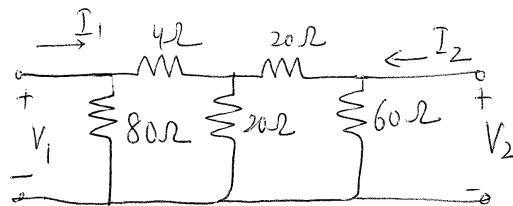


EEL 3123C TEST 3 - PART A - MANDATORY DURATION: 60 minutes  
Dec 7, 2007

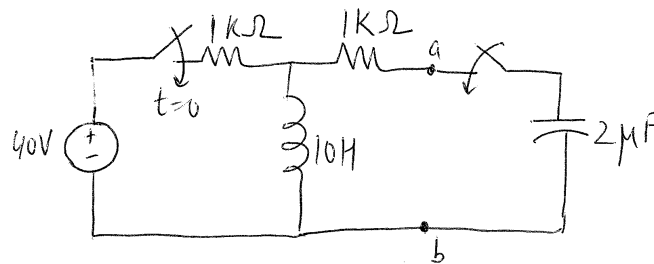
- The voltage across a  $50 \Omega$  resistor is  $v(t) = 4t \exp(-|t|)$ . What is the total energy dissipated in the resistor? What percentage of the total energy dissipated in the resistor can be associated with the the frequency band  $0 \leq \omega \leq 10$  rad/s? Hint:  $\int \cos^4 \theta \sin^2 \theta d\theta = \frac{1}{16}\theta + \frac{1}{64}\sin 2\theta - \frac{1}{64}\sin 4\theta - \frac{1}{192}\sin 6\theta$  (50)
- Find the z parameters for the following circuit (30)



- The following signals are sampled at 1KHz: (a)  $\sin(100\pi t)$ , and (b)  $\exp(-5t)$ . Which ones would exhibit aliasing? Justify your answer. (20)

EEL 3123C TEST 3 - PART B - OPTIONAL DURATION: 60 minutes

- Compute the transfer function of a series RC low-pass filter that has a load resistor  $R_L$  in parallel with its capacitor. Find an expression for  $\max|H(j\omega)|$  and use that to find the cut-off frequency. (50)
- For the circuit given below, there is no energy stored in the circuit at the time the switches close. Find the s-domain Thevenin equivalent of the circuit to the left of the terminals a,b. Use that to find an expression for  $i(t)$  for  $t \geq 0^+$ . (50)



Part A

$$R = 50 \Omega$$

$$v(t) = 4t \exp(-|t|)$$

$$V(\omega) = 4j \frac{d}{d\omega} \mathcal{F}\{\exp(-|t|)\} = 4j \frac{d}{d\omega} \left( \frac{2}{1+\omega^2} \right)$$

$$= 8(-1) \frac{1}{(1+\omega^2)^2} \cdot 2\omega j = \frac{-16\omega j}{(1+\omega^2)^2}$$

Energy consumed in resistor is  $\int_{-\infty}^{\infty} \frac{(4t \exp(-|t|))^2}{50} dt$

$$= \frac{1}{50\pi} \int_0^{\infty} |F(\omega)|^2 d\omega = \frac{1}{50\pi} \int_0^{\infty} \frac{256\omega^2}{(1+\omega^2)^4} d\omega$$

• let  $\omega = \tan \theta$   
 $d\omega = \sec^2 \theta d\theta$

$$\text{Energy} = \frac{256}{50\pi} \int_0^{\pi/2} \frac{\tan^2 \theta \cdot \sec^2 \theta \cdot d\theta}{\sec^8 \theta} = \frac{256}{50\pi} \int_0^{\pi/2} \frac{\sin^2 \theta}{\sec^4 \theta} d\theta$$

$$= \frac{256}{50\pi} \int_0^{\pi/2} \cos^4 \theta \sin^2 \theta d\theta = \frac{5}{50} = \frac{1}{10} = 0.1 = 0.16$$

Energy consumed in band  $0 \leq \omega \leq 10$

$$= \frac{1}{50\pi} \int_0^{10} |F(\omega)|^2 d\omega = \frac{256}{50\pi} \int_0^{1.47} \cos^6 \theta \sin^2 \theta d\theta$$

$$\approx 0.16$$

Approximately 100% of the energy lies in the band  $0 \leq \omega \leq 10$  rad/s.

2.

$$z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} = (((60 + 20) \parallel 20) + 4) \parallel 80 = 16$$

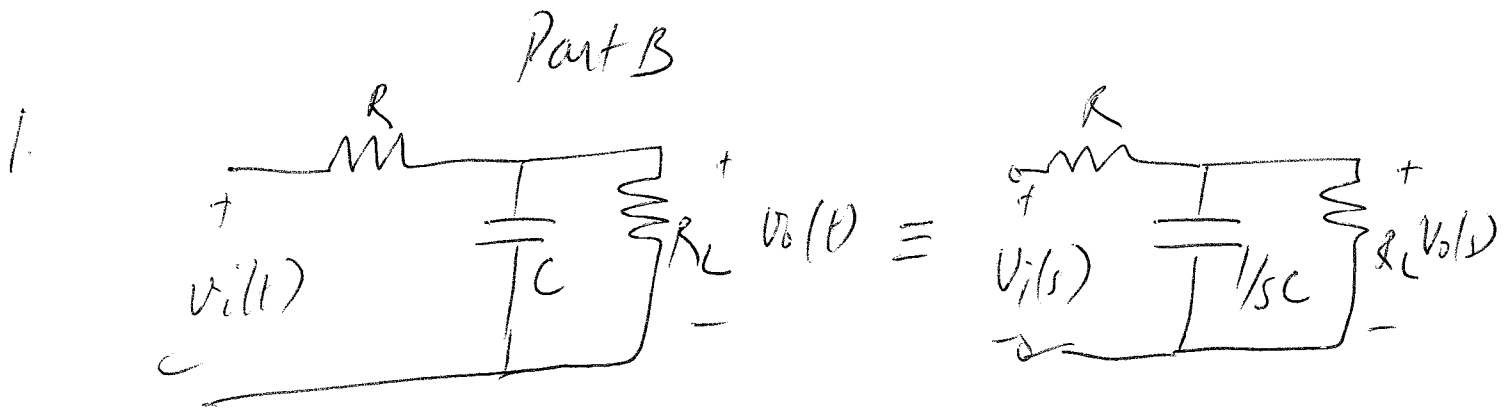
$$z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0} = (((80 + 4) \parallel 20) + 20) \parallel 60 = 22.56$$

$$z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0} = \frac{((80 + 4) \parallel 20)V_2}{((80 + 4) \parallel 20) + 20} \frac{80}{(80 + 4)} \frac{z_{22}}{V_2} = 9.6\Omega$$

$$z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0} = \frac{((60 + 20) \parallel 20)V_1}{(((60 + 20) \parallel 20) + 4)} \frac{60}{(60 + 20)} \frac{z_{11}}{V_1} = 9.6\Omega$$

3: (a)  $\sin(100\pi t)$  has a frequency of 50 Hz, so any sample rate above 100 Hz is OK, so this will not get aliased.

(b)  $\exp(-5t)$  has <sup>finite</sup> no maximum frequency, so it will get aliased.



$$\frac{V_o(s)}{V_i(s)} = H(s) = \frac{R_L \parallel 1/sC}{R + R_L \parallel 1/sC} = \frac{R_L \cdot \frac{1}{sC}}{\left(\frac{R_L + 1}{sC}\right) \left(R + \frac{R_L \cdot \frac{1}{sC}}{R_L + 1}\right)}$$

$$= \frac{R_L \cdot 1/sC}{R R_L + \frac{R}{sC} + \frac{R_L}{sC}} = \frac{R_L}{R + R_L + R R_L \cdot sC}$$

$$= \frac{1/RC}{s + \frac{R + R_L}{R R_L C}} = \frac{1/RC}{s + \frac{1}{KRC}}$$

where  $K^{-1} = (R + R_L)/R_L$

$$|H_{\max}(j\omega)| = K$$

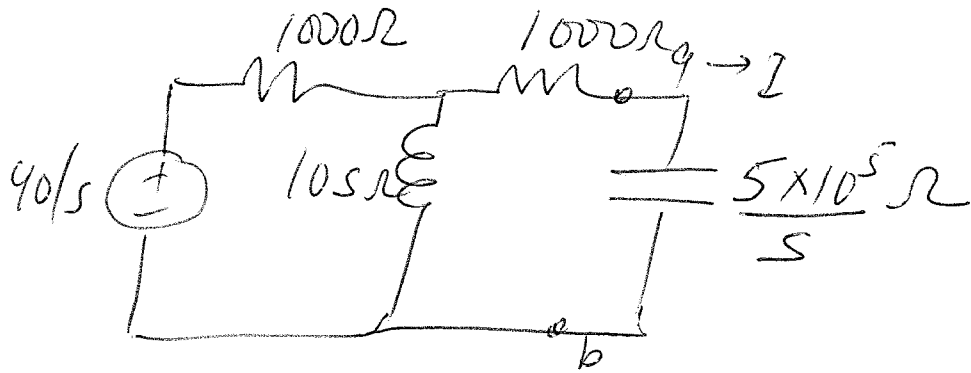
$$\frac{1/RC}{\sqrt{j\omega_c^2 + \frac{1}{K^2 R^2 C^2}}} = \frac{1}{\sqrt{2}} K$$

$$\Rightarrow \frac{1}{R^2 C^2} = \frac{K^2}{2} \cdot \left(\omega_c^2 + \frac{1}{K^2 R^2 C^2}\right) \Rightarrow \omega_c^2 = \frac{2}{K^2} \left(\frac{1}{R^2 C^2} - \frac{1}{2R^2 C^2}\right)$$

$$= \frac{1}{K^2 R^2 C^2}$$

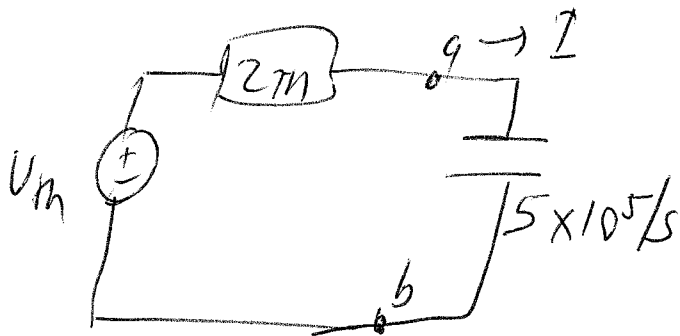
$$\Rightarrow \omega_c = \frac{1}{KR C}$$

2.



$$V_{Th} = \frac{10s}{10s + 1000} \cdot \frac{40}{s} = \frac{40}{s + 100}$$

$$Z_{Th} = 1000 + 1000 \parallel 10s = \frac{2000(s + 100)}{s + 100}$$



$$I = \frac{0.02s}{s^2 + 300s + 25,000}$$

$$i(t) = 63.25 e^{-150t} \cos(50t + 71.5^\circ) u(t) \text{ mA}$$