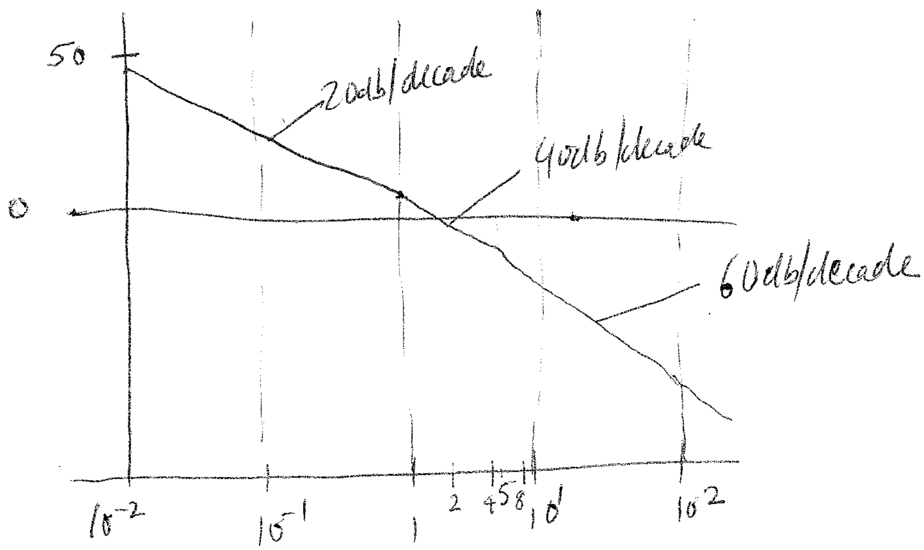
Find K_c by mag cond

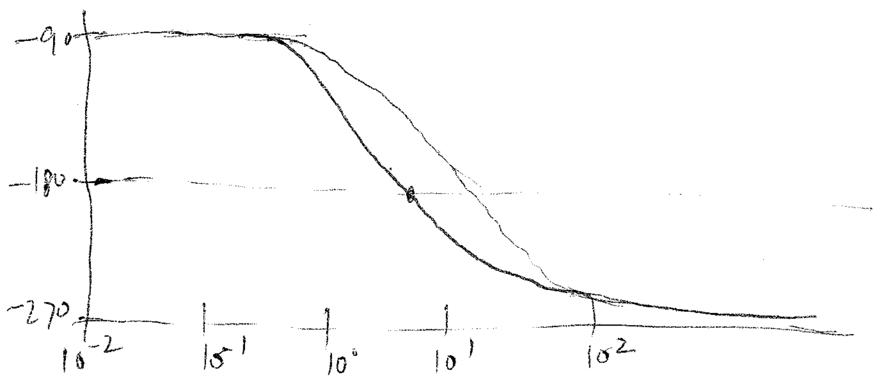
$$K_c = \left|(s+1)(s+3)\right|_{s=-2+j2}$$

$$= \left|(-1+j2)(1+j2)\right| = 5$$

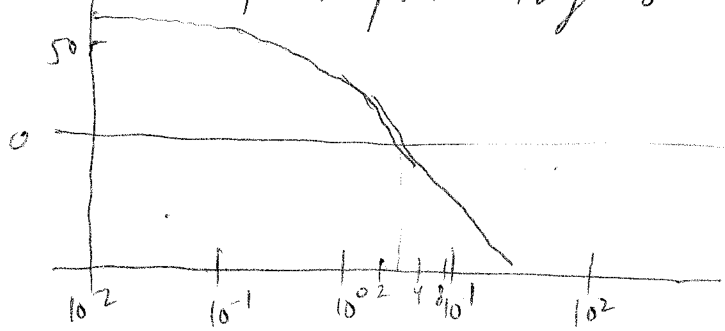
$$G_c = \frac{5}{s+3}$$

2. $K = 10$, $GM = 9.54\,dB$, $PM = 25.4^\circ$

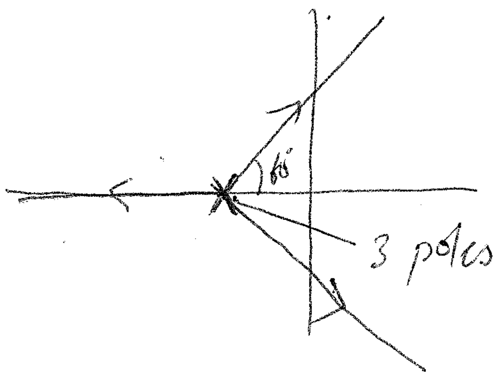




For K = 100, raise the previous gain plot by 20dB
phase plot stays same



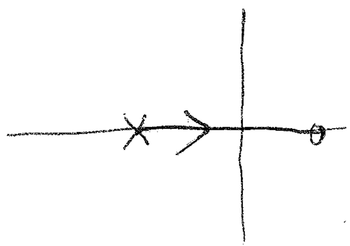
5.



$$G = \frac{K}{(s+1)^3}$$

4. $180°$

6.

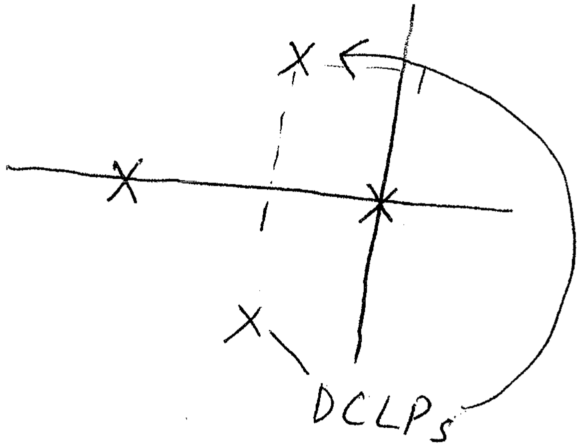


with a $\underbrace{\text{RHP}}_{\text{right half plane}}$ zero

6.

$z = 94x + 90y - 636$ (About $x=3$, $y=11$)

7.



Place a pole at $s=-2$, the point $-1\pm j1$ will be on root locus if you vary K.

$$K\cdot\left|\frac{1}{s(s+2)}\right|_{s=-1+j1}=1$$

$$\Rightarrow\quad K\cdot\left|\frac{1}{(-1+j1)(1+j1)}\right|=1$$

$$\Rightarrow\quad K\cdot\frac{1}{2}=1\ \Rightarrow\ K=2.$$

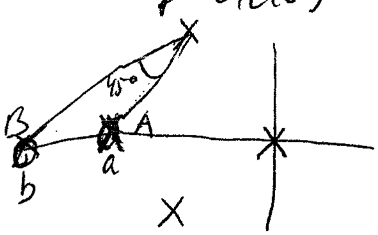
So, compensator is $\frac{2}{s+2}$.

or use systematic design technique taught in class.

$$\left.\angle\frac{1}{s}\right|_{-1+j1}=-135^\circ$$

Angle excess is 45°, so need to lag via compensator to the tune of -45°. let

$G_c(s)=K_c\frac{s+b}{s+a}$, where $b>a$ and



$\angle APB=45^\circ$. If $a=+1$ and and $b=+2$, then angle subtended by pole is 90° and that by zero is 45°. Then, angle deficiency is -45°.

$$G_c(s) = K_c \frac{s+2}{s+1}$$

$$G_cG = K_c \frac{(s+2)}{s(s+1)}$$

To find K_c, then use mag condition

$$\Rightarrow K_c = \left|\frac{s(s+1)}{s+2}\right|_{-1+j1} = \left|\frac{(-1+j1)\,j1}{(1+j1)}\right| = 1$$

$$\text{So } G_c = 1 \cdot \frac{s+2}{s+1}$$

2. $$G(j\omega) = \frac{1}{1 + 2\zeta(j\omega/\omega_n) + (j\omega/\omega_n)^2}$$

$$|G(j\omega)| = \frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}} \triangleq g(\omega)$$

When denominator is minimum, then $|G(j\omega)|$ has a maximum which is the resonant peak.

$$g(\omega) = \left(\frac{\omega^2 - \omega_n^2(1-2\zeta^2)}{\omega_n^2}\right)^2 + 4\zeta^2(1-\zeta^2)$$

Solve $\frac{d\,g(\omega)}{d\omega} = 0 \Rightarrow \omega = \omega_n\sqrt{1-2\zeta^2} \Rightarrow \omega_r = \omega_n\sqrt{1-2\zeta^2}$

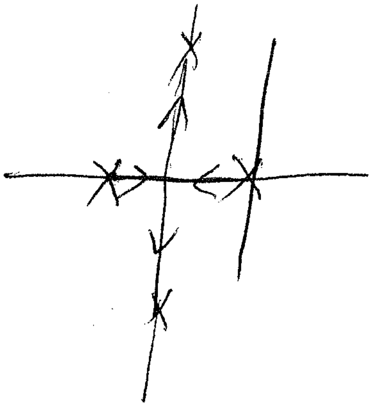
of course ω_r is real only when $1-2\zeta^2 > 0 \Rightarrow \zeta < 1/\sqrt{2}$

Then resonance peak $M_r = |G(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$

For $0.707 < \zeta < 1$, $M_r = 1$. Since $|G(j\omega)|$ just keeps decreasing as ω goes from 0 to ∞.

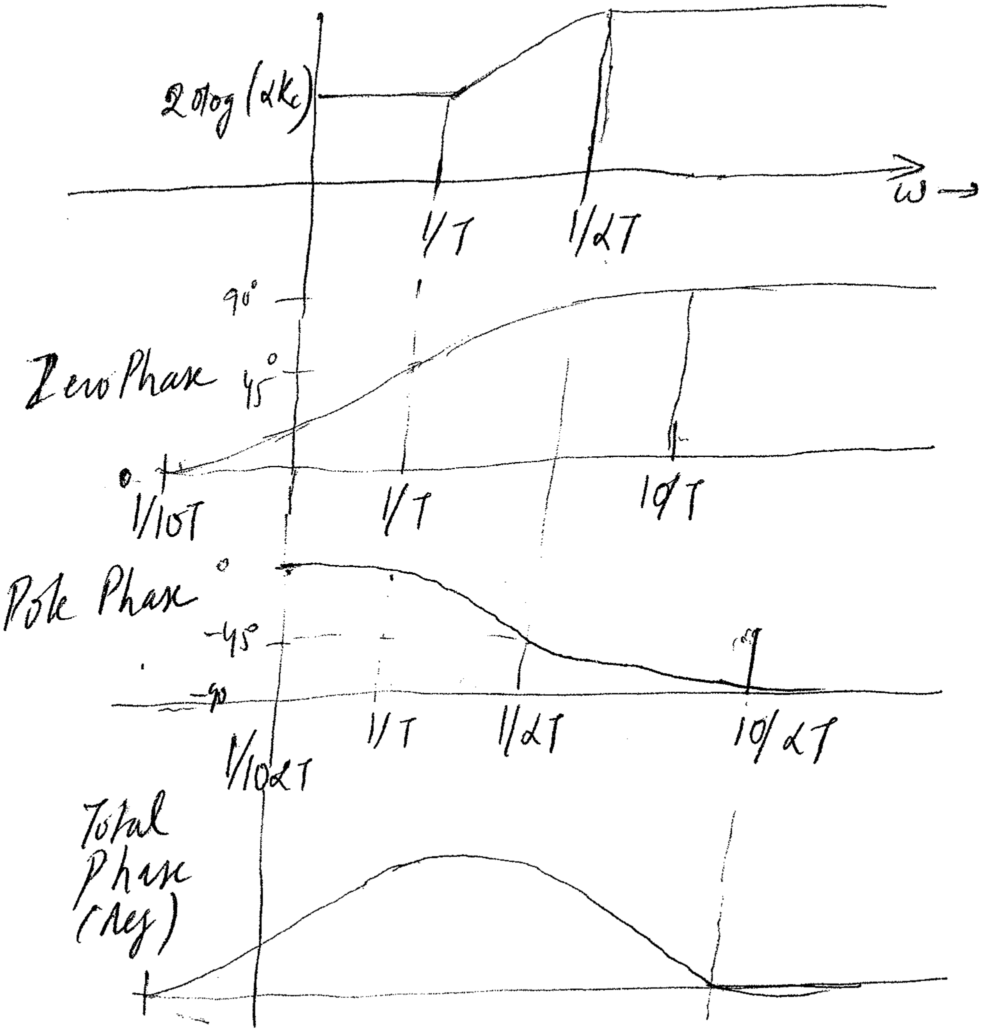
3. $|G(j\omega)| = 1$ or $20\log|G(j\omega)| = 0$

8.

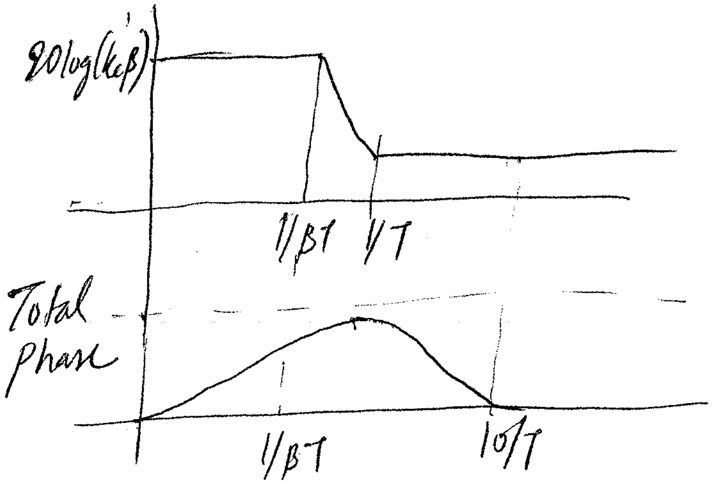


From the RL, it is obvious that a gain change is sufficient.

5. Lead: $G(s) = K_c \dfrac{s+1/T}{s+1/\alpha T}$ $\quad 0<\alpha<1$



Lag: $G(s) = K_c \frac{s + 1/T}{s + 1/\beta T}$, $\beta > 1$



Q9. $\angle \frac{1}{(s+1)^2} \Big|_{s=-2 \pm j2} = -116.565 \times 2$

$= -233.13^\circ$

Angle deficiency is $\phi = 53.13^\circ$