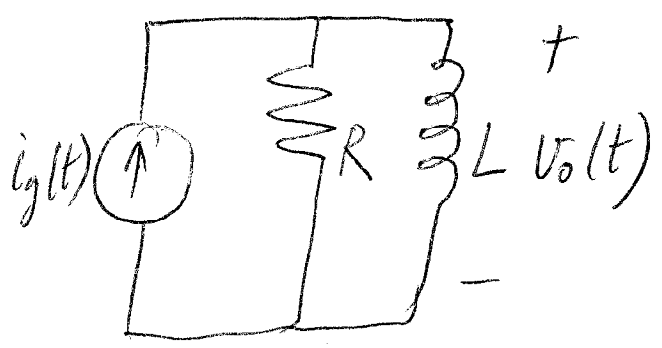
10/13/06 Networks and Systems - Midterm 1 (in class)

Duration: 1 hour

1. Find the inverse Laplace transform for $F(s) = \dfrac{1}{s^3+5s^2+8s+4}$. Use the fact that one of the roots of the denominator is at $s=-1$. (40)

2. A system has a transfer function given by $H(s) = \dfrac{1}{(s+1)^2}$. Find the final value of the output (if it exists) when (a) unit impulse input is applied, (b) unit step input is applied.

3.



Find the expression for the sinusoidal steady-state value for $v_o(t)$ when input applied is $i_g(t) = I\cos(\omega t)$

Networks and Systems Mid-term I solution

1. $$F(s) = \frac{1}{s^3+5s^2+8s+4} = \frac{1}{(s+1)(s+2)^2}$$

$$= \frac{A_1}{s+1} + \frac{A_{21}}{(s+2)^2} + \frac{A_{22}}{s+2}$$

$$A_1 = F(s)\cdot(s+1)\Big|_{s=-1} = \frac{1}{s^2+4s+4}\Big|_{s=-1} = 1$$

$$A_{21} = F(s)(s+2)^2\Big|_{s=-2} = \frac{1}{s+1}\Big|_{s=-2} = -1$$

$$A_{22} = \left[\frac{d}{ds}\left(F(s)(s+2)^2\right)\right]\Big|_{s=-2} = -\frac{1}{(s+1)^2}\Big|_{s=-2} = -1$$

$$f(t) = 1\cdot e^{-1\cdot t} - 1\cdot t\cdot e^{-2t} - 1\,e^{-2t}$$

2. $$H(s) = 1/(s+1)^2$$

(a) $$Y(s) = H(s)\cdot 1 = \frac{1}{(s+1)^2}$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} \frac{s\cdot 1}{(s+1)^2} = 0$$

(b) $$Y(s) = H(s)\cdot\frac{1}{s} = \frac{1}{s(s+1)^2}$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} \frac{1}{(s+1)^2} = 1$$

3. $H(s) = \dfrac{V_o(s)}{I_g(s)}$

$$V_o(s) = I_g(s)\, Z_{eq}(s) = I_g(s) \cdot \frac{R \cdot sL}{R + sL}$$

$$\Rightarrow \quad H(s) = \frac{sLR}{R + sL}$$

$$H(j\omega) = \frac{j\omega LR}{R + j\omega L} = \frac{\omega LR(\omega L + jR)}{R^2 + \omega^2 L^2}$$

$$|H(j\omega)| = \frac{\omega LR}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\theta(\omega) = \angle H(j\omega)$$

$$\cot\theta = \frac{\omega L}{R} \Rightarrow \tan(90^\circ - \theta) = \frac{\omega L}{R}$$

$$\Rightarrow \theta = 90^\circ - \tan^{-1}\frac{\omega L}{R}$$

or $\tan\theta = \dfrac{R}{\omega L} \Rightarrow \theta = \tan^{-1}\left(\dfrac{R}{\omega L}\right)$

$$V_{o_{ss}}(t) = \frac{I \cdot \omega LR}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t + 90^\circ - \tan^{-1}\frac{\omega L}{R}\right)$$