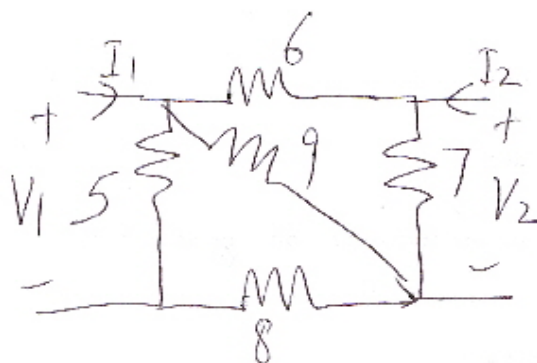


EEL 3123C TEST 3 - PART A - MANDATORY DURATION: 60 minutes

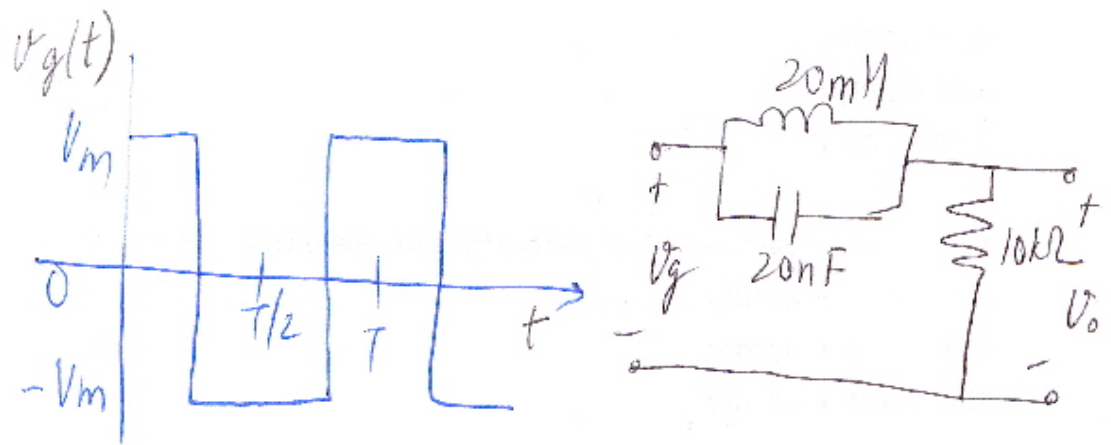
1. The current through a 50Ω resistor is $i(t) = 4t \exp(-t) u(t)$. What percentage of the total energy dissipated in the resistor can be associated with the frequency band $0 \leq \omega \leq \sqrt{3}$ rad/s? (50)
2. Find the z parameters for the following circuit (30)



3. The following signals are sampled at 50 Hz: (a) $\sin(100\pi t)$, and (b) $\cos(100\pi t)$. Which ones would exhibit aliasing? Justify your answer. (20)

EEL 3123C TEST 3 - PART B - OPTIONAL DURATION: 60 minutes

1. Find the inverse Laplace transform of $F(s) = \frac{1}{(s+1)^2}$. (20)
2. Given a series RLC circuit, compute the component values that yield a bandreject filter with a bandwidth of 250 Hz and a center frequency of 750 Hz. Use a 100 nF capacitor. (20)
3. Compute the transfer function of a series RC low-pass filter ($R = 100 \Omega$, $C = 10 \mu F$). Compute the expression for the steady state output to an input $v_i = 10 \sin(100\pi t)$. (20)
4. The periodic square wave shown below is applied to the circuit shown below. Derive the first four nonzero terms in the Fourier series that represents the steady-state voltage v_o . Which frequency component is eliminated from the output voltage and why? (40)



AP 17.8

$$v(t) = 4te^{-t}u(t); \quad V(\omega) = \frac{4}{(1 + j\omega)^2}$$

$$\text{Therefore } |V(\omega)| = \frac{4}{1 + \omega^2}$$

$$\begin{aligned} W_{1\Omega} &= \frac{1}{\pi} \int_0^{\sqrt{3}} \left[\frac{4}{(1 + \omega^2)} \right]^2 d\omega \\ &= \frac{16}{\pi} \left\{ \frac{1}{2} \left[\frac{\omega}{\omega^2 + 1} + \tan^{-1} \frac{\omega}{1} \right]_0^{\sqrt{3}} \right\} \\ &= 16 \left[\frac{\sqrt{3}}{8\pi} + \frac{1}{6} \right] = 3.769 \text{ J} \end{aligned}$$

$$W_{1\Omega}(\text{total}) = \frac{8}{\pi} \left[\frac{\omega}{\omega^2 + 1} + \tan^{-1} \frac{\omega}{1} \right]_0^{\infty} = \frac{8}{\pi} \left[0 + \frac{\pi}{2} \right] = 4 \text{ J}$$

$$\text{Therefore } \% = \frac{3.769}{4}(100) = 94.23\%$$

~~Q~~

$$2. \quad Z_{11} = \frac{((13119) + 8)115}{13.32115} = 3.64$$

$$Z_{22} = \frac{((13119) + 6)117}{11.32117} = 4.33$$

$$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0} = \frac{5.32 \times 5 \times 4.33}{5.32 + 6 \times 13} = 0.78$$

$$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0} = \frac{5.32 \times 7 \times 3.64}{5.32 + 8 \times 13} = 0.78$$

3. Both are sinusoids at 50 Hz. Both would exhibit aliasing when sampled at 50 Hz.

1. $f(t) = te^{-t} u(t)$

2. $\frac{1}{L(100 \times 10^{-9})} = 22206609.9 \Rightarrow L = 2.22 \text{ H}$

$$\beta = \frac{R}{L} \Rightarrow R = 2\pi(250)(2.22) = 3.5 \text{ k}\Omega$$

3. $H(s) = \frac{1000}{s+1000}$

$$H(j\omega) = \frac{1000}{j\omega+1000} \Rightarrow |H(j1000\pi)| = \frac{1000}{\sqrt{1000^2 + (1000\pi)^2}}$$

$$= 1000/1048 = 0.95$$

$$\theta(\omega) = -\tan^{-1}\left(\frac{1000\pi}{1000}\right) = -17.4^\circ$$

$$V_{o_s}(t) = (10)(0.95)\sin(1000\pi t - 17.4^\circ)$$

4. See next page

P 16.28 [a] For the circuit in Fig. P16.28

$$H(s) = \frac{s^2 + (1/LC)}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{1}{LC} = 25 \times 10^8; \quad \frac{1}{RC} = 5000$$

$$H(s) = \frac{s^2 + 25 \times 10^8}{s^2 + 5000s + 25 \times 10^8}$$

$$H(j\omega) = \frac{25 \times 10^8 - \omega^2}{(25 \times 10^8 - \omega^2) + j5000\omega}$$

$$H_1 = \frac{24 \times 10^8}{24 \times 10^8 + j5 \times 10^7} = 0.99978 / \underline{-1.19^\circ}$$

$$H_3 = \frac{16 \times 10^8}{16 \times 10^8 + j15 \times 10^7} = 0.99563 / \underline{-5.36^\circ}$$

$$H_5 = \frac{0}{j25 \times 10^7} = 0$$

$$H_7 = \frac{-24 \times 10^8}{-24 \times 10^8 + j35 \times 10^7} = 0.98953 / \underline{8.30^\circ}$$

From Assessment Problem 16.6

$$\mathbf{V}_{g1} = 840 / \underline{0^\circ} \text{ V}; \quad \mathbf{V}_{g3} = 280 / \underline{180^\circ} \text{ V}$$

$$\mathbf{V}_{g5} = 168/\underline{0^\circ} \text{ V}; \quad \mathbf{V}_{g7} = 120/\underline{180^\circ} \text{ V}$$

Thus,

$$\mathbf{V}_{o1} = 840/\underline{0^\circ} H_1 = 839.82/\underline{-1.19^\circ} \text{ V}$$

$$\mathbf{V}_{o3} = 280/\underline{180^\circ} H_3 = 278.78/\underline{174.64^\circ} \text{ V}$$

$$\mathbf{V}_{o5} = 168/\underline{0^\circ} H_5 = 0 \text{ V}$$

$$\mathbf{V}_{o7} = 120/\underline{180^\circ} H_7 = 118.74/\underline{-171.70^\circ} \text{ V}$$

$$\begin{aligned} v_o &= 839.82 \cos(10,000t - 1.19^\circ) + 278.78 \cos(30,000t + 174.64^\circ) \\ &= +0 + 118.74 \cos(70,000t - 171.70^\circ) + \dots \text{ V} \end{aligned}$$

- [b] The 5th harmonic, that is, the voltage having a frequency of 50 krad/s. The circuit is a passive bandreject filter with a center frequency of 50 krad/s.