Test 3 – Networks and Systems 12/4/06

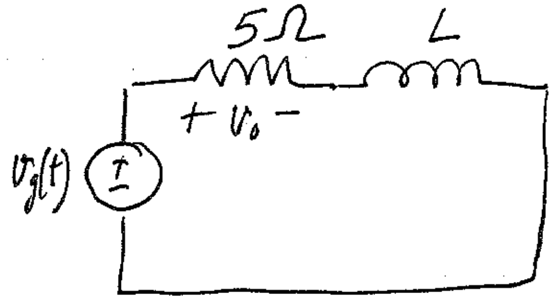
Part A – Covers chapter 17

1. Prove that:

(a) $\mathcal{F}\{f_1(t)f_2(t)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)F_2(\omega-u)\,du$ (25)

(b) $j^2 \frac{d^2F(\omega)}{d\omega^2} = \mathcal{F}\{t^2 f(t)\}$ (25)

2.



Given that $v_g(t) = \mathrm{sgn}(t)$ is the input to the system above, an output voltage $v_o(t)$ is recorded as $v_o(t) = \mathrm{sgn}(t) - 2\exp(-t)u(t)$. Find the value of the inductor L. (50)

Part B (Optimal)

1. Given $F(s) = \dfrac{5s^2 + 29s + 32}{s^2 + 6s + 8}$, find $f(t)$.

   Find $\lim_{t \to 0^+} f(t)$ and $\lim_{t \to \infty} f(t)$ by using, respectively, the initial and final value theorem if the limit exists. Verify your answer by computing the limit from your $f(t)$ computation. (50)

2. A high-pass RL filter with a cutoff frequency of 25 krad/s needs to be constructed. A 5 mH inductor is available, find the value of the required resistor. Next, find the smallest load resistor that can be connected across the output terminals of the filter such that the cutoff frequency does not drop below 24 krad/s. (50)

   Hint: With load resistor in the circuit, $|H(j\omega)|_{max}$ is less than 1, use this fact to compute the cut-off frequency.

(1) Solutions to Test 3 - Networks & Systems 12/4/06

Part A

1. (a) $\mathcal{F}\{f_1(t)f_2(t)\} = \int_{-\infty}^{\infty} \underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)e^{jut}du}_{f_1(t)} \cdot f_2(t)e^{-j\omega t}dt$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)\left\{\int_{-\infty}^{\infty} f_2(t)e^{-j(\omega-u)t}dt\right\}du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)F_2(\omega-u)\,du$$

(b) $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$

$$\frac{dF}{d\omega} = \int_{-\infty}^{\infty} \frac{d}{d\omega}\left(f(t)e^{-j\omega t}dt\right) = -j\int_{-\infty}^{\infty} t f(t)e^{-j\omega t}dt$$

$$\frac{d^2F}{d\omega^2} = (-j)(-j)\int_{-\infty}^{\infty} t^2 f(t)e^{-j\omega t}dt$$

$$\Rightarrow j^2\frac{d^2F}{d\omega^2} = \mathcal{F}\{t^2 f(t)\}$$

2. $v_g(t) = \mathrm{sgn}(t) \Rightarrow V_g(j\omega) = 2/j\omega$

$$\frac{V_0(j\omega)}{V_g(j\omega)} = \frac{1}{1+(j\omega)(L/R)} \Rightarrow V_0(j\omega) = \frac{1}{1+j\omega(L/5)}\cdot\frac{2}{j\omega}$$

$$\Rightarrow V_0(j\omega) = \frac{A}{1+j\omega(L/5)} + \frac{B}{j\omega}$$

$$A = \left.\frac{2}{j\omega}\right|_{j\omega=-5/L} = \frac{-2L}{5}, \quad B = \left.\frac{2}{1+j\omega}\right|_{j\omega=0} = 2$$

(2)

$$\Rightarrow V_o(j\omega) = \frac{-2L}{5}\cdot\frac{1}{1+j\omega\cdot L/5} + \frac{2}{j\omega}$$

$$= \frac{-2}{5/L + j\omega} + \frac{2}{j\omega}$$

$$\Rightarrow v_o(t) = \operatorname{sgn}(t) - 2\exp(-5/L\, t)\,u(t)$$

Compare with $v_o(t) = \operatorname{sgn}(t) - 2\exp(-t)\,u(t)$

$$\Rightarrow 5/L = 1 \Rightarrow L = 5\,H$$

Part B

1. $$F(s) = 5 + \frac{-s-8}{s^2+6s+8} = 5 + \frac{A}{s+4} + \frac{B}{s+2}$$

$$A = \left.\frac{-(s+8)}{(s+2)}\right|_{s=-4} = \frac{-4}{-2} = 2$$

$$B = \left.\frac{-(s+8)}{(s+4)}\right|_{s=-2} = -\frac{6}{2} = -3$$

$$f(t) = 5\delta(t) + \left[2\exp(-4t) - 3\exp(-2t)\right]u(t) \quad (1)$$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \frac{5s^3+29s^2+32s}{s^2+6s+8} = 0$$ which squares with (1)

Cannot apply IVT because $F(s)$ is not strictly proper.
This squares with (1) because we see an impulse at $t=0$ and therefore, initially value is not defined.

2. $$R/L = 25{,}000 \Rightarrow R = 25{,}000 \times 5\times 1e{-3} = 125\,\Omega$$

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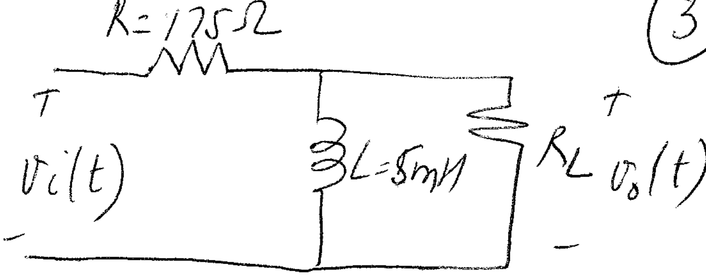

$$\frac{V_o(s)}{V_i(s)} = H(s) = \frac{\left(\frac{R_L}{R+R_L}\right)s}{s+\left(\frac{R_L}{R+R_L}\right)\frac{R}{L}} = \frac{Ks}{s+\omega_0}$$

$$|H(j\omega)|_{max} = \max_{\omega} \frac{K\omega}{\sqrt{\omega^2+\omega_0^2}} = K$$

To find the cutoff frq, at $\omega = \omega_c$

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}} \Rightarrow \frac{K\omega_c}{\sqrt{\omega_c^2+\omega_0^2}} = \frac{K}{\sqrt{2}} \Rightarrow \omega_c = \omega_0$$

$$\Rightarrow \frac{R_L}{(125+R_L)} \times \frac{125}{.005} = 24000 \Rightarrow 25R_L = 24R_L + 24 \times 125$$

$$\Rightarrow R_L = 3000\,\Omega$$