EE 3657 Final - Dec 8, 2008

1. A unity feedback system has a feedforward transfer function given by $G(s) = \dfrac{1}{s(s+1)}$. A series compensator of the form $\dfrac{s+3}{s+5}$ is designed. Calculate for both the uncompensated and compensated system the location of the dominant closed-loop poles and the static velocity error constant K_v. (Hint: Non-dominant root for compensated system is at $s = -4.9006$) (40)

Uncomp: $s^2+s+1=0 \Rightarrow s = (-1 \pm \sqrt{1-4})/2$, $K_v = \lim_{s\to 0} sG = 1$

Comp: $s(s+1)(s+5)+(s+3)=0 \Rightarrow s^3+6s^2+6s+3=0$

$\Rightarrow s_1 = -4.9,\ s_{2,3} = -0.55 \pm 0.557j$

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = \cancel{s}\cdot\frac{1}{\cancel{s}(s+1)}\frac{s+3}{s+5} = \frac{3}{5}$$

2. In a sample question, we saw that the resonant frequency ω_r for an underdamped system $G(j\omega) = \dfrac{1}{1+2\zeta\left(j\frac{\omega}{\omega_n}\right)+\left(j\frac{\omega}{\omega_n}\right)^2}$ is given by $\omega_r = \omega_n\sqrt{1-2\zeta^2}$. Write a general expression for the phase angle of $G(j\omega)$ and then find its expression at the resonant frequency. (20)

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{2\zeta\,\omega/\omega_n}{1-\omega^2/\omega_n^2}\right)$$

$$\angle G(j\omega)\Big|_{\omega=\omega_r} = -\tan^{-1}\left(\frac{2\zeta\,\omega_n\sqrt{1-2\zeta^2}/\omega_n}{1-\cancel{\omega_n^2}(1-2\zeta^2)/\cancel{\omega_n^2}}\right) = -\tan^{-1}\left(\frac{\cancel{2}\zeta\sqrt{1-2\zeta^2}}{\cancel{2}\zeta^2}\right)$$

$$= -\tan^{-1}\left(\frac{\sqrt{1-2\zeta^2}}{\zeta}\right)$$

3. Two systems have transfer functions given by: (1) $G(s) = \dfrac{s+1}{(s^2+2s+2)(s+3)}$, and (2) $G(s) = \dfrac{s-1}{(s^2+2s+2)(s+3)}$. For both the cases, what is the slope (express in dB/decade) and the phase angle of the bode-plot as $\omega \to \infty$. (20)

(1) Slope $= n - m = (2)(20) =$ 40dB/decade

Phase angle $= -180° = -270° + 90°$

(2) Slope $= n - m = (3-1)(20) =$ 40dB/decade

Phase angle $= -270° - 90° = -360°$

4. Label gain and phase margin on an arbitrary (a) Bode and (b) Nyquist plot. (20)

(a)