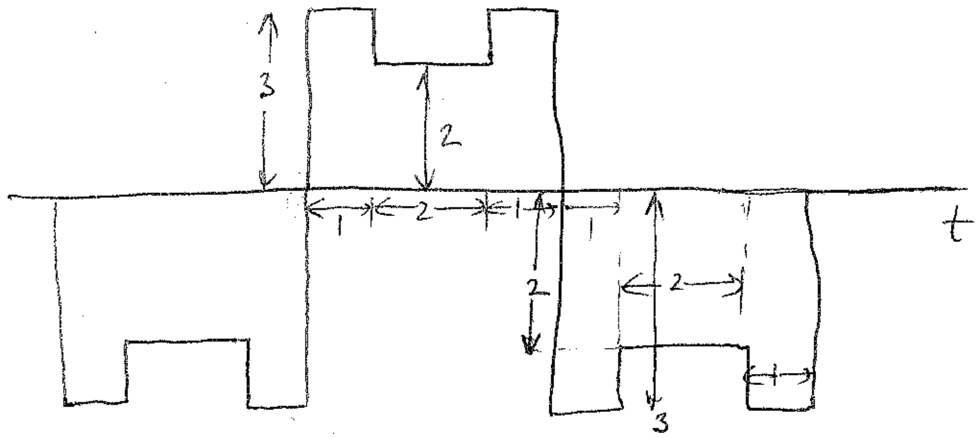
Duration: 1 hr Test 2 — Networks and Systems 11/13/06

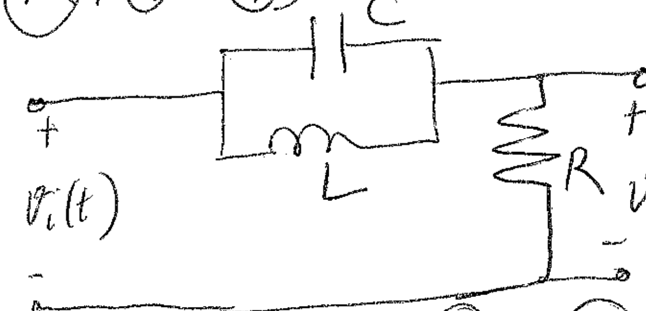
1.



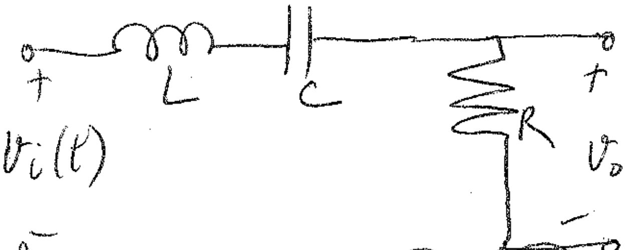
(10) {What kind of symmetries does the function above show? {Fix $t=0$ to make the function (a) odd, and (b) even} (10) For both cases, use symmetries to calculate a_v, a_k, and b_k with the least amount of computation.

(12) + (4) + (4)} × 2 (60)

2.



(a) (8) + (2)

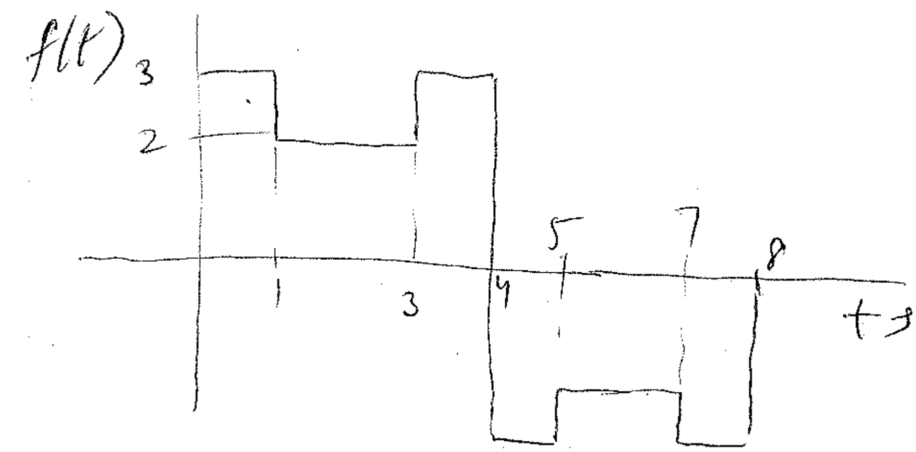


(8) + (12)

Classify the filters in (a) and (b) as low-pass, high-pass, band-pass, band-stop or none of the above. Give justification based on mathematics and/or circuit element characteristics based reasoning. (40)

Solutions, Test 2 (pg 1)

1. Odd Function:



$a_v = 0$ (odd function)

$a_k = 0$ (all k, ∵ odd function)

$b_k = 0$ (k even, HW symmetry)

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\cdot\underbrace{\frac{2\pi}{T}}_{\omega_0}\cdot t\,dt \quad, k \text{ odd})$$

$T = 8$

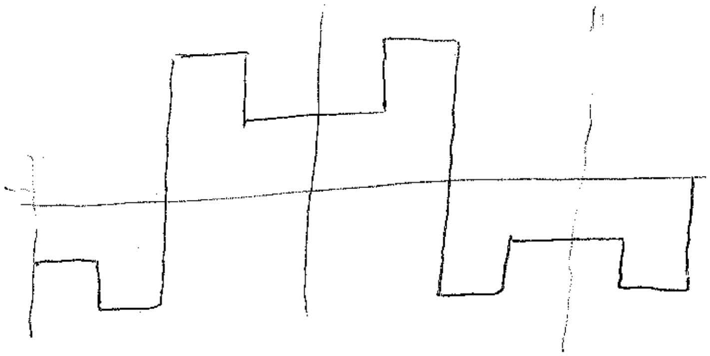
$$= 1\int_0^2 f(t)\sin\left(\frac{k\cdot\pi}{4}\cdot t\right)dt$$

$$= \frac{4}{k\pi}\left[3\cdot\left[\cos\left(\frac{k\pi}{4}\cdot 0\right) - \cos\left(\frac{k\pi}{4}\cdot 1\right)\right] + 2\left[\cos\left(\frac{k\pi}{4}\cdot 1\right) - \cos\left(\frac{k\pi 2}{4}\right)\right]\right]$$

$$= \frac{4}{k\pi}\left(3 - \cos\left(\frac{k\pi}{4}\right) - \cancel{2\cos\left(\frac{k\pi}{2}\right)}^{\,0}\right)$$

$$= \frac{4}{k\pi}\left(3 - \cos(k\pi/4)\right) \quad, k \text{ odd}$$

Even function: Solutions (p2)



$a_v = 0$, (HW)

$a_k = 0$, (k even) (HW)

$$a_k = \frac{8}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\,dt \qquad (k \text{ odd})$$

$$= \frac{4}{k\pi}\left(2\left[\sin\left(\frac{k\pi\cdot 1}{4}\right) - \cancel{\sin\left(\frac{k\pi\cdot 0}{4}\right)}\right] + 3\left[\sin\left(\frac{k\pi}{2}\right) - \sin\left(\frac{k\pi\cdot 1}{4}\right)\right]\right)$$

$$= \frac{4}{k\pi}\left(\cancel{2}\sin\left(\frac{k\pi}{4}\right) + 3\sin\left(\frac{k\pi}{2}\right)\right)$$

$b_k = 0$ for all k (even function)

2. (a) is bandstop filter. Equivalent impedance for LC combination is $Z_{eq} = \dfrac{j\omega L \cdot 1/j\omega C}{j(\omega L - 1/\omega C)}$

$$Z_{eq} = \frac{L/C \times \omega C}{j(\omega^2 LC - 1)} = \frac{-j\omega L}{(\omega^2 LC - 1)}$$

At low and high ω, Z_{eq} is small and input voltage drops across resistor, At $\omega = 1/\sqrt{LC}$, all voltage drops across LC combo and none across resistor.

Thus, there is output voltage at low and high frequencies and small output along the center frequency. Thus, this is a bandpass.

(5) is a bandpass. $Z_{eq} = j(\omega L - \frac{1}{\omega C})$
At low frequencies, input voltage drop across ~~inductor~~ capacitor, at high frequencies, input drops across inductor. And the center frequency, the inductive and capacitive impedances cancel each other out and most input voltage drops across the resistor.